$$\lim_{x \to a} f(x) = L$$

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The limit of $f(x)$, as x approaches a, is L

$f(x)$ gets "closer and closer" to L
as x gets "closer and closer" to a.

How close can we get $f(x)$ to L?

$$\lim_{x \to a} f(x) = L$$

$f(x)$ can be made arbitrarily close to L by making x sufficiently close to a.

$$\lim_{x \to a} f(x) = L$$

Let ϵ be an arbitrary positive number. We can get $f(x)$ within a distance of ϵ of L just be taking x sufficiently close to a.

In other words, it doesn't matter how small a distance ϵ that you take, we can always get x close enough to a so that:

$$|f(x) - L| < \epsilon$$

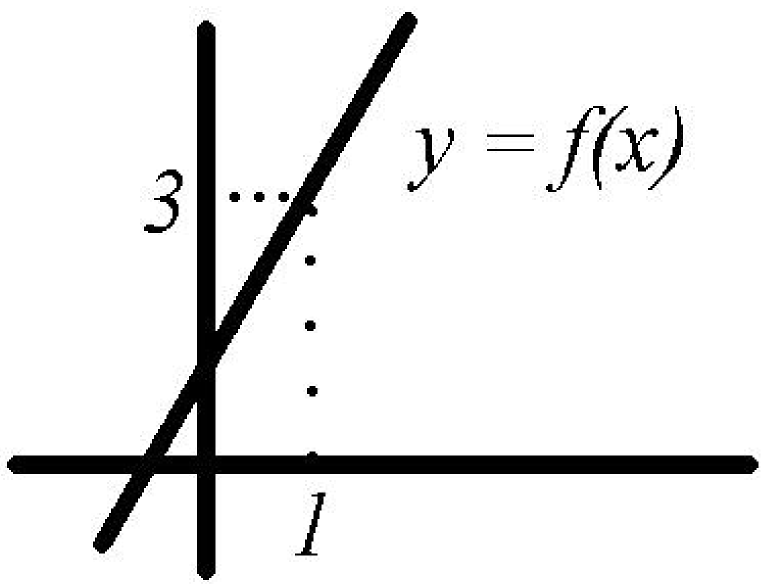
How do we calculate limits?

$$\lim_{x \to 1} (2x + 1)$$

$$\lim_{x\to 1}(2x+1)$$

Just substitute $x = 1$ into $f(x) = 2x + 1$

$$\lim_{x\to 1} f(x) = f(1) = (2)(1) + 1 = 3$$



$$g(x) = \frac{2x^2 - x - 1}{x - 1}$$

Find $\lim_{x \to 1} g(x)$

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$$\text{Find } \lim_{x \to 1} g(x)$$

We can't get the answer by substituting $x = 1$ into $g(x)$ because $g(1)$ is not defined.

$$g(x) = \frac{2x^2 - x - 1}{x - 1}$$

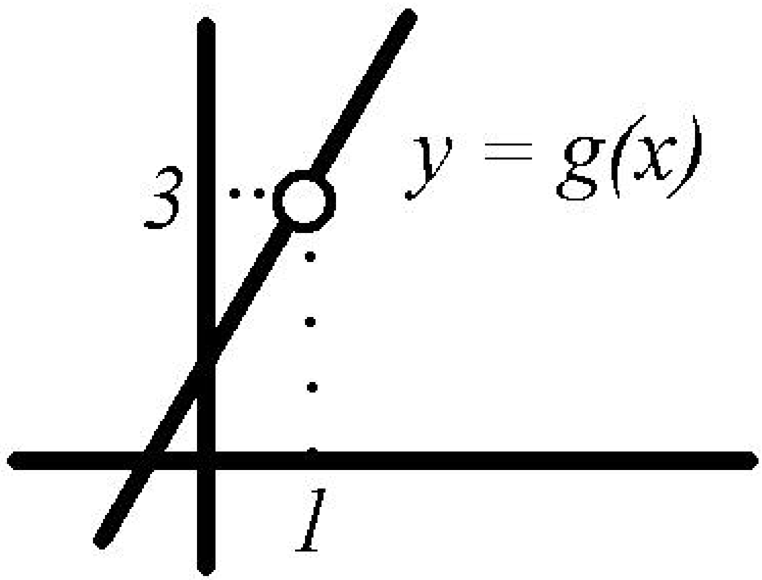
Find $\lim_{x \to 1} g(x)$

$$\lim_{x \to 1} \frac{2x^2 - x - 1}{x - 1} = \lim_{x \to 1} \frac{(2x + 1)(x - 1)}{x - 1}$$

Note that this is not quite the same as $f(x) = 2x + 1$

$$g(x) = \frac{2x^2 - x - 1}{x - 1}$$

$g(x)$ is *discontinuous* at $x = 1$



$$g(x) = \frac{2x^2 - x - 1}{x - 1}$$

Find $\lim_{x \to 1} g(x)$

$$\begin{aligned} \lim_{x \to 1} \frac{2x^2 - x - 1}{x - 1} &= \lim_{x \to 1} \frac{(2x+1)(x-1)}{x-1} \\ &= \lim_{x \to 1} (2x+1) \\ &= 3 \end{aligned}$$

Note: A function $f(x)$ is said to be *continuous* at a point $x = a$ if f is defined at $x = a$ and:

$$\lim_{x \to a} f(x) = f(a)$$