Interval Notation

The notation $[a,\ b]$ refers to the set of all numbers x such that $a \leq x \leq b$. This is referred to as the *closed interval* between a and b

A function f is said to have a *maximum value* on an interval $[a,\ b]$ if there is a number c in $[a,\ b]$ with the property that $f(c) \geq f(x)$ for all x in $[a,\ b]$

A function f is said to have a *minimum value* on an interval $[a,\ b]$ if there is a number c in $[a,\ b]$ with the property that $f(c) \leq f(x)$ for all x in $[a,\ b]$

The Extreme Value Theorem

If f is continuous on the interval $[a,\ b]$ then it will have a maximum value on this interval. It will also have a minimum value on this interval.

If the derivative of a function exists at a maximum point then the derivative will be zero. The same will also be true at a minimum point.

Critical Points

A point is a *critical point* if either one of the following statements are true:

1. The derivative does not exist at that point.

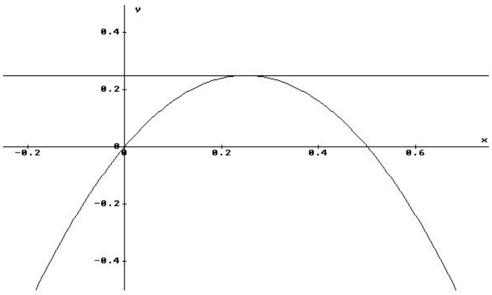
2. The derivative equals 0 at that point

Maximum points and minimum points are critical points.

Rolle's Theorem

Suppose f is a function that is continuous in $[a,\ b]$ and differentiable in $(a,\ b)$. Then if $f(a) = f(b) = 0$, there must be a point c in $[a,\ b]$ with the property that:

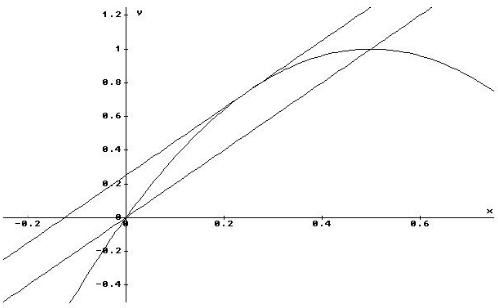
$$f'(c) = 0$$



Mean Value Theorem

Suppose f is a function that is continuous in $[a,\ b]$ and differentiable in $(a,\ b)$. Then there is a point c in $[a,\ b]$ with the property that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



A motorist drives onto the Florida Turnpike and is given a card at the entrance ramp showing the exit number and the time that of entry. An hour later, the motorist exits the Turnpike. The toll taker examines the card in order to determine how much of a toll to collect. The toll taker sees from the data stamped on the card that the motorist had traveled 80 miles in one hour. The toll taker calls over a state trooper who also examines the data and then arrests the motorist for speeding. When the motorist comes before the judge, he pleads innocent and points out that nobody had actually seen him speeding. How can the prosecutor use the Mean Value Theorem to convince the judge that the driver had to have been going 80 miles per hour at some point during the trip?

If the derivative of a function is positive for all values of x in an interval then the function is *increasing* throughout the interval.

If the derivative of a function is positive for all values of x in an interval then the function is *increasing* throughout the interval.

Proof: If the function decreased anywhere in the interval, then there would be points $x = a$ and $x = b$ where $b > a$ but $f(a) > f(b)$. This means that $\frac{f(b)-f(a)}{b-a}$ is a negative number. The Mean Value Theorem says that there is a point c between a an b such that $f'(c) = \frac{f(b)-f(a)}{b-a}$ and this would mean that f' is negative at $x = c$. This would contradict the premise that the derivative of the function is positive at all points in the interval.

If the derivative of a function is negative for all values of x in an interval then the function is *decreasing* throughout the interval.

The First Derivative Test

Suppose $f'(a) = 0$. If $f'(x) > 0$ in the region immediately to the left of $x = a$ and $f'(x) < 0$ in the region immediately to the right of $x = a$ then $f(x)$ must have a *maximum value* at $x = a$.

The First Derivative Test

Suppose $f'(a) = 0$. If $f'(x) > 0$ in the region immediately to the left of $x = a$ and $f'(x) < 0$ in the region immediately to the right of $x = a$ then $f(x)$ must have a *maximum value* at $x = a$.

Suppose $f'(a) = 0$. If $f'(x) < 0$ in the region immediately to the left of $x = a$ and $f'(x) > 0$ in the region immediately to the right of $x = a$ then $f(x)$ must have a *minimum value* at $x = a$.

The Second Derivative Test

If $f'(a) = 0$ and $f''(a) < 0$ then $x = a$ *maximizes* $f(x)$

If $f'(a) = 0$ and $f''(a) > 0$ then $x = a$ *minimizes* $f(x)$

What does it mean if $f'(a) = 0$ and $f''(a) = 0$?