Practice Problems - Solutions

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1. Calculate the following limit:

$$\lim_{h\to 0} \frac{2^{-1} - (2-h)^{-1}}{h}$$

a) $-\frac{1}{4}$ **b)** $\frac{1}{4}$ **c)** $-\frac{1}{2}$ **d)** 1 **e)** 0

2. Calculate the derivative of $\ln(4\sqrt{x})$ at the point $x = 1$

a) 0 **b)** $\frac{1}{2}$ **c)** 1 **d)** 2 **e)** 4

3. If $y = x^2 e^{-x}$, calculate $\frac{dy}{dx}$ at $x = 2$.

a) 0 **b)** $\frac{4}{e^2}$ **c)** $\frac{8}{e^2}$ **d)** $\frac{-4}{e^2}$ **e)** $\frac{-1}{e^2}$

4. Calculate the following integral:

$$\int_0^1 \frac{x}{\sqrt{1+x^2}}\, dx$$

a) $\frac{1}{2}\ln 2$ **b)** $\sqrt{2}$ **c)** $\frac{1}{\sqrt{2}}$ **d)** $\sqrt{2}-1$ **e)** $1 - \frac{1}{\sqrt{2}}$

5. If the perimeter of a square is growing at the rate of 4 meters per minute, find the rate that the area is changing at the point in time when the perimeter is 8 meters?

Note: The units of these choices will be square meters per minute

a) 2 **b)** 4 **c)** 8 **d)** 16 **e)** 32

6. Simplify the following sum:

$$\sum_{i=1}^{8} \left(i^2 - (i-1)^2\right)$$

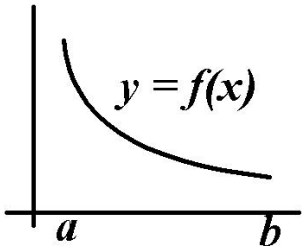
a) 7 **b)** 8 **c)** 15 **d)** 32 **e)** 64

7. Calculate the following area function:

$$F(x) = \int_0^x \frac{t}{t+1}\, dt$$

a) $\frac{x}{x+1}$ **b)** $\ln(x+1)$ **c)** $1 + \ln(x+1)$ **d)** $\frac{2x}{x+2}$ **e)** $x - \ln(x+1)$

8. The following is a graph of a function $y = f(x)$ on an interval from a to b

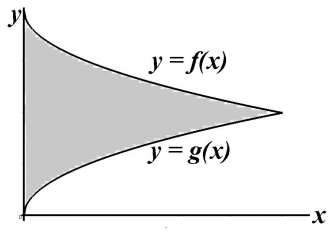


Which of the following would you expect to be true for all x between a and b ?

a) $f'(x) = 0$ and $f''(x) > 0$ **b)** $f'(x) < 0$ and $f''(x) > 0$ **c)** $f'(x) > 0$ and $f''(x) > 0$

d) $f'(x) < 0$ and $f''(x) < 0$ **e)** $f'(x) > 0$ and $f''(x) < 0$

9. Let $f(x) = 2 - \sqrt{x}$ and $g(x) = \sqrt{x}$. Find the area of the region bounded by $y = f(x)$, $y = g(x)$ and the y-axis.



a) $\frac{1}{2}$ **b)** $\frac{2}{3}$ **c)** $\frac{3}{4}$ **d)** $\frac{4}{3}$ **e)** 2

10. Calculate the derivative of $\frac{2x+1}{x+1}$

a) 2 **b)** $\frac{1}{(x+1)^2}$ **c)** $\frac{1}{x+1}$ **d)** $\frac{-1}{x+1}$ **e)** $\frac{4x+3}{(x+1)^2}$

11. At what value of x will $y = (x-1)e^x$ have a point of inflection?

a) -1 **b)** 0 **c)** 1 **d)** 2 **e)** This function has no points of inflection

12. Which of the following limits of Riemann sums represents $\int_2^3 x^2\,dx$?

a) $$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^2 \frac{1}{n}$$

b) $$\lim_{n\to\infty} \sum_{i=1}^{n} \left(2+\frac{i}{n}\right)^2 \frac{1}{n}$$

c) $$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i}{n}\right)^2 \frac{1}{n}$$

d) $$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^2 \frac{2}{n}$$

e) $$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{3}\left(\frac{i}{n}\right)^3 \frac{1}{n}$$

13. Find the area of the region bounded between the parabolas $y = x - x^2$ and $y = 7x - 7x^2$.

a) 1 **b)** 2 **c)** 4 **d)** 6 **e)** 7

14. Which one of the following functions is discontinuous at some point?

a) $f(x) = \frac{x}{x^2+1}$ **b)** $f(x) = (x-1)^{1/3}$ **c)** $f(x) = \frac{x^2+1}{(x-1)^2}$
d) $f(x) = |x-1|$ **e)** None of these

15. Calculate the following limit. You may assume that x is some positive number.

$$\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$$

a) 0 **b)** $\frac{2}{3}x^{3/2}+C$ **c)** $\frac{1}{\sqrt{x}}$ **d)** $\frac{1}{2\sqrt{x}}$ **e)** undefined

16. Calculate the following integral:

$$\int_0^3 \left(\frac{x}{3}+1\right)^2 dx$$

a) $\frac{1}{3}$ **b)** 3 **c)** 7 **d)** 8 **e)** 9

17. The area of a square can be expressed as a function of the length of its diagonal by the formula $A = \frac{1}{2}x^2$. Suppose the diagonal length is increased from $x = 5$ cm to 5.2 cm. Then the value of the differential dA is given by:

a) $\frac{1}{5}$ cm^2 **b)** $\frac{1}{2}$ cm^2 **c)** 1 cm^2 **d)** $\frac{51}{50}$ cm^2 **e)** 2 cm^2

18. Suppose the acceleration of an object is given by $a = \cos(\pi t)$. If the velocity is 0 at $t = 0$, find the formula for velocity at time t.

a) $\sin(\pi t)$ **b)** $\pi \sin(\pi t)$ **c)** $-\pi \sin(\pi t)$ **d)** $-\frac{1}{\pi}\sin(\pi t)$ **e)** $\frac{1}{\pi}\sin(\pi t)$

19. Which of the following gives the derivative of $f(x) = \frac{1}{x}$?

a) $\dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$

b) $\lim\limits_{h\to 0} \dfrac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$

c) $\lim\limits_{h\to 0} \dfrac{1}{1+h}$

d) $\lim\limits_{h\to 0} \dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$

e) None of the above

20. Find the derivative of the following function:

$$f(x) = \ln\left(\frac{x}{x+1}\right)$$

a) $\frac{1}{x(x+1)}$ **b)** $\frac{4x}{(x+1)^2}$ **c)** $\frac{x+1}{2x}$ **d)** $\ln\left(\frac{2}{(x+1)^2}\right)$ **e)** $\frac{1}{2x} - \frac{1}{x+1}$

21. Find the derivative of $y = e^{2\sqrt{x}}$

a) $\frac{e^{2\sqrt{x}}}{\sqrt{x}}$ **b)** $e^{1/\sqrt{x}}$ **c)** $(2\sqrt{x})\, e^{2\sqrt{x}-1}$ **d)** $(\sqrt{x})\, e^{2\sqrt{x}}$ **e)** $e^{2\sqrt{x}}$

22. If we make the substitution $u = 1 + \sin\frac{x}{2}$ into the integral $\int_0^{\pi} \frac{\cos\frac{x}{2}}{1+\sin\frac{x}{2}}\, dx$, which of the following integrals will result?

a) $\frac{1}{2}\int_1^2 \frac{du}{u}$ **b)** $\frac{1}{2}\int_0^{\pi} \frac{du}{u}$ **c)** $2\int_1^2 \frac{du}{u}$ **d)** $2\int_0^{\pi} \frac{du}{u}$ **e)** $\int_0^{\pi} \frac{du}{u}$

23. Calculate the integral:

$$\int \frac{e^x}{(e^x+1)^2}\, dx$$

a) $\frac{-1}{e^x+1} + C$ **b)** $2\ln(e^x+1) + C$ **c)** $\frac{-e^x}{e^x+1}$
d) $\frac{3e^x}{(e^x+1)^3}$ **e)** $\frac{1}{3(e^x+1)^3}$

24. Calculate the derivative:

$$y = \tan^2 x$$

a) $\sec^4 x$ **b)** $2\sec^4 x$ **c)** $2\tan^2 x \sec x$ **d)** $2\tan x \sec^2 x$ **e)** $\tan^2 x \sec^2 x$

25. The equation of a line tangent to a curve at a point is sometimes referred to as the *linear approximation* of the function. Find the linear approximation of $f(x) = \ln\left(\frac{1}{2}(e^x+1)\right)$ at $x = 0$.

a) $y = x$ **b)** $y = \frac{1}{2}x$ **c)** $y = 2x$ **d)** $y = 2x + 1$ **e)** $y = \frac{1}{2}x + 1$