A conveyor belt is dropping sand in a conical pile.

$$V = \frac{1}{3}\pi x^2 h$$

Let us suppose that the height is always twice as big as the base radius

$$h = 2x$$

$$V = \frac{1}{3}\pi x^2 h = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 y = \frac{\pi}{12}h^3$$

Suppose the volume of sand is increasing at 8 cubic meters per minute. How fast is the height of the sandpile increasing at the point in time when the height is 4 meters.

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$$\frac{dV}{dt} = 8 \qquad \text{Find } \frac{dh}{dt}$$

$$V = \frac{\pi}{12} h^3$$

$$\frac{dV}{dt} = \frac{d}{dt} \left(\frac{\pi}{12} h^3 \right)$$

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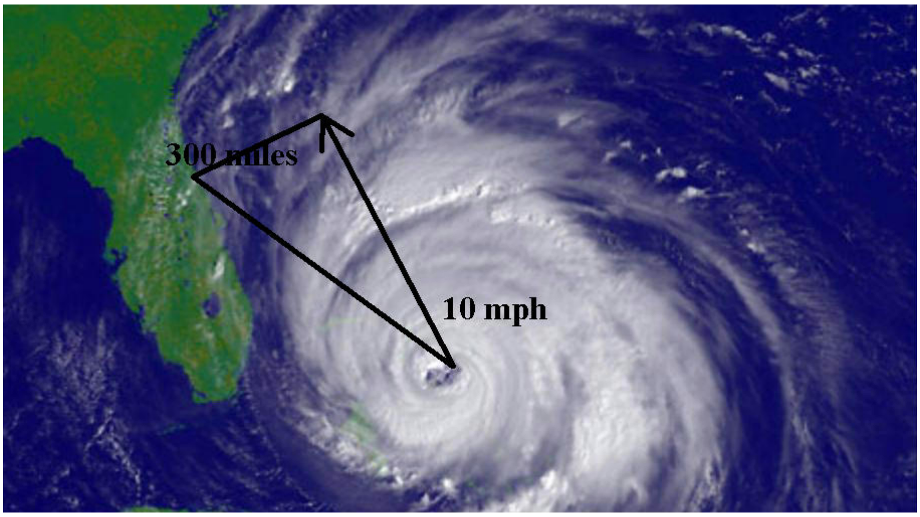
$$\frac{dh}{dt} = \frac{4}{\pi h^2} \frac{dV}{dt}$$

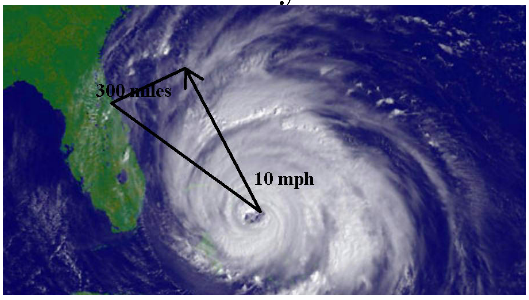
If $\frac{dV}{dt} = 8\ \mathrm{m}^3/\mathrm{min}$ and $h = 4$ meters then,

$$\frac{dh}{dt} = \frac{4}{\pi h^2}\frac{dV}{dt} = \frac{4}{\pi \cdot 16\ \mathrm{m}^2} \cdot 8\frac{\mathrm{m}^3}{\mathrm{min}} = \frac{2}{\pi}\frac{\mathrm{m}}{\mathrm{min}}$$

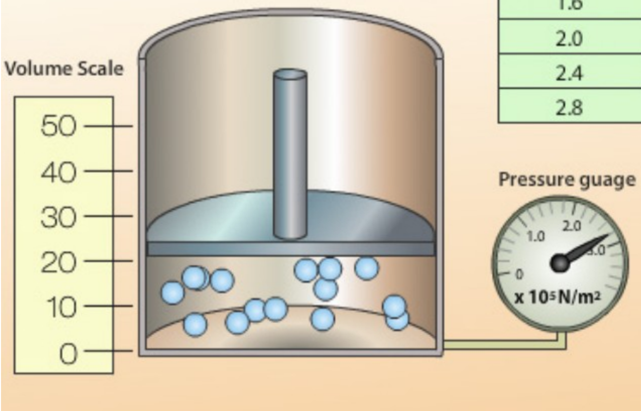

300 miles
10 mph

Suppose that the closest point of approach to Daytona Beach is 300 miles and that the storm is moving along the coast at 10 miles per hour.

At the point in time when the storm is 500 miles from Daytona Beach, calculate the rate at which the distance between the storm and Daytona Beach is changing.



Pressure (p)	Volume (V)	p x V
1.2	50	60
1.6	37.5	60
2.0	30	60
2.4	25	60
2.8	21.4	60



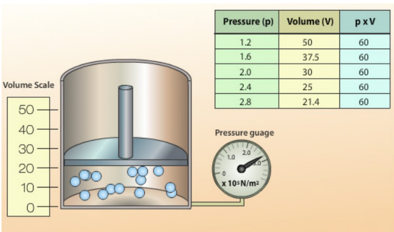
Let P be the pressure (in atmospheres)
Let V be the volume (in liters)

$$PV = nRT$$

If n, R and T are constant then

$$PV = \textit{(constant)}$$

Let us suppose that the constant is 4 liter-atmospheres



$$PV = 4$$

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If the piston is being compressed and the pressure is increasing at $\frac{1}{8}$ atmospheres per minute, find the rate at which the volume is changing at the point in time when $P = 1$ atmosphere and $V = 4$ liters.

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$$\frac{dP}{dt} = \frac{1}{8} \qquad \text{Find} \quad \frac{dV}{dt}$$

$$PV = 4$$

$$\frac{d}{dt}(PV) = \frac{d}{dt}(4)$$

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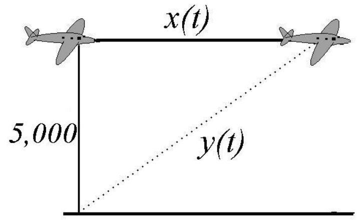
At $V = 4$ liters and $P = 1$ atmosphere, $\frac{dV}{dt}$ will be:

$$\frac{dV}{dt} = -\frac{4 \text{ liters}}{1 \text{ atm}} \cdot \frac{1}{8}\frac{\text{atm}}{\text{min}} = -\frac{1}{2}\frac{\text{liters}}{\text{min}}$$

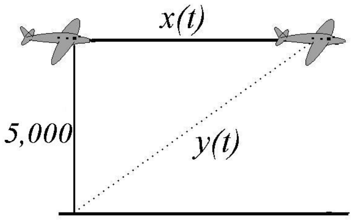
An observer is tracking a plane flying at 5,000 feet. The plane flies directly over the observer on a horizontal path at a fixed rate of speed of 520 feet per second. As time goes by, the distance from the plane to the observer changes.

What is the rate of change in the distance between the observer and the plane at the point in time when the plane has traveled 12,000 feet from the point above the observer.

Let $x(t)$ be the horizontal distance traveled by the plane.
Let $y(t)$ be the distance from the observer to the plane.
Given $\frac{dx}{dt} = 520$ feet/sec. Find $\frac{dy}{dt}$



$$(5,000)^2 + x^2 = y^2$$



$$(5,000)^2 + x^2 = y^2$$

$$\frac{d}{dt}\left((5,000)^2 + x^2\right) = \frac{d}{dt}\left(y^2\right)$$

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$$0 + 2x\frac{dx}{dt} = 2y\frac{dy}{dt}$$

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$$\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt}$$

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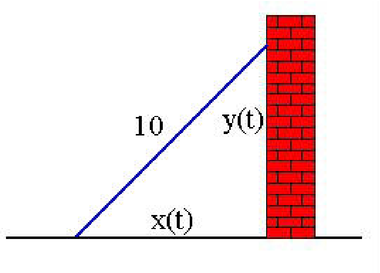
$$\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt}$$

We are given that $\frac{dx}{dt} = 520$. Find $\frac{dy}{dt}$ when $x = 12,000$.

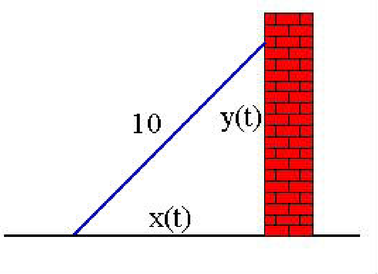
$$\frac{dy}{dt} = \frac{12,000}{13,000}\frac{\text{feet}}{\text{feet}} \cdot 520\frac{\text{ft}}{\text{sec}} = 480\frac{\text{ft}}{\text{sec}}$$

A ladder that is 10 feet long is leaning against a wall. The top of the ladder is slipping down the wall and the base of the ladder is sliding away from the wall at $\frac{1}{3}$ ft/min. Find how fast the top of the ladder is slipping down the wall at the point in time when the bottom of the ladder is 6 feet away from the wall.



Given that $\frac{dx}{dt} = \frac{1}{3}$, find $\frac{dy}{dt}$

$$x^2 + y^2 = 10^2$$



$$x^2 + y^2 = 10^2$$

$$\frac{d}{dt}\left(x^2 + y^2\right) = \frac{d}{dt}\left(10^2\right)$$

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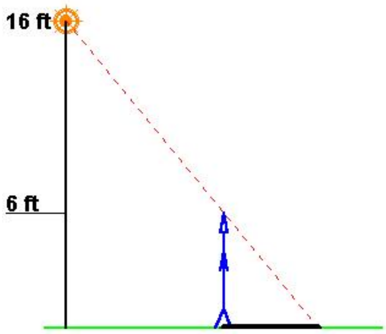
$$x^2 + y^2 = 10^2$$

$$\frac{d}{dt}\left(x^2 + y^2\right) = \frac{d}{dt}\left(10^2\right)$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

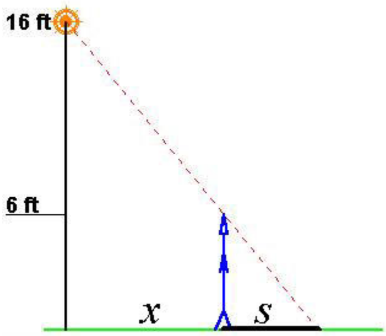
$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt} = -\frac{6}{8}\cdot\frac{1}{3} = -\frac{1}{4}\ \frac{\mathrm{ft}}{\mathrm{min}}$$

A man 6 ft. tall is directly under a light which is 16 feet high. He walks to the right at a steady speed of 2 feet per second. As time goes by the length of his shadow increases. How fast is the length of the shadow increasing?

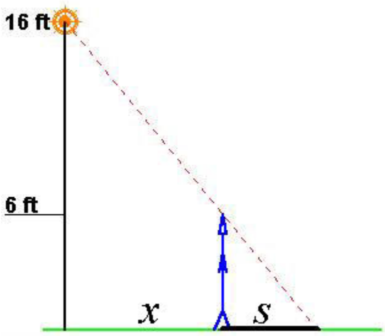


Let x be the distance from the base of the light to the man. We are given that $\frac{dx}{dt} = 2$.

Let s be the length of the shadow. Find $\frac{ds}{dt}$



Use similar triangles



$$\frac{16}{x+s} = \frac{6}{s}$$

$$\frac{16}{x+s} = \frac{6}{s}$$

$$16s = 6(x+s)$$

$$16s = 6x + 6s$$

$$\frac{16}{x+s} = \frac{6}{s}$$

$$16s = 6(x+s)$$

$$16s = 6x + 6s$$

$$10s = 6x$$

$$s = \frac{6}{10}x = \frac{3}{5}x$$

$$s = \frac{3}{5}x$$

$$\frac{ds}{dt} = \frac{3}{5}\frac{dx}{dt} = \frac{3}{5} \cdot 2\frac{\text{ft}}{\text{sec}} = 1.2\ \frac{\text{ft}}{\text{sec}}$$

A man in a hot-air balloon is ascending at the rate of .002 miles per second. How fast is the distance from the balloon to the horizon increasing when the balloon is $\frac{1}{5}$ of a mile high.

Assume that the earth is a ball of radius 4,000 miles.

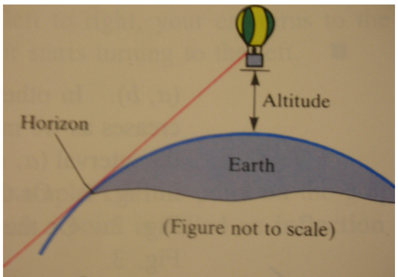
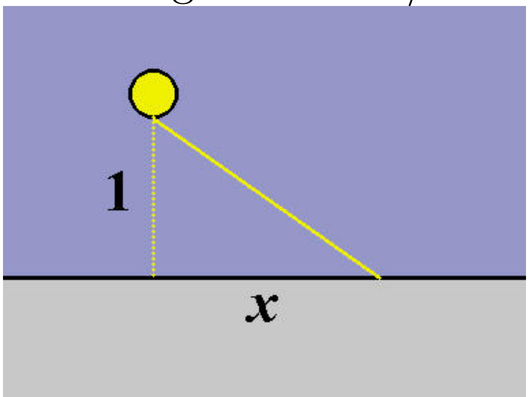


Figure 8

The lighthouse is 1 mile offshore. It is revolving at ω radians per minute. The beam of light from the lighthouse is moving along the shoreline as the light rotates. When $x = \frac{1}{2}$, the light is moving at 5 miles/minute. Calculate ω



A small ferris wheel has a radius of 10 meters and rotates at the rate of 1 revolution every 2 minutes. Let $(x,\ y)$ be the coordinates of a point on the wheel. Find $\frac{dy}{dt}$ at the point in time when the point is 16 meters above the ground.