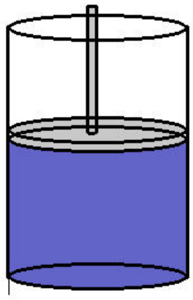
Let V denote the volume in a piston
Let P denote the pressure of the gas inside the piston



The value of P depends on the value of V
P is a **function** of V

$$P = f(V)$$

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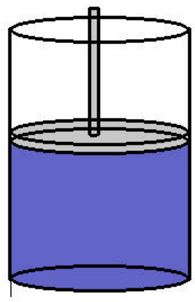


The value of P depends on the value of V
P is a **function** of V

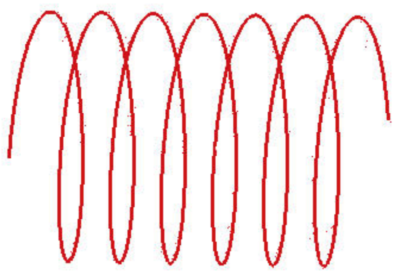
$$P = f(V)$$

$$P = \frac{nRT}{V}$$

Let x = the distance (in meters) a spring is stretched.

Let E = the energy it takes to stretch the spring x meters

$$E = f(x)$$



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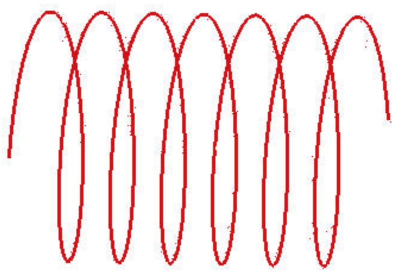
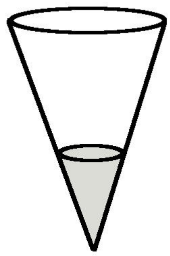


$$E = \frac{1}{2}kx^2$$

Water is being poured into a conical container.



Let h = the height of the water level

Let M = the mass of water inside this container

$$M = f(h)$$

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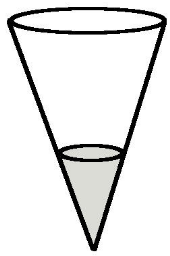


Let h = the height of the water level

Let M = the mass of water inside this container

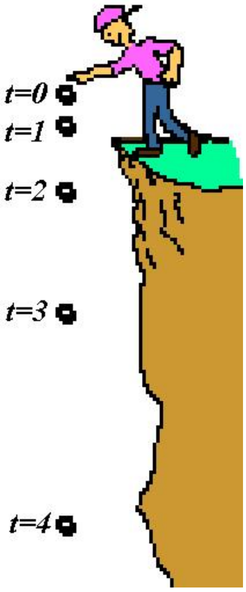
$$M = f(h)$$

$$M = \frac{1}{3}\pi\rho\frac{a^2}{b^2}h^3$$

A ball is dropped from cliff.

Let t be the elapsed time (in seconds) that the ball is falling.

Let s be the distance (in meters) that the ball has fallen after t seconds.

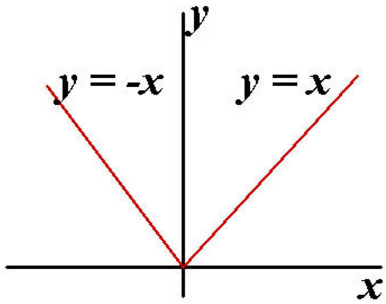


Time (in sec) t	Distance (in feet) $f(t)$
0	0
1	16
2	64
3	144
4	256

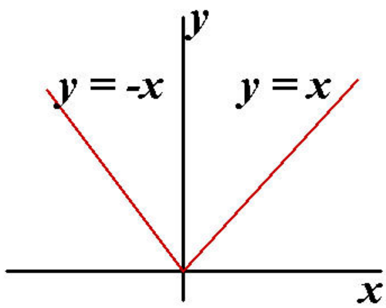
Time (in sec) t	Distance (in feet) $f(t) = 16t^2$
0	0
1	16
2	64
3	144
4	256

Some functions require more than one equation.

$$f(x) = \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases}$$



$$f(x) = |x|$$



An equation defining a function can be regarded as an *instruction* for converting an *input* to an *output*

$$y = f(x) = x^2 + 2x$$

$$output = f(input) = (input)^2 + 2(input)$$

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Examples:

$$f(a) = a^2 + 2b$$

$$f(h) = h^2 + 2h$$

$$f(a+h) = (a+h)^2 + 2(a+h)$$

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Examples:

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$$f(a+h) = (a+h)^2 + 2(a+h)$$

Please note that $f(a) + f(h)$ is not the same as $f(a+h)$

$$f(a) + f(h) = a^2 + 2a + h^2 + 2h$$

$$f(a+h) = (a+h)^2 + 2(a+h) = a^2 + 2ah + h^2 + 2a + 2h$$

We will be particularly interested in expressions of the form:

$$\frac{f(x+h)-f(x)}{h}$$

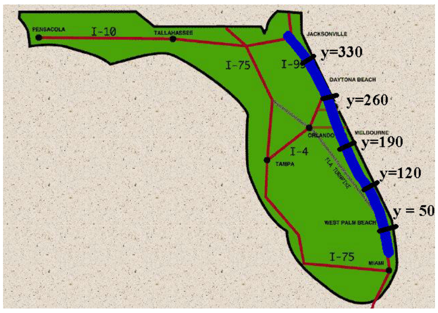
Example:

$$f(x) = x^2 + 2x$$

$$\begin{aligned} f(x+h) - f(x) &= \big((x+h)^2 + 2(x+h)\big) - \big(x^2 + 2x\big) \\ &= \big(x^2 + 2xh + h^2 + 2x + 2h\big) - \big(x^2 + 2x\big) \\ &= 2xh + h^2 + 2h \end{aligned}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{2xh + h^2 + 2h}{h} = 2x + h + 2$$

The car begins at Mile Marker 50.



Let t denote the time of travel (in hours)
Let y denote the mile marker that the car is up to

$$\text{Distance} = (\text{Rate})(\text{Time})$$

$$y - 50 = 70t$$

$$y = 70t + 50$$

Let x be time.
Let $f(x)$ be the mile marker that the car is up to.

$$f(x) = 70x + 50$$

$$f(x+h) = 70(x+h) + 50$$

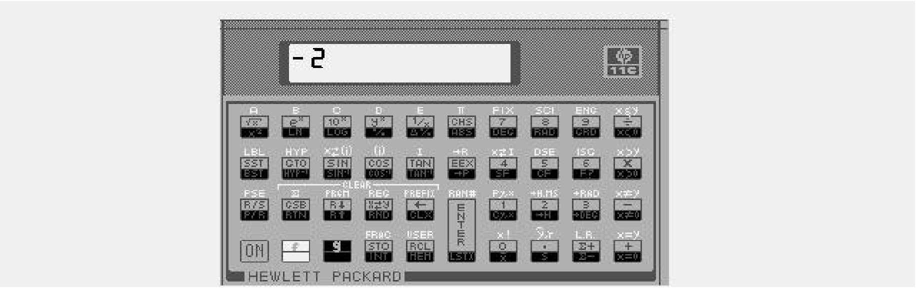
$$\frac{f(x+h)-f(x)}{h} = \frac{(70(x+h)+50)-(70x+50)}{h} = \frac{70h}{h} = 70$$

The set of all allowable *inputs* into a function is its *domain*
The set of all possible *outputs* from a function is its *range*

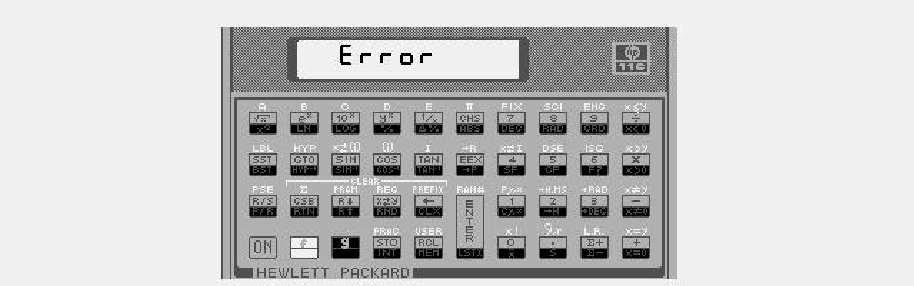
$$f(x) = \sqrt{x}$$

What is the domain of this function?

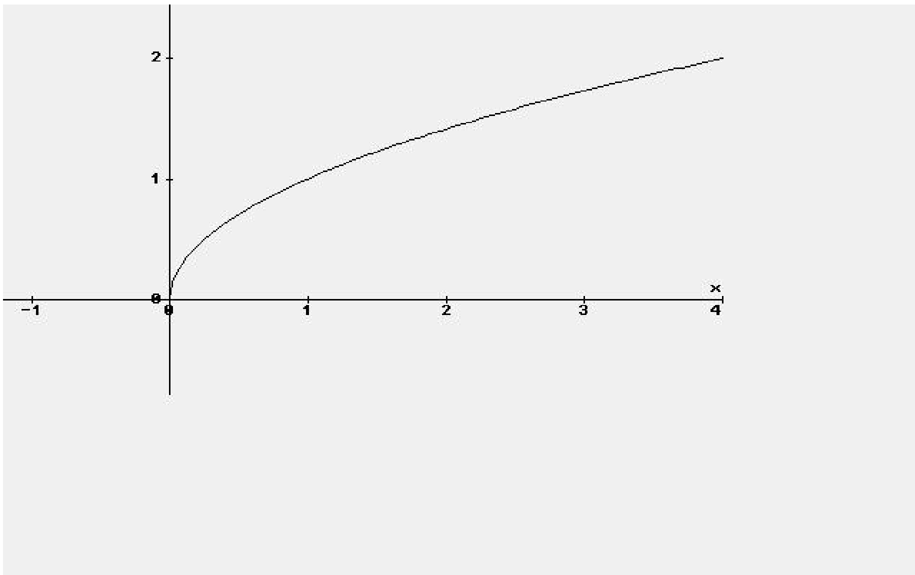
Suppose we enter the number -2 on a calculator and press the square root button?



A negative number is not an allowable input.



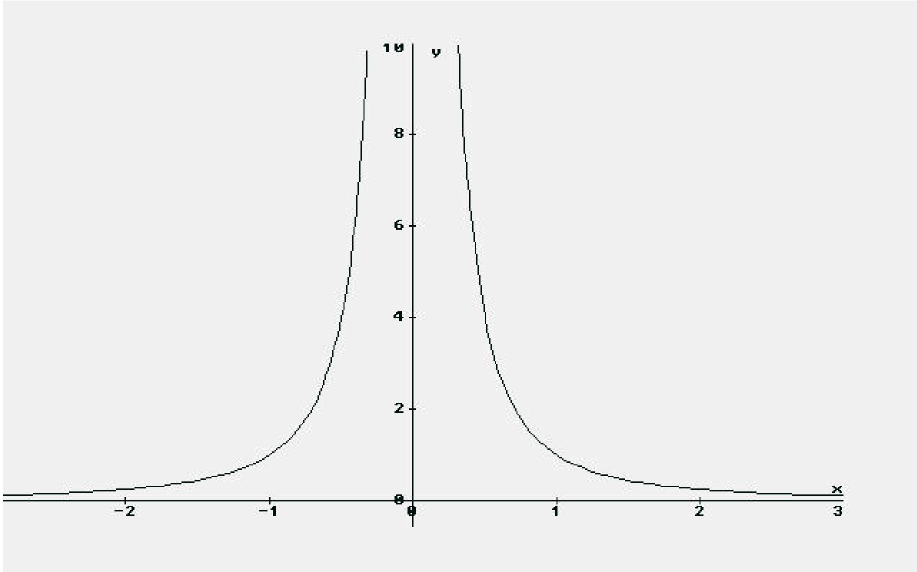
$$\text{Domain} = \{x \mid x \geq 0\}$$



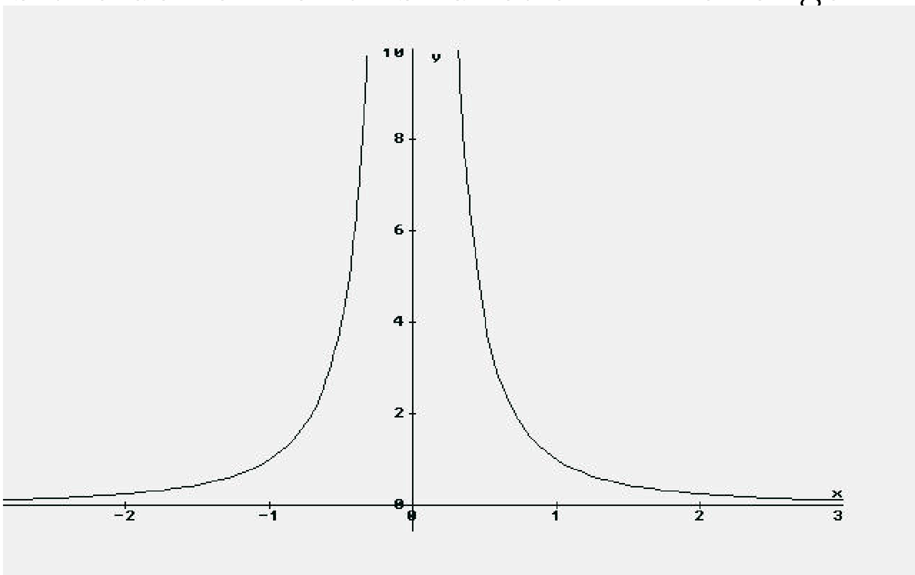
2
1
0
x
−1
0
1
2
3
4

$$y = \frac{1}{x^2}$$

$$f(x) = x^{-2}$$

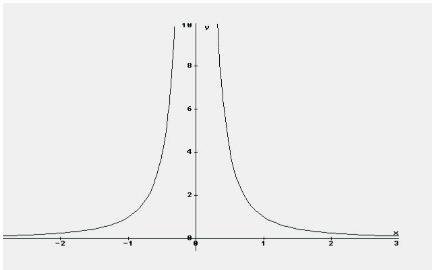


What is the domain of this function? The range?

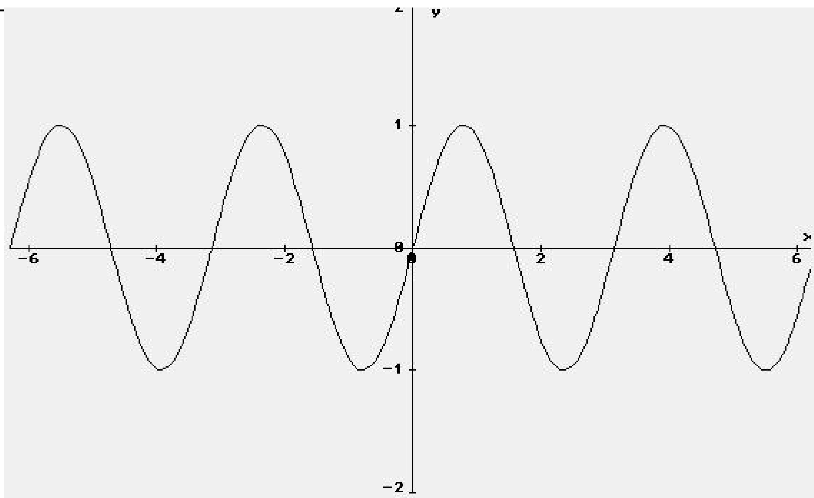


$$\text{Domain} = \{x \,|\, x \neq 0\}$$

$$\text{Range} = \{y \,|\, y > 0\}$$



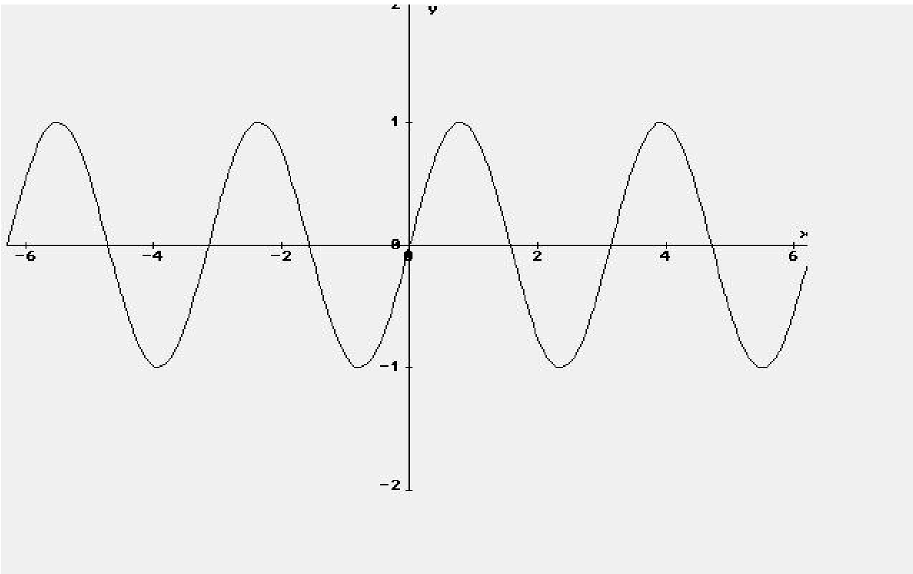
$$y = f(x) = \sin 2x$$



domain? range?

$$
\begin{aligned}
\text{Domain} &= \text{The set of all real numbers} \\
&= \{x \,\big|\, -\infty < x < \infty\} \\
&= \mathbb{R}
\end{aligned}
$$

$$\text{Range} = \{y \,\big|\, -1 \le y \le 1\}$$



Interval Notation

$$[a,\, b] = \{x \,\big|\, a \le x \le b\}$$ Closed Interval

$$(a,\, b) = \{x \,\big|\, a < x < b\}$$ Open Interval

$$(a,\, b] = \{x \,\big|\, a < x \le b\}$$

$$[a,\, b) = \{x \,\big|\, a \le x < b\}$$

$$y = f(x) = \sin 2x$$

$$\text{Domain} = I\!R = \{x \,\big|\, -\infty < x < \infty\} = (-\infty,\ \infty)$$

$$\text{Range} = \{y \,\big|\, -1 \le y \le 1\} = [-1,\ 1]$$

$$f(x) = 1 + \frac{1}{(x-2)^2}$$

What is the domain of this function?
What is the range of this function?

$$f(x) = 1 + \frac{1}{(x-2)^2}$$

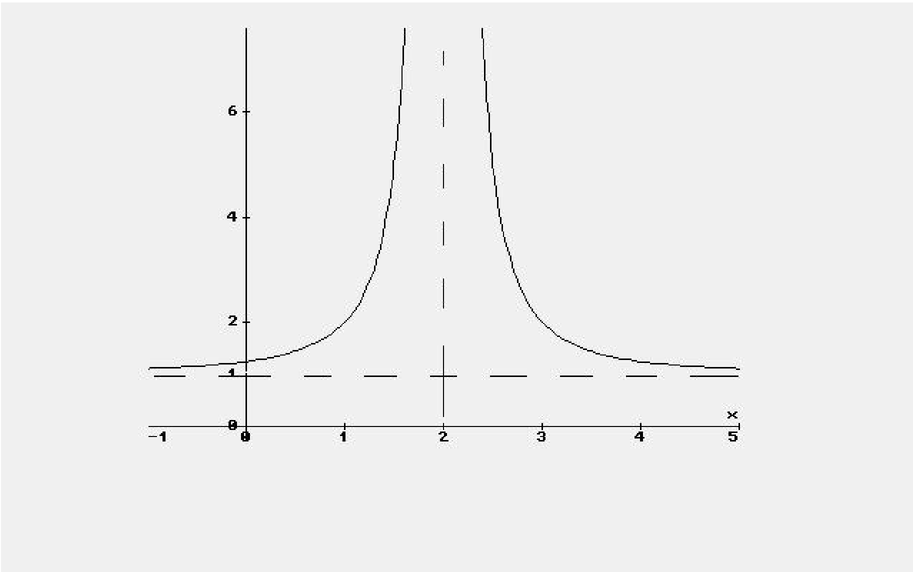
$$\text{Domain} = \{x \,|\, x \neq 2\}$$

$$\text{Range} = \{y \,|\, y > 1\} = (1,\ \infty)$$

$$f(x) = 1 + \frac{1}{(x-2)^2}$$

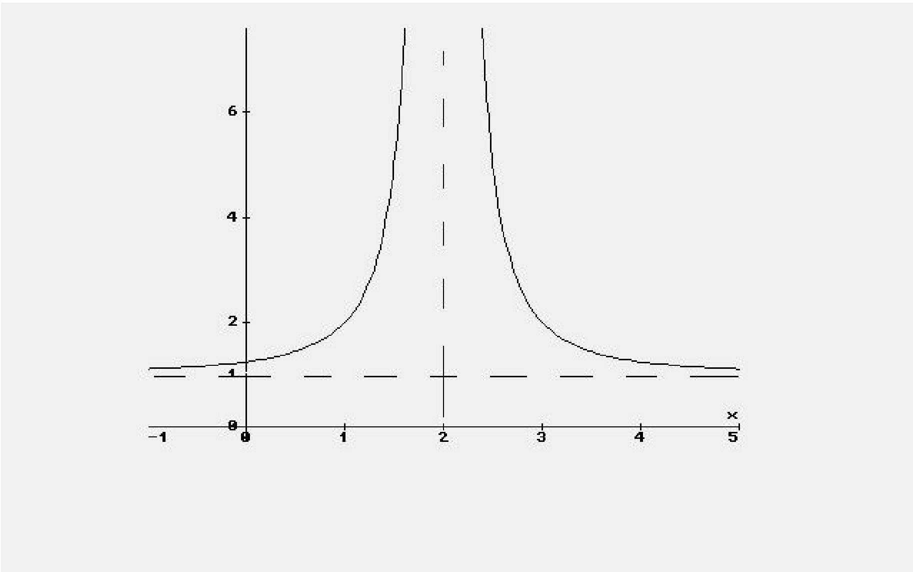
$$\text{Domain} = \{x \,|\, x \neq 2\}$$

$$\text{Range} = \{y \,|\, y > 1\} = (1, \infty)$$



$$f(x) = 1 + \frac{1}{(x-2)^2}$$

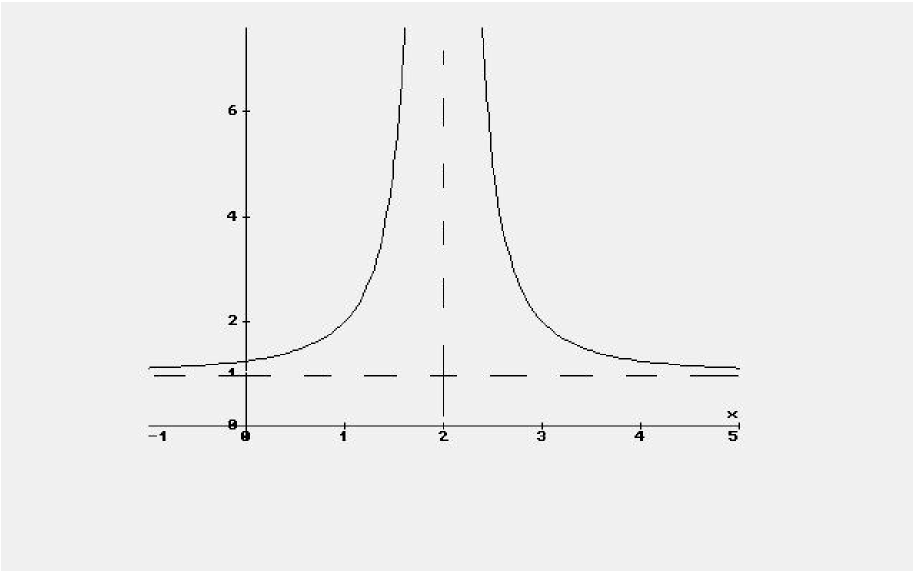
$f(x)$ gets closer and closer to 1 as x gets larger and larger



$$f(x) = 1 + \frac{1}{(x-2)^2}$$

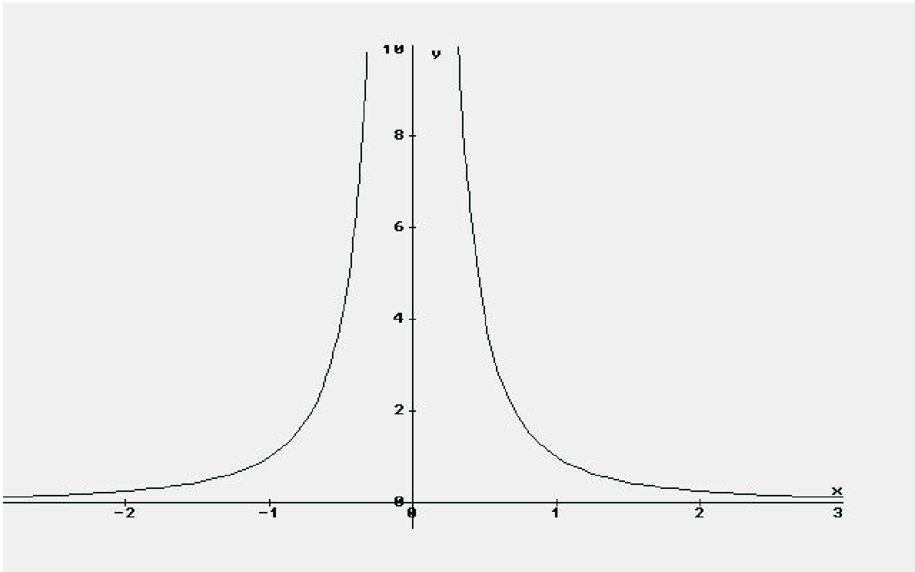
$f(x)$ gets closer and closer to 1 as x gets larger and larger

$$\lim_{x \to \infty} f(x) = 1$$

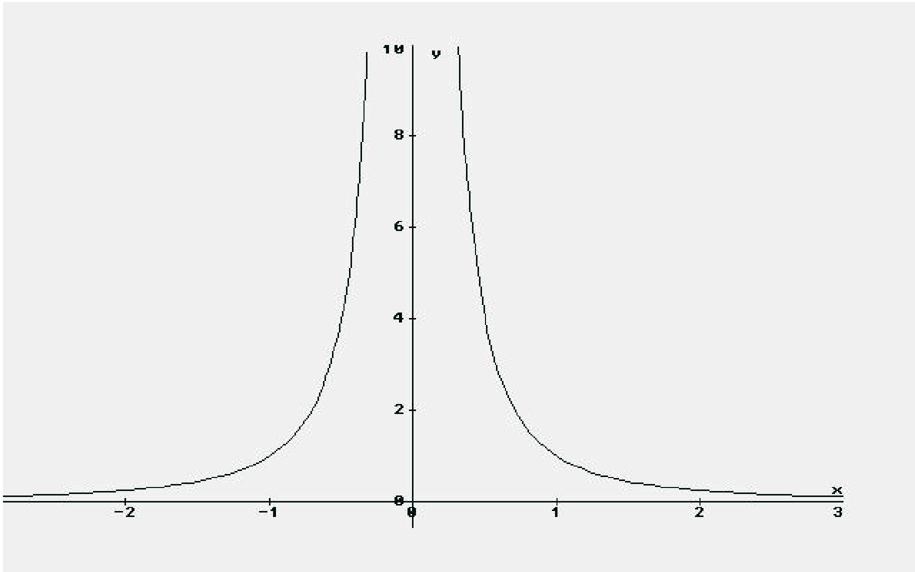


$$\lim_{x \to \infty} \frac{1}{x^2}$$

$$\lim_{x \to \infty} \frac{1}{x^2} = 0$$



$f(x)$ gets larger and larger (without bound) as x gets closer and closer to 0



$$\lim_{x \to 0} \frac{1}{x^2} = \infty$$