Assignment 1. *Lines and Functions*

Read Chapter 1.1
You should be able to do the following problems:

Chapter 1.1/Problems 27 - 37

Hand in the following problems:

1. Find the equation of the line through $(2, 4)$ and $(3, 0)$

2. Find the equation of the line that passes through the point $(0,\ 4)$ and is parallel to the line in problem 1.

Problems 3 - 5. The functions f, g, p, ϕ and ψ are defined as follows:

$$f(x) = x^2 + 4x \qquad g(x) = \frac{1}{x^2} \qquad p(x) = \frac{x}{2} \qquad \phi(x) = \frac{2}{x} \qquad \psi(x) = \frac{x}{2} + \frac{2}{x}$$

3. State the domain of each of these functions.

4. Calculate and simplify each of the following:

$$f(-2) \qquad g\left(\tfrac{1}{2}\right) \qquad g(p(x)) \qquad \phi(x+2)$$

5. Calculate and simplify each of the following:

$$\tfrac{f(x+h)-f(x)}{h},\ \tfrac{g(x+h)-g(x)}{h},\ \tfrac{p(x+h)-p(x)}{h},\ \tfrac{\phi(x+h)-\phi(x)}{h},\ \tfrac{\psi(x+h)-\psi(x)}{h}$$

Assignment 2. *Limit Calculations*

Read 2.1 - 2.5
You should be able to do the following problems:

Section 2.3/Problems 10 - 32, Section 2.5/Problems 1 - 24

Hand in the following problems:

Problems 1 - 5. Calculate the limits. Show all algebraic steps. If the limit does not exist, then state this explicitly.

1. $$\lim_{x\to 0}\left(\frac{4x + 3x^2}{x}\right)$$

2. $$\lim_{x\to 4}\left(\frac{x-4}{x^2-16}\right)$$

3. $$\lim_{x\to 0}\frac{x}{(x-1)^2}$$

4. $$\lim_{x\to 1}\frac{x}{(x-1)^2}$$

5. $$\lim_{x\to -1}\left(\frac{x+1}{x^2-x-2}\right)$$

Assignment 3. *Continuity. Additional Limit Problems.*

Review Chapter 2 sections 3, 4 and 5.

For problems 1 - 3, calculate the indicated limit. If the limit does not exist, then state this explicitly.

1.
$$\lim_{x\to 1}\left(\frac{x-1}{\sqrt{x}-1}\right)$$

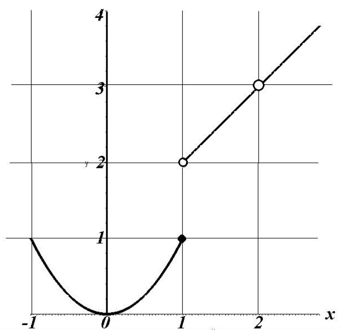
2.
$$\lim_{x\to 1}\left(\frac{x-1}{1+\frac{1}{1-2x}}\right)$$

3.
$$\lim_{x\to 4}\frac{\frac{1}{4}-\frac{1}{x}}{x-4}$$

4. Let $f(x)$ be given by the following equations:

$$f(x)=\begin{cases} x^2, & x\le 1\\ \frac{x^2-x-2}{x-2}, & x>1\end{cases}$$

This function is graphed below. Note that the function is defined for $x=1$ but it is not defined for $x=2$.



Calculate each of the following:

$$\lim_{x\to 1^-} f(x) \qquad \lim_{x\to 1^+} f(x) \qquad \lim_{x\to 2^-} f(x) \qquad \lim_{x\to 2^+} f(x)$$

5. Let $f(x)$ be the function defined in problem 4.

Does $\lim_{x\to 1} f(x)$ exist? If so, what does it equal?

Does $\lim_{x\to 2} f(x)$ exist? if so, what does it equal?

Assignment 4. *Introduction to Differentiation*

Read 2.1, 2.7, 2.8, 3.1
You should be able to do the following problems:

Section 2.8/Problems 21 - 31

Hand in the following problems:

Problems 1 - 5. Use limits to calculate the expressions for the derivatives of the functions f, g, p, ϕ and ψ defined in Assignment 1.

Assignment 5. *Techniques of Differentiation*

Read 2.7, 2.8, 3.1
You should be able to do the following problems:

Section 3.1/Problems 3 - 28

Hand in the following problems:

Problems 1 - 6. Find the derivative $\frac{dy}{dx}$ and simplify your answer as much as possible.

1.
$$y = 4 - 4x$$

2.
$$y = \frac{\sqrt{x}}{2} + \frac{x}{4}$$

3.
$$y = x - \frac{1}{\sqrt{x}}$$

4.
$$y = 6x + \frac{1}{x^3}$$

5.
$$y = \frac{(3x)^2}{\pi}$$

6.
$$y = \frac{x^4}{4} + \frac{1}{\sqrt{2}}$$

7. Determine any points on the curve $y = x^2 + 4x$ where the tangent line is horizontal.

8. Find the equation of the line that is tangent to the curve $f(x) = x^3 + 3x - 1$ at the point $(1,\ 3)$. Then, find the coordinates of the point where this tangent line intersects the x-axis.

9. Find the equation of the line perpendicular to the curve $y = 2\sqrt{x}$ at the point $(1,\ 2)$

10. The formula for the force of gravity exerted by an object of mass m_1 on an object of mass m_2 at a distance of r is:

$$F = \frac{Gm_1m_2}{r^2}$$

where G is the universal gravitation constant.
Find the expression for the rate at which the force changes with respect to distance. You may assume that G, m_1 and m_2 are constants.

Assignment 6. *Introduction to Integration*

Read 4.9

You should be able to do the following problems:

Section 4.9/Problems 1 - 15

Hand in the following problems:

For problems 1 - 8, calculate the given integrals.

1. $\displaystyle\int x^6\,dx$ **2.** $\displaystyle\int (4-4x)\,dx$ **3.** $\displaystyle\int \sqrt[3]{x}\,dx$ **4.** $\displaystyle\int \frac{20}{x\sqrt{x}}\,dx$

5. $\displaystyle\int \frac{1}{2x^3}\,dx$ **6.** $\displaystyle\int \left(6x+\frac{1}{x^3}\right)dx$ **7.** $\displaystyle\int \frac{x^2+1}{x^2}\,dx$ **8.** $\displaystyle\int (1+3t)t^2\,dt$

9. Let $F(x)$ be the antiderivative of $f(x)$. In symbols, $F(x) = \int f(x)\,dx$. The *definite integral* $\int_a^b f(x)\,dx$ is defined by the formula:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Calculate the following definite integral:

$$\int_1^2 \frac{1}{x^2}\,dx$$

10. An object is moving along a straight line. It is subject to a variable force and its velocity after t seconds is given by:

$$v(t) = 6(t^2+t)+1$$

a) If its initial displacement is $x(0) = 3$ meters, find its displacement after 1 second. *(Hint:* $v = \frac{dx}{dt}$ *)*

b) Find the general expression for the acceleration after t seconds.

Assignment 7. *Product and Quotient Rules*

Read 3.2

You should be able to do the following problems:

Section 3.2/Problems 1 - 26

Hand in the following problems:

Problems 1 - 4. Find the derivative. Simplify as much as possible.

1. $y = (x-1)e^x$ **2.** $y = \dfrac{x^2+1}{x}$ **3.** $y = \dfrac{2x+1}{x+1}$ **4.** $y = \dfrac{e^x}{1+e^x}$

5. Find the equation of the line tangent to the curve $y = \frac{x+2}{x-2}$ at the point $(0, -1)$

Assignment 8. *Derivatives of Trigonometric Functions*

Read 3.3
You should be able to do the following problems:

Section 3.3/Problems 1 - 24

Hand in the following problems:

Problems 1 - 3. Find the derivative. Simplify as much as possible.

1.
$$y = \sin x \cos x$$

2.
$$y = \frac{\sin x}{1 + \cos x}$$

3.
$$y = \sec x - \tan x$$

4. Prove the following differentiation formula:

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

5. Write the equation of the line tangent to $f(x) = x \cos x$ at $x = \pi$.

Assignment 9. *The Chain Rule*

Read 3.4
You should be able to do the following problems:

Section 3.4/Problems 1 - 54

Hand in the following problems:

Problems 1 - 9. Find $\frac{dy}{dx}$. Show work where appropriate. Simplify your answers as much as possible.

1. $y = \left(x^2 + 3x + 1\right)^4$

2. $y = \frac{1}{(2x-1)^3}$

3. $y = x^6(2x - 1)^3$

4. $y = x\, e^{-x^2/2}$

5. $y = \left(\frac{x-1}{x}\right)^3$

6. $y = x\sqrt{1 + x^2}$

7. $y = \sin\sqrt{x} + \sqrt{\sin x}$

8. $y = (\sin x + \cos x)^2$

9. $y = \left(\dfrac{ax + b}{cx + d}\right)^6$ where a, b, c and d are constants

10. The period of oscillation of a simple pendulum of length x (in feet) is given by:

$$f(x) = \frac{\pi}{2}\sqrt{\frac{x}{2}}$$

Find $f'(8)$ (the rate of change of period with respect to pendulum length at $x = 8$)

Assignment 10. *Implicit Differentiation*

Read 3.5
You should be able to do the following problems:
Section 3.5/Problems 1 - 12, 25 - 28, 44 - 45
Hand in the following problems:
Problems 1 - 2. Use the method of *implicit differentiation* to find the derivative $\frac{dy}{dx}$.

1. $$x^3 + y^5 = 6$$

2. $$\frac{1}{y} - \frac{1}{x} = 1$$

3. The point $(2,\ 2)$ is one of the points on the curve described by the equation:

$$x^2 + 4y^2 + 4 = 4x + 8y$$

Use the method of *implicit differentiation* to find the slope of the tangent line at the point $(2,\ 2)$.
4. Find the equation of the line that is tangent to the curve

$$y^2 + 2xy - x^2 = 2$$

at the point $(0,\ \sqrt{2})$.
5. Calculate the equation of the line that is tangent to the curve described by the following equation at the point $(\sqrt{\pi},\ \sqrt{\pi}\,)$.

$$\sin(xy) = x^2 - y^2$$

Assignment 11. *The Natural Logarithm*

Read 3.6
You should be able to do the following problems:
Section 3.6/Problems 1 - 26, 33 - 34, 39 - 50
Hand in the following problems:
Problems 1 - 4. Find the derivative $\frac{dy}{dx}$. Simplify your answer as much as possible.

1. $$y = -x + x \ln x$$

2. $$y = \ln \frac{1}{x}$$

3. $$y = \ln\left(\sin^2 x\right)$$

4. $$y = \ln\left(e^x + 1\right)$$

5. Find the equation of the line that is tangent to $y = x^2 \ln x$ at the point $(1,\ 0)$

Assignment 12. *Related Rates, Higher Derivatives*

Read 3.9

You should be able to do the following problems:

Section 3.9/Problems 1 - 37

Hand in the following problems:

1. Suppose the included angle of two sides of equal length x in an isosceles triangle is θ. The area of the triangle is given by the formula

$$A = \frac{x^2}{2} \sin\theta$$

For the purposes of this problem, let's take x to be 4 centimeters. If θ is increasing at the rate of $\frac{1}{2}$ radian per minute, find the rate of change of the area when $\theta = \frac{\pi}{6}$ and $\theta = \frac{3\pi}{4}$.

2. Suppose a resistor of resistance x ohms and another resistor of resistance y ohms are connected in parallel. If R is the total resistance, then R is related to x and y by the formula:

$$\frac{1}{R} = \frac{1}{x} + \frac{1}{y}$$

If x and y are increasing at rates of 1 and 1.5 ohms per second, respectively, at what rate is R changing when $x = 50$ ohms and $y = 75$ ohms?

3. The sides of an equilateral triangle are increasing at the rate of 0.2 cm. per hour. At what rate is the area changing when the side length is 4 cm. ?

4. If a tree trunk adds $\frac{1}{96}$ of a foot to its radius and 1 foot to its height each year, how rapidly is its volume changing when its radius is 1.5 feet and its height is 50 feet?
(Assume that the tree trunk is a circular cylinder)

5. A car is driving towards Kennedy Space Center at 50 miles per hour. The driver sees a rocket rising vertically from the Space Center. At the point in time when the car is 4 miles from the Space Center, the rocket is 3 miles above the Space Center and traveling at 500 miles per hour. At this point in time, how fast (in miles per hour) is the distance from the car to the rocket changing?

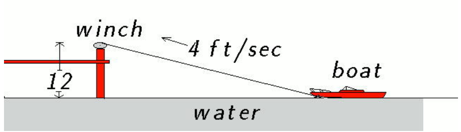
Assignment 13 *Additional Related Rates Problems*

Hand in the following problems:

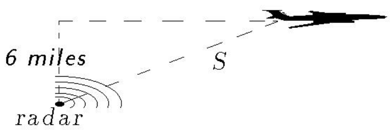
1. An aircraft is climbing at a 30° angle to the horizontal. How fast is the aircraft gaining altitude if its airspeed is 500 miles per hour?

2. A woman 6 feet tall is walking toward a wall at a rate of 4 feet per second. Directly behind her on the ground and 40 feet from the wall is a spotlight. How fast is the length of the woman's shadow on the wall changing when she is halfway from the spotlight to the wall? Is the shadow lengthening or shortening?

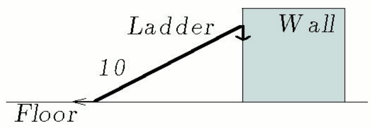
3. A boat is pulled in by means of a winch on the dock 12 feet above a boat (see figure below). The winch pulls in rope at the rate of 4 feet per second. Determine the speed of the boat when there is 13 feet of rope out.



4. An airplane is flying at an altitude of 6 miles and passes directly over a radar antenna (see figure below). When the plane is 10 miles away ($s = 10$), the radar detects that the distance s is changing at the rate of 240 miles per hour. What is the speed of the plane?



5. A ladder that is 10 meters long is leaning against a wall. Suppose the ladder begins to slip and the top of the ladder is moving down the wall at $\frac{1}{4}$ meter/second.
a) Find the rate that the bottom of the ladder is moving away from the wall at the point in time when the bottom of the ladder is 6 meters away from the wall.
b) Consider the triangle made by the ladder, the wall and the floor. Find the rate at which the area of this triangle is changing at the point in time when the bottom of the ladder is 6 meters away from the wall.



Assignment 14. *Mean Value Theorem, Applications to Graphing*

Read 4.1 to 4.3

You should be able to do the following problems:

Section 4.1/Problems 16 - 44, Section 4.2/Problems 1 - 12
Section 4.3/Problems 9 - 21

Problems 1 - 6. For each of the functions below, use the first and second derivatives to find and classify all maximums, minimums and points of inflections. Use this information to graph the curves.

1. $y = x^3 + 3x^2$ **2**. $y = 3\left(x^2 - 1\right)^2$ **3**. $y = 2x^3 - 9x^2 + 12x + 7$

4. $y = \dfrac{x}{1+x^2}$ **5**. $y = \dfrac{(x-2)^2}{4} + \dfrac{4}{(x-2)^2}$ **6**. $y = 2x\, e^{-x^2/2}$

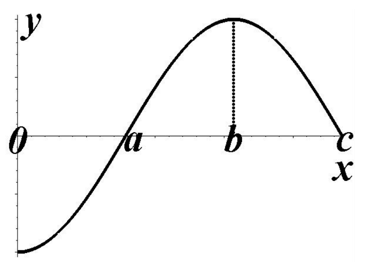
7. Rolle's Theorem says that if f is continuous on the closed interval $[a,\ b]$ and differentiable on the open interval $(a,\ b)$ and $f(a) = f(b)$ then there is at least one number c in $(a,\ b)$ such that $f'(c) = 0$.

Let $f(x) = x^2 - 3x$ on the interval $[0,\ 3]$. Determine if Rolle's Theorem can be applied to this function. If so, find all values of c in the interval $(0,\ 3)$ so that $f'(c) = 0$.

8. The Mean Value Theorem says that if f is continuous on the closed interval $[a,\ b]$ and differentiable on the open interval $(a,\ b)$ then there is at least one number c in $(a,\ b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

Determine whether the Mean Value Theorem can be applied to the function $f(x) = \sqrt{x}$ on the interval $[1,\ 4]$. If the Mean Value Theorem can be applied, find all values of c in the interval such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

Problems 9 - 10. Both problems refer to the function $y = F(x)$ that is sketched below. You are not given the formula for $F(x)$



9. For $a < x < b$, which of the following is true?

a. $F'(x) < 0$ and $F''(x) < 0$ **b.** $F'(x) > 0$ and $F''(x) > 0$
c. $F'(x) < 0$ and $F''(x) > 0$ **d.** $F'(x) > 0$ and $F''(x) < 0$

10. For $b < x < c$, which of the following is true?

a. $F'(x) < 0$ and $F''(x) < 0$ **b.** $F'(x) > 0$ and $F''(x) > 0$
c. $F'(x) < 0$ and $F''(x) > 0$ **d.** $F'(x) > 0$ and $F''(x) < 0$

Assignment 15. *Limits at Infinity. Introduction to Applied Maximum-Minimum*

Read 2.6, 4.5, 4.7

You should be able to do the following problems:

Section 2.6/Problems 1 - 32

Hand in the following problems:

1. A retailer has determined that the total cost T for ordering and storing x units of a product is:

$$T = 2x + \frac{2,000,000}{x} \qquad x > 0$$

Find the value of x that minimizes the total cost T.

2. Coughing forces the trachea (windpipe) to contract, which affects the velocity v of the air passing through the trachea. Suppose the velocity of the air during coughing is:

$$v = k(a-r)r^2, \qquad 0 \le r \le a$$

where k is a constant, a is the normal radius of the trachea (a is also a constant) and r is the radius during coughing. What radius will produce the maximum air velocity?

3. The equation:

$$E = \frac{kqx}{(x^2+a^2)^{3/2}}$$

gives the electric field intensity on the axis of a uniformly charged ring, where q is the total charge on the ring, k is a constant, and a is the radius of the ring. At what value of x is E maximum?

4. A force directed at an angle of θ to the horizontal pulls an object horizontally. For a particular coefficient of friction, the force necessary to do this is given by the formula:

$$F = \frac{1}{\sqrt{2}\sin\theta + \sqrt{6}\cos\theta} \qquad \text{where} \quad 0 \le \theta \le \frac{\pi}{2}$$

Find the angle θ that minimizes the force F.

5. The distance from a point $(0, a)$ to a line $y = mx$ is given by:

$$p = \sqrt{(mx-a)^2 + x^2}$$

Assume that the point $(0, a)$ is *not* on the line. Use calculus to find the value of x that minimizes the distance p. Your answer will be in terms of the constants a and m.

For problems 6 - 7, find the vertical and horizontal asymptotes (if any) of the given function. Sketch each of these functions.

6. $f(x) = 1 - \dfrac{4}{x^2}$ **7.** $f(x) = \dfrac{4x}{x^2+4}$

For problems 8 - 10, calculate the given limits.

8. $\displaystyle\lim_{x\to\infty}\left(2x - \frac{1}{x^2}\right)$ **9.** $\displaystyle\lim_{x\to\infty}\frac{x}{\sqrt{2x^2+3}}$ **10.** $\displaystyle\lim_{x\to\infty}\frac{\cos x}{x^2}$

Assignment 16. *Applied Maximum-Minimum Problems*

Read 4.7
You should be able to do the following problems:

Section 4.7/Problems 1 - 14

Hand in the following problems:

1. A rancher wants to construct a rectangular corral. He also wants to divide the corral by a fence parallel to one of the sides. The constraint is that he wants the total area of the corral to be 5,400 square meters. The fence itself costs 1 dollar per meter. Find the dimensions of the least expensive corral.

2. If a rectangle has a perimeter p (where p is a constant), calculate the dimensions of the rectangle with maximum area.

3. A church window consisting of a rectangle topped by a semicircle is to have a perimeter p. Find the radius of the semicircle if the area of the window is to be a maximum.

4. An open topped box has a square base and a volume of 144 cubic centimeters. Use calculus to find the dimensions of the box so that the surface area of the box is minimized.

5. A rectangular sheet of paper contains 88 square inches. The margins at the top and bottom are 2 inches, and those at the sides are 1 inch. What are the dimensions of the maximum printed area?

Assignment 17. *Additional Applied Maximum-Minimum Problems*

Read 4.7 again

You should be able to do the following problems:

Section 4.7/Problems 15 - 76

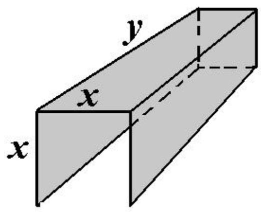
Hand in the following problems:

1. The two equal sides of an isosceles triangle are each L units long. Let ϕ be the angle between the two equal sides. Use calculus to find the angle ϕ that maximizes the area of the triangle.

2. A straight east-west highway along the coast of an enemy country connects the weapons depot to the army barracks. The depot and the barracks are 10 miles apart. There is an American aircraft carrier in the ocean 3 miles north of the army barracks. There is an American warplane over the ocean 2 miles north of the weapons depot. The pilot of the plane is given the order to disrupt transportation by dropping a bomb somewhere along the highway and then flying directly to the aircraft carrier. How many miles along the highway from the weapons depot should the pilot bomb if he wants to make the total length of this trip as short as possible.

3. The material in the top of a beer can costs \$.0002 per square inch, while the lateral surface and bottom costs \$.0001 per square inch. If the can is to hold twelve ounces (1.8 in^3 = 1 oz) find the dimensions of the least expensive can.

4. A net enclosure for practicing golf has square ends with one end open (see figure below). Note that the bottom does not require any material. Find the dimensions that require the least amount of netting if the volume of the enclosure is to be 144 cubic inches.



Assignment 18. *The Differential*

Read 3.10, 4.6, 4.8
You should be able to do the following problems:

Section 3.10/Problems 1 - 22

Hand in the following problems:

1. Begin by finding the equation of the line tangent to $y = x^2 - 6x + 8$ at the point $(4, 0)$.

a) When x is changed from 4 to 4.05, the height of the parabola changes. Calculate this change in height.

b) When x is changed from 4 to 4.05, the height of the tangent line changes. Calculate the change in the height of the tangent line.

c) Explain what this problem has to do with the *differential* of the function $y = x^2 - 6x + 8$

Problems 2 - 3. In each of the following two problems, you are given a function $f(x)$ and a point x_0 at which $f(x_0)$ can be calculated easily. You must use the differential df to approximate $f(x_1)$ where x_1 is a point nearby x_0. Compare your estimate from the differential to the actual value of $f(x_1)$.

2. $$f(x) = \sqrt{x^2 - 9} \qquad x_0 = 5 \qquad x_1 = 5.1$$

3. $$f(x) = \tan\left(\frac{\pi x}{4}\right) \qquad x_0 = 1 \qquad x_1 = 1.001$$

4. The surface area of a sphere is given by the formula

$$S = 4\pi r^2$$

where r is the radius of the sphere. Suppose the radius is increased from $r = 2$ to 2.003 cm. Use the differential to approximate the change in the surface area.

5. Under certain conditions, due to the presence of an electric charge, the electric potential at a position x along a line is given by:

$$V = \frac{1}{\sqrt{x^2 + 4}}$$

If the position is varied from $x = 2$ to 2.08, the potential V is going to change. Use the **differential** dV to estimate the change in the potential.

Assignment 19. *Summation Notation. Areas as Limits.*

Read Appendix E and Sections 5.1, 5.2

You should be able to do Appendix E/Problems 1 - 35 and Section 5.2/Problems 1 - 4

Problems 1 - 3. Evaluate the following sums:

1.
$$\sum_{j=1}^{3} \frac{6}{j}$$

2.
$$\sum_{n=1}^{3} 6$$

3.
$$\sum_{k=1}^{5} 20k$$

Problems 4 - 6. Express each of the following expressions in sigma notation ($\sum$ notation) but do not evaluate.

4.
$$2 + 4 + 6 + 8 + \ldots + 200$$

5.
$$\frac{1}{3} + \frac{2}{4} + \frac{3}{5} + \frac{4}{6} + \ldots + \frac{100}{102}$$

6.
$$x^4 + x^3y + x^2y^2 + xy^3 + y^4$$

Problems 7 - 9. Evaluate the following sums. Your answers will be expressions that depend on n.

7.
$$\sum_{i=1}^{n} (12i - 1)$$

8.
$$\sum_{i=1}^{n} \left(i^3 - (i-1)^3\right)$$

9.
$$\sum_{i=1}^{n} \left(\frac{2i}{n^2} + \frac{1}{n}\right)$$

10. Let $f(x) = x + 1$. Calculate the area under the curve $y = f(x)$ between $x = 0$ and $x = 2$ by evaluating the limit:

$$\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\, \Delta x_i$$

Assignment 20. *The Fundamental Theorem of Calculus*

Read 5.3, 5.4
You should be able to do the following problems:

Section 5.3/Problems 19 - 37, Section 5.4/Problems 1 - 39

Hand in the following problems:

Problems 1 - 5. Evaluate using the Fundamental Theorem of Calculus:

1.
$$\int_1^2 \left(\frac{6}{x^4} - \frac{1}{x^2}\right) dx$$

2.
$$\int_1^4 \frac{x+1}{\sqrt{x}}\, dx$$

3.
$$\int_0^{\pi/4} \sec^2 \theta \, d\theta$$

4.
$$\int_0^{\pi/3} \cos x \tan x \, dx$$

5.
$$\int_0^1 14x^2\sqrt{x}\ dx$$

For problems 6 and 7, calculate $F(x)$ and $F'(x)$.

6.
$$F(x) = \int_0^x \sqrt{t}\ dt$$

7.
$$F(x) = \int_{\pi/4}^x \sec t \tan t\ dt$$

8. The average value of a function $f(x)$ on the interval $a \le x \le b$ is given by $\frac{1}{b-a}\int_a^b f(x)\,dx$. Suppose the density (in gm/meter) along a rod between $x = 0$ and $x = 2$ is given by $f(x) = x^2(x+3)$. Calculate the average density.

9. Find the area of the region above the x axis that is below the curve $y = 4x - x^2$.

10. Find the area of the region that is below $y = 3\sin x$ and above $y = 2\sin x$ for $0 \le x \le \pi$.

Assignment 21. *The Method of Variable Substitution*

Read 5.5

You should be able to do the following problems:

Section 5.5/Problems 1- 30, 39 - 73

Hand in the following problems:

Problems 1 - 5. Do the following integrals using the method of variable substitution. Be careful to change your limits of integration to conform to your new variable when you do the definite integrals.

1.
$$\int 8x\left(1+x^2\right)^3\,dx$$

2.
$$\int \cos \pi x\,dx$$

3.
$$\int \frac{dt}{(1+t)^2}$$

4.
$$\int \tan^2\theta\sec^2\theta\,d\theta$$

5.
$$\int_2^3 \frac{x}{x-1}\,dx$$

Assignment 22. *More Variable Substitutions*

Problems 1 - 5. Do the following integrals using the method of variable substitution. Be careful to change your limits of integration to conform to your new variable when you do the definite integrals.

1.
$$\int_2^5 \frac{3x}{\sqrt{x-1}}\,dx$$

2.
$$\int_0^{\pi/4} \sin^2(2\theta)\,\cos(2\theta)\,d\theta$$

3.
$$\int_{\pi^2}^{4\pi^2} \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

4.
$$\int \frac{e^x}{(1-e^x)^2}\,dx$$

5.
$$\int \frac{(\ln x)^2}{x}\,dx$$