Embry-Riddle Aeronautical University
MA241 Sample Final Exam (Fall 2016)
Fall 2016

Directions:

There are twenty-five questions, each worth 4 points for a total of 100 points.

Scientific calculators are permitted, but not graphing calculators.

Time allotted: two hours.

1.

If $f(x) = \begin{cases} -x^2, & x < 3 \\ -x^3 + 18, & x \geq 3 \end{cases}$ then which statement about the function is *false*?

a. $\lim_{x \to 3} f(x) = -9$ **b.** $f(3) = -9$

c. continuous at $x = 3$ **d.** differentiable at $x = 3$

2. Calculate the following limit:

$$\lim_{x \to 0} \frac{x \cos x}{\sin 2x}$$

a. 0 **b.** $\frac{1}{2}$ **c.** 1 **d.** $\frac{1}{4}$ **e.** does not exist

3. The tangent line to the curve $y = x^3 + x$ at the point $(1,\ 2)$ is given by:

a. $y + 2 = 4(x + 1)$ **b.** $y + 2 = 3(x - 4)$ **c.** $y - 1 = 4(x - 2)$

d. $y - 4 = 2(x - 1)$ **e.** $y - 2 = 4(x - 1)$

4. Evaluate the limit, if it exists:

$$\lim_{x \to \infty} \frac{3x^4 + 5x + 10}{x^4 + 10x^3 + 2}$$

a. Does not exist **b.** 3 **c.** 0 **d.** 4

5. Find the derivative of $f(x) = \frac{\ln x}{e^x}$

a. $\frac{\frac{1}{x} - \ln x}{e^x}$ **b.** $\frac{\frac{1}{x} + \ln x}{e^x}$ **c.** $\frac{\ln x - \frac{2}{x}}{e^x}$ **d.** $\frac{e^x - 1}{\ln x + 1}$ **e.** $\frac{e^x}{e^x + 1}$

6. Find the derivative of $f(x) = \ln\left(x^2 + x\right)$

a. $\frac{2x+1}{x^2+x}$ **b.** $\frac{x+1}{x^2+x}$ **c.** $\frac{x}{x+1}$ **d.** $\frac{1}{x^2+x}$ **e.** $\frac{x^2}{x-1}$

7. Given $f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{x}$ then:

a. $f(x) = \sqrt{x} + \frac{1}{x^2} + C$ **b.** $f(x) = \sqrt{x} - \ln|x| + C$

c. $f(x) = -\frac{1}{4x^{3/2}} + \frac{1}{x^2} + C$ **d.** $f(x) = \sqrt{x} + \ln|x| + C$

8. Find the following limit:

$$\lim_{h \to 0} \frac{\sin\left(\frac{\pi}{3} + h\right) - \sin\frac{\pi}{3}}{h}$$

a. $\frac{\sqrt{2}}{2}$ **b.** $\frac{\sqrt{3}}{2}$ **c.** $\frac{1}{2}$ **d.** $-\frac{1}{2}$ **e.** does not exist

9. Find $\frac{dy}{dx}$ at the point $(1,\ 2)$ given $x^4 - xy + y^3 = 7$

a. $-\frac{1}{5}$ **b.** $-\frac{2}{11}$ **c.** $-\frac{1}{6}$ **d.** $-\frac{2}{13}$ **e.** $-\frac{1}{14}$

10. A 10 foot ladder is leaning against a house. When the top of the ladder is 8 feet above the ground, it is sliding down the wall at 2 feet per second. How fast is the base of the ladder sliding away from the wall?

a. $\frac{5}{3}$ **b.** 2 **c.** $\frac{7}{3}$ **d.** $\frac{8}{3}$ **e.** 3

11. A farmer has 24 feet of fencing to build a chicken coop which will be shaped like a rectangle split into two pens. What is the maximum possible area (square feet) of the chicken coop?

a. 12 **b.** 16 **c.** 18 **d.** 20 **e.** 24

12. Find the linear approximation of the function $f(x) = x^2 + x$ at $x = 1$

a. $L(x) = 2x$ **b.** $L(x) = 3x - 1$ **c.** $L(x) = 4x - 2$ **d.** $L(x) = 5x - 3$ **e.** $L(x) = 6x - 4$

13. Let $f(x) = \sqrt{2x+1}$ for $4 \leq x \leq 12$. According to the Mean Value Theorem, there exists a number $x = c$ such that $f'(c) =$?

a. $\frac{1}{2}$ **b.** $\frac{1}{3}$ **c.** $\frac{1}{4}$ **d.** $\frac{1}{5}$ **e.** $\frac{1}{6}$

14. Find the derivative of $f(x) = e^{\sin^2 x}$

a. $2\sin x \cos x e^{\cos^2 x}$ **b.** $2\sin x \cos x e^{\sin x}$ **c.** $2\sin x e^{\sin^2 x}$ **d.** $2\sin x \cos x e^{\sin^2 x}$

15. Find the absolute maximum value and the absolute minimum value of the function $f(x) = 5 + 54x - 2x^3$ on the interval $[0,\ 4]$

a. abs. max: 113, abs. min: 5 **b.** abs. max: 100, abs. min: 0

c. abs. max: 103, abs. min: 5 **d.** abs. max: 113, abs. min: 0

16. Calculate the following integral:

$$\int_0^{\sqrt{\frac{\pi}{2}}} x \sin\left(x^2\right)\, dx$$

a. $\frac{1}{3}$ **b.** $\cos\left(x^2\right) + C$ **c.** $-\frac{1}{4}$ **d.** $\frac{1}{2}$

17. What is the derivative $g'(x)$ if $g(x)$ is defined by the following integral:

$$g(x) = \int_0^x \sin\left(t^2\right)\, dt$$

a. $\cos\left(x^2\right)$ **b.** $2x\sin\left(x^2\right)$ **c.** $2x\cos\left(x^2\right)$ **d.** $\sin(2x)$ **e.** $\sin\left(x^2\right)$

18. Find the derivative of $g(x) = \ln\left(e^{2x}\right)$

a. $\frac{1}{e^{2x}}$ **b.** $2x$ **c.** $\frac{1}{x}$ **d.** 2 **e.** 1

19. Calculate the average value of the function $f(x) = \sin 3x$ on the interval $\left[\frac{\pi}{2}\ ,\ \pi\right]$

a. $-\frac{2}{3\pi}$ **b.** $\frac{5}{\pi}$ **c.** $\frac{3}{2\pi}$ **d.** $\frac{2}{\pi}$ **e.** $\frac{2}{3\pi}$

20. Find: $\int \frac{x+1}{x^2+2x}\, dx$

a. $\ln\left|x^2+2x\right| + C$ **b.** $2\ln\left|x^2+2x\right| + C$ **c.** $\frac{1}{2}\ln\left|x^2+2x\right| + C$ **d.** $\frac{1}{2}\ln|x+1| + C$

21. Find the value of the integral: $\int_0^1 \frac{4x}{1+x^2}\, dx$

a. 2 **b.** $\ln 2$ **c.** $2\ln 2$ **d.** $2\ln 4$ **e.** 4

22. Estimate the integral $\int_1^7 \frac{30}{x}\,dx$ using three equal subintervals with the left endpoints.

a. 46 **b.** 23 **c.** 96 **d.** $30\ln 7$ **e.** 92

23. Find the area between the curves $y = x^2 + 3$ and $y = 5 - x^2$

a. $\frac{2}{3}$ **b.** $\frac{1}{3}$ **c.** $\frac{10}{3}$ **d.** $\frac{4}{3}$ **e.** $\frac{8}{3}$

24. Given the velocity of a particle $v(t) = -32t - 10$ and the position $s(3) = 0$, find $s(0)$.

a. 194 **b.** 150 **c.** 174 **d.** 200 **e.** 19

25. Which of the following conditions will *guarantee* that the point $(a,\ f(a))$ is a relative maximum point?

a. $f'(a) = 0$ and $f''(a) < 0$ **b.** $f'(a) = 0$ and $f''(a) > 0$ **c.** $f'(a) > 0$ and $f''(a) = 0$

d. $f'(a) < 0$ and $f''(0) = 0$ **e.** $f'(a) = 0$ and $f''(a) = 0$