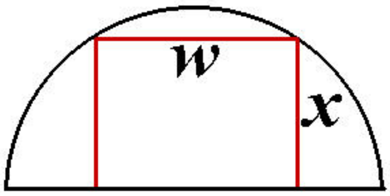
A rectangle is inscribed in a semicircle of radius 10 cm. What is the area of the largest rectangle we can inscribe?

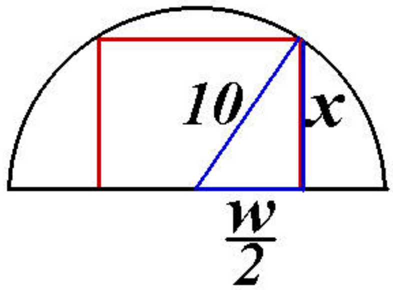
$$A = xw$$



$$\left(\frac{w}{2}\right)^2 + x^2 = 10^2$$

$$w = 2\sqrt{100 - x^2}$$

$$A = xw = x \cdot 2\sqrt{100 - x^2}$$



$$A = 2x\left(100 - x^2\right)^{1/2}$$

$$\frac{dA}{dx} = 2x \cdot \frac{1}{2}\left(100 - x^2\right)^{-1/2}(-2x) + 2\left(100 - x^2\right)^{1/2}$$

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&= \frac{-2x^2}{\sqrt{100 - x^2}} + 2\sqrt{100 - x^2}
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$$x = 5\sqrt{2} \qquad w = 2\sqrt{100 - x^2} = 2\sqrt{100 - (5\sqrt{2})^2} = 10\sqrt{2}$$

$$A_{max} = xw = (5\sqrt{2})(10\sqrt{2}) = 100$$

A poster is supposed to have margins of 1 inch on the left and right and 1.5 inches on top and on bottom. The printed area is supposed to be 54 square inches.