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The derivative of $f(x)g(x)$ is found by calculating the limit:

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This limit is:

$$f(x)g'(x) + g(x)f'(x)$$

# The Product Rule

The derivative of the product $f(x)g(x)$ is given by:

$$f(x)g'(x) + g(x)f'(x)$$

$$(fg)' = fg' + gf'$$

$$\frac{d}{dx}(fg) = f\frac{dg}{dx} + g\frac{df}{dx}$$