Find the equation of the curve that passes through the point $(0,\ 4)$ and has the following derivative:

$$\frac{dy}{dx} = \frac{2x}{x^2 + 1}$$

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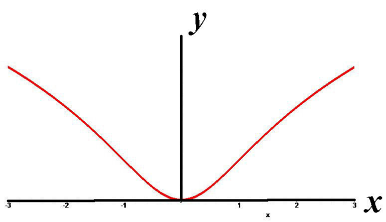
$$y = \int \frac{2x}{x^2 + 1}\, dx$$

$$\frac{dy}{dx} = \frac{2x}{x^2 + 1}$$

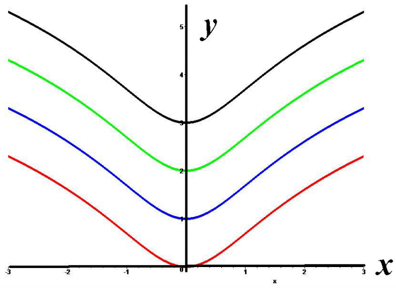
$$y = \int \frac{2x}{x^2 + 1}\, dx = \ln\left(x^2 + 1\right) + C$$

This is called the *general solution*.

$$y = \ln\left(x^2 + 1\right)$$



$$y = \ln\left(x^2+1\right) + C$$



$$y = \ln\left(x^2 + 1\right) + C$$

If $y(0) = 4$ then:

$$4 = \ln(0 + 1) + C = C$$

$$y = \ln\left(x^2 + 1\right) + 4$$

This is called a *particular solution*

An equation involving the derivative of an unknown function is called a *differential equation.*

$$\frac{dy}{dx} = \frac{2x}{x^2 + 1}$$

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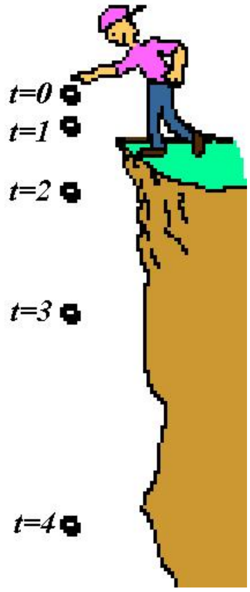
$$\frac{dy}{dx} = \frac{2x}{x^2 + 1}$$

$$dy = \frac{2x}{x^2 + 1}\, dx$$

Separation of variables.

Find the velocity of a falling object.



If $v$ is the velocity of a falling object then

$$\frac{dv}{dt} = -32$$

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$$v = \int -32\,dt = -32t + C$$

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$$v = \int -32\,dt = -32t + C$$

If we impose the initial condition that $v(0) = 2$ then

$$2 = (-32)(0) + C \quad \text{so} \quad C = 2$$

$$v = -32t + 2$$

$$\frac{dy}{dx} = \frac{-x}{4y}$$

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$$y = \int \frac{-x}{4y}\,dx = \mathbf{?\ ?\ ?}$$

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$$4y\,dy = -x\,dx$$

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$$\int 4y\,dy = -\int x\,dx$$

$$2y^2 = -\frac{1}{2}x^2 + C$$

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$$\int 4y\,dy = -\int x\,dx$$

$$2y^2 = -\frac{1}{2}x^2 + C$$

$C$ is determined if some initial condition is given. For example, $y(0) = 1$.

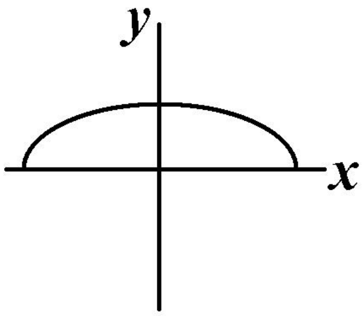
$$\frac{dy}{dx} = \frac{-x}{4y} \qquad y(0) = 1$$

$$2y^2 = -\frac{1}{2}x^2 + C$$

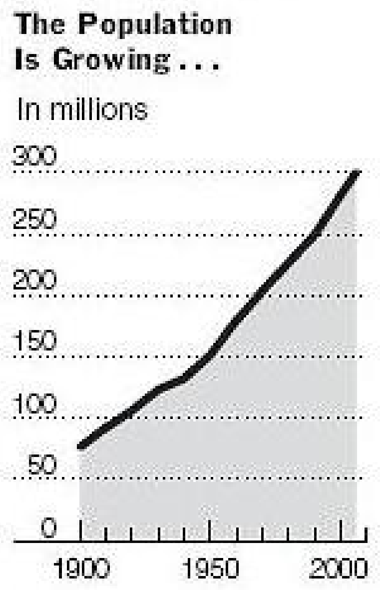
$$2 \cdot 1^2 = -\frac{1}{2} \cdot 0^2 + C \quad \text{implies} \quad C = 2$$

$$2y^2 = -\frac{1}{2}x^2 + 2$$

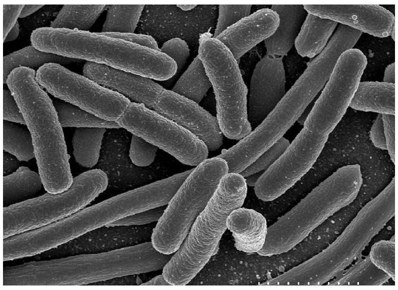
$$y = \sqrt{1 - \frac{x^2}{4}}$$



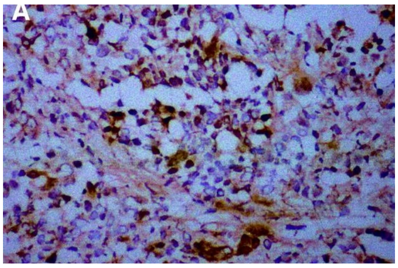
Differential equations come up in population growth problems



Differential equations come up in population growth problems



Differential equations come up in population growth problems



Malthusian Model of Population Growth:

Population grows at a rate proportional to the size of the population at any point in time.

Malthusian Model of Population Growth:

Population grows at a rate proportional to the size of the population at any point in time.

Let $P(t)$ denote the population at time $t$

$\frac{dP}{dt}$ is proportional to $P$ at any point in time.

$$\frac{dP}{dt} = kP \qquad \text{where } k \text{ is a constant}$$

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$$\frac{1}{P}\, dP = k\, dt$$

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$$\ln P = kt + C$$

$$P = e^{kt+C}$$

$$P = e^{kt+C} = e^{C}e^{kt}$$

Let $a = e^{C}$

$$P(t) = ae^{kt}$$

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Let $P_0$ denote the *initial population*

$$P_0 = ae^{k \cdot 0} = ae^0 = a \cdot 1 = a$$

$$P(t) = P_0 e^{kt}$$

A truck arrives at a flour storage facility. Hidden in the flour on the truck are 100 flour beetles which then find a home in the flour storage facility.

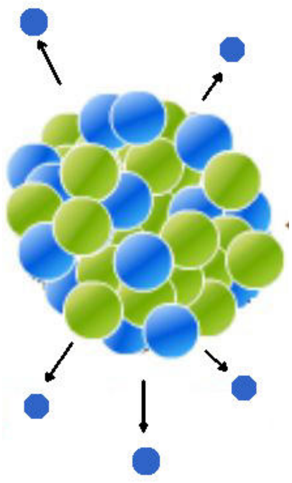
The population of beetles after $t$ days is given by:

$$P(t) = 100e^{kt}$$

For flour beetles, the value of $k$ is known to be 0.10

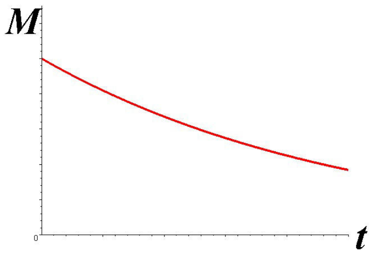
How long will it take for the beetle population to double?

The mass of a radioactive object decreases with time:

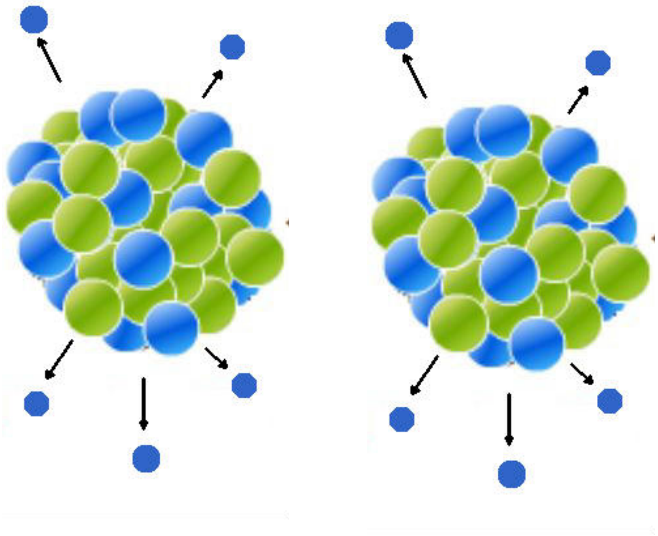


The mass of a radioactive object decreases with time:

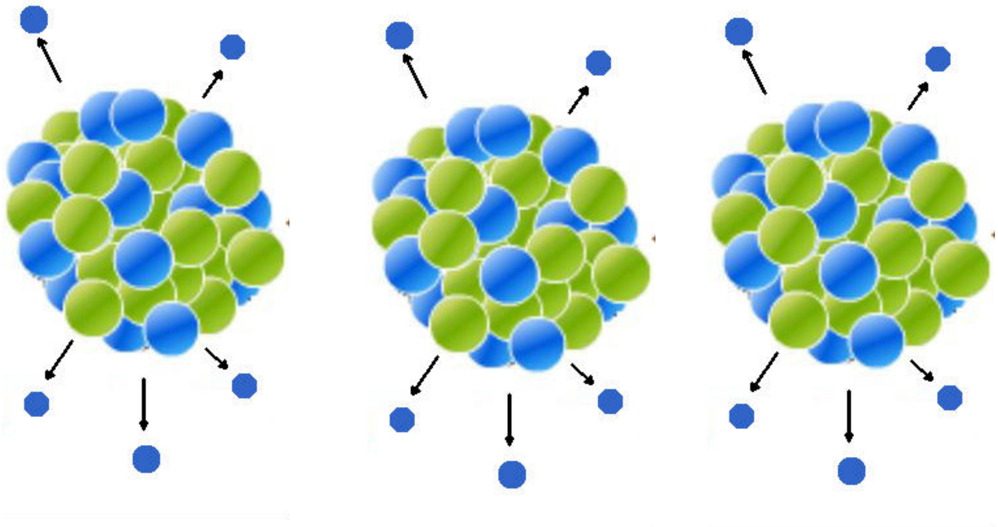
$$M = M(t)$$



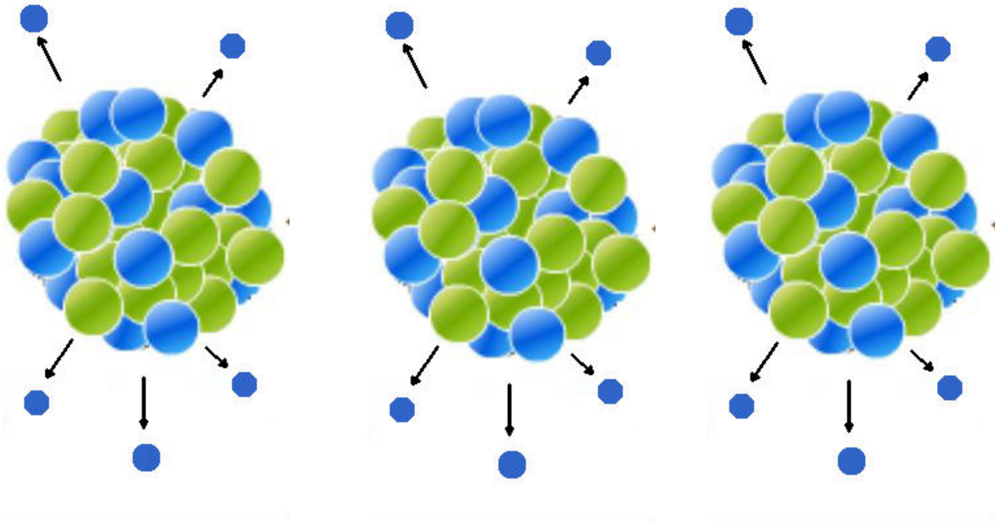
If we double the mass, we double the rate at which the mass decreases.



If we triple the mass, we triple the rate at which the mass decreases.

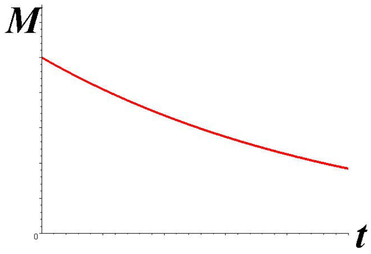


The rate at which the mass decreases is proportional to the mass at any given time.



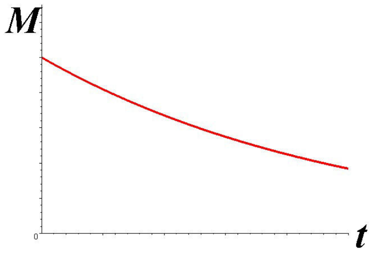
The rate at which the mass decreases is proportional to the mass at any given time.

$$\frac{dM}{dt} = \lambda M$$



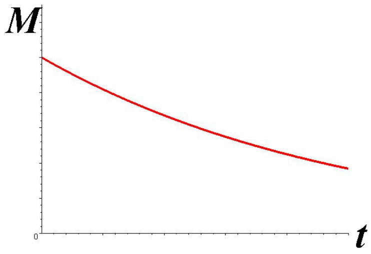
The rate at which the mass decreases is proportional to the mass at any given time.

$$\frac{dM}{dt} = \lambda M \qquad (\lambda \text{ is a negative number})$$



The rate at which the mass decreases is proportional to the mass at any given time.

$$\frac{dM}{dt} = -\lambda M \qquad (\lambda \text{ is a positive number})$$



The rate at which an object cools is proportional to the temperature difference between the object and its environment

The rate at which an object cools is proportional to the temperature difference between the object and its environment

Let $y(t)$ denote the temperature at time $t$. Let us suppose that the temperature of the environment is 25 (degrees Centigrade)

$$\frac{dy}{dt} \quad \text{is proportional to} \quad y - 25$$

$$\frac{dy}{dt} = k(y - 25)$$

$$\frac{dy}{dt} = k(y - 25)$$

$k$ must be a negative constant

$$\frac{dy}{dt} = -k(y - 25)$$

This way, $k$ will stand for a positive constant

$$\frac{dy}{dt} = -k(y - 25)$$

Suppose the temperature of an object is $100^\circ\, C$ at $t = 0$ and it cools to $75^\circ\, C$ at $t = 8$. Find the formula for the temperature $y = y(t)$