

If  $P(x)$  and  $Q(x)$  are polynomials, then  $\frac{P(x)}{Q(x)}$  is called a *rational function*.

If  $\deg(P(x)) \geq \deg(Q(x))$  then use the long division algorithm to simplify.

$$\frac{2x^2 + 6}{x^2 + 1} = 2 + \frac{4}{x^2 + 1}$$

$$2$$

$$x^2 + 1 \overline{) 2x^2 + 6}$$

$$\begin{array}{r} 2x^2 + 2 \\ \hline 4 \end{array}$$

$$\frac{2x^2 + 6}{x^2 + 1} = 2 + \frac{4}{x^2 + 1}$$

$$\begin{array}{r}
 2 \\
 \text{multiply} \swarrow \\
 x^2 + 1 \overline{) 2x^2 + 6} \\
 \phantom{x^2 + 1} \underline{2x^2 + 2} \phantom{0} \\
 \phantom{x^2 + 1} \phantom{2x^2 + 2} 4
 \end{array}$$

subtract

$$\frac{P(x)}{Q(x)} = M(x) + \frac{R(x)}{Q(x)}$$

The diagram illustrates the steps of polynomial long division:

- Initial Setup:**  $\mathbf{Q(x)} \overline{) P(x)}$ . An arrow labeled "multiply" points from  $\mathbf{M(x)}$  to  $\mathbf{Q(x)}$ .
- Multiplication:** An arrow points from  $\mathbf{Q(x)}$  to  $\mathbf{M(x)Q(x)}$ .
- Subtraction:** A curved arrow labeled "subtract" points from  $\mathbf{M(x)Q(x)}$  to the final result  $\mathbf{R(x)}$ .

$$\frac{P(x)}{Q(x)} = M(x) + \frac{R(x)}{Q(x)}$$

multiply

$$\begin{array}{r} \mathbf{M(x)} \\ \mathbf{Q(x)} \overline{) P(x)} \end{array}$$

$\mathbf{M(x)Q(x)}$

subtract

$$\mathbf{R(x) = P(x) - M(x)Q(x)}$$

$$P(x) - M(x) \cdot Q(x) = R(x)$$

$$P(x) = M(x) \cdot Q(x) + R(x)$$

$$\frac{P(x)}{Q(x)} = M(x) + \frac{R(x)}{Q(x)}$$

If  $\deg P(x) \geq \deg(Q(x))$

then you can use the *long division algorithm* to produce a polynomial  $R(x)$  with

$$\deg(R(x)) < \deg(Q(x))$$

such that

$$\frac{P(x)}{Q(x)} = M(x) + \frac{R(x)}{Q(x)}$$

Integration of Rational Functions:

$$\int \frac{P(x)}{Q(x)} dx = \int \left( M(x) + \frac{R(x)}{Q(x)} \right) dx$$