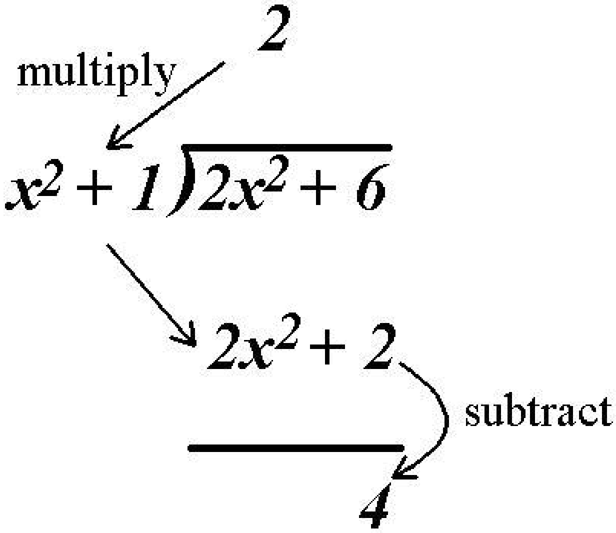
If $P(x)$ and $Q(x)$ are polynomials, then $\frac{P(x)}{Q(x)}$ is called a *rational function*.

If $\deg(P(x)) \geq \deg(Q(x))$ then use the long division algorithm to simplify.

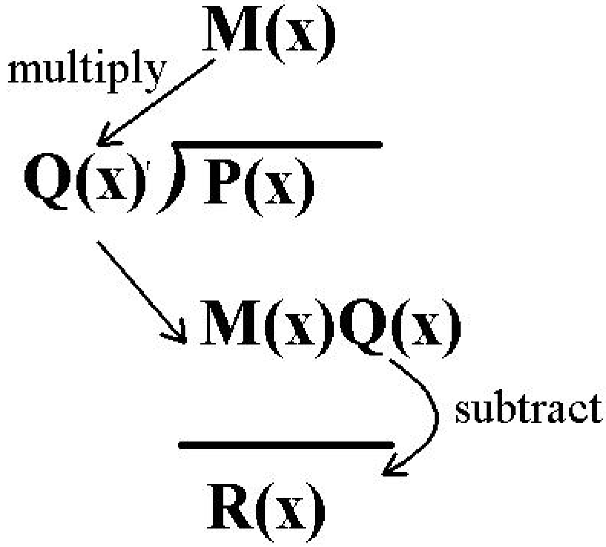
$$\frac{2x^2+6}{x^2+1} = 2 + \frac{4}{x^2+1}$$

$$\begin{array}{r l}
 & \quad\;\; 2 \\
x^2+1 & \big)\overline{2x^2+6} \\
 & \;\; 2x^2+2 \\
\hline
 & \qquad\quad 4
\end{array}$$

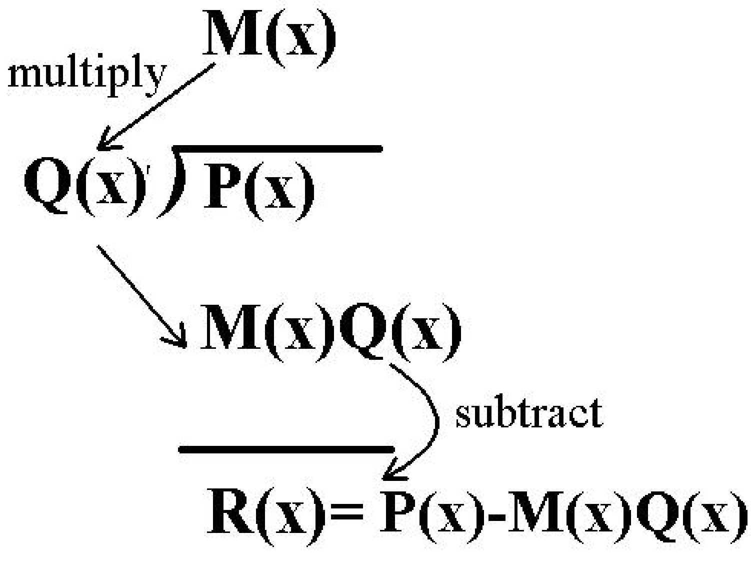
$$\frac{2x^2+6}{x^2+1} = 2 + \frac{4}{x^2+1}$$



$$\frac{P(x)}{Q(x)} = M(x) + \frac{R(x)}{Q(x)}$$



$$\frac{P(x)}{Q(x)} = M(x) + \frac{R(x)}{Q(x)}$$



$$P(x) - M(x) \cdot Q(x) = R(x)$$

$$P(x) = M(x) \cdot Q(x) + R(x)$$

$$\frac{P(x)}{Q(x)} = M(x) + \frac{R(x)}{Q(x)}$$

$$\text{If } \deg P(x) \geq \deg(Q(x))$$

then you can use the *long division algorithm* to produce a polynomial $R(x)$ with

$$\deg(R(x)) < \deg(Q(x))$$

such that

$$\frac{P(x)}{Q(x)} = M(x) + \frac{R(x)}{Q(x)}$$

Integration of Rational Functions:

$$\int \frac{P(x)}{Q(x)}\,dx = \int \left(M(x) + \frac{R(x)}{Q(x)} \right)\,dx$$