



Suppose we measure a tumor to be 2.00 cm along the major axis and 1.00 cm along the minor axis.

$$a_1 = 1.00$$
 $b_1 = 0.50$

Two months later, the tumor is measured to be 2.08 cm along the major axis and 1.10 cm along the minor axis.

$$a_2 = 1.04$$
 $b_2 = 0.55$

How long will it take for the mass of the tumor to double?

The Malthusian Law

$$\frac{dM}{dt} = \lambda M$$
$$M(t) = M_0 e^{\lambda t}$$

If τ is the doubling time then:

$$2M_0 = M_0 e^{\lambda \tau}$$
$$\tau = \frac{\ln 2}{\lambda}$$

$$\frac{M}{V} = \rho \quad (\text{density in gm/cm}^3)$$

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 (density in gm/cm³)
 $M = \rho V$

Let M_1 be the mass of the tumor at volume V_1 Let M_2 be the mass of the tumor at volume V_2

$$\frac{M_2}{M_1} = \frac{\rho V_2}{\rho V_1} = \frac{V_2}{V_1}$$

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$$\frac{M_0 e^{\lambda t_2}}{M_0 e^{\lambda t_1}} = \frac{\frac{4}{3}\pi a_2 b_2^2}{\frac{4}{3}\pi a_1 b_1^2}$$
$$e^{\lambda (t_2 - t_1)} = \frac{a_2 b_2^2}{a_1 b_1^2}$$

 $e^{\lambda(t_2-t_1)} = \frac{a_2 b_2^2}{a_1 b_1^2}$ $t_2 - t_1 = 2 \text{ months}$ $a_1 = 1.00 \qquad b_1 = 0.50$ $a_2 = 1.04 \qquad b_2 = 0.55$ $e^{2\lambda} = \frac{(1.04)(0.55)^2}{(1.00)(0.50)^2}$

$$e^{2\lambda} = \frac{(1.04)(0.55)^2}{(1.00)(0.50)^2}$$
$$\lambda = \frac{1}{2} \ln \left(\frac{(1.04)(0.55)^2}{(1.00)(0.50)^2} \right)$$
$$\tau = \frac{\ln 2}{\lambda} \approx 6.03 \text{ months}$$