Geometric Series

$$\sum_{n=0}^{\infty} r^n = 1 + r + r^2 + r^3 + r^4 + \cdots$$

The geometric series converges only if $-1 < r < 1$

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However:

$\lim_{n\to\infty} a_n = 0$ does not guarantee convergence

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$$

Converge or diverge?

The Integral Test (Published by Colin Maclaurin in 1742 *Treatise on Fluxions*)

If $f(x)$ is a continuous, positive and decreasing on $[1, \infty)$ then the following equivalence is true:

$$\sum_{n=1}^{\infty} f(n) < \infty \text{ if and only if } \int_1^{\infty} f(x)\, dx < \infty$$