Assignment 1. *Integration Review*

Read Section 5.5
You should be able to do the following problems:
Section 5.5/Problems 1 - 7, 9 - 15, 18 - 20
Hand in the following problems:

Problems 1 - 5. Do the following integrals using the method of variable substitution. Be careful to change your limits of integration to conform to your new variable when you do the definite integrals.

1.
$$\int \frac{1}{(x+1)^2}\, dx$$

2.
$$\int_0^1 \frac{x}{(x+1)^3}\, dx$$

3.
$$\int_0^{1/2} x \, \cos\left(2\pi x^2\right)\, dx$$

4.
$$\int_0^{\pi/2} \sin\theta \cos\theta \, d\theta$$

5.
$$\int_0^{\sqrt{2}} \frac{x}{\sqrt{4-x^2}}\, dx$$

Assignment 2. *The Natural Logarithm - Review*

Read Section 3.6

You should be able to do the following problems:

Section 3.6/Problems 1 - 22, Section 5.5/Problems 20 - 21, 56, 69

Hand in the following problems:

Problems 1 - 5. Find $\frac{dy}{dx}$ for each of the following functions. Simplify each answer as much as possible.

1.
$$y = \ln\left(2\sqrt{x}\right)$$

2.
$$y = -x + x\ln x$$

3.
$$y = \ln(\csc x + \cot x)$$

4.
$$y = \ln\sqrt{\frac{1+x}{1-x}}$$

5.
$$y = \frac{\ln\, x}{x}$$

Problems 6 - 10. Evaluate the integrals:

6.
$$\int_0^{\pi/4} \frac{\cos 2x}{1+\sin 2x}\, dx$$

7.
$$\int \frac{3\sqrt{x}}{1+x\sqrt{x}}\, dx$$

8.
$$\int_1^e \frac{2x^2+1}{x}\, dx$$

9.
$$\int_1^e \frac{\ln x}{x}\, dx$$

10.
$$\int_0^{\pi} \frac{x}{x+\pi}\, dx$$

Assignment 3. *The Exponential Function - Review*

Read Section 3.6

You should be able to do the following problems:

Section 3.4/Problems 23 - 25, 28 - 29, 33 - 36, 38 - 39, 49 - 50, 83 - 84, Section 3.6/Problems 34, 39 - 50

Hand in the following problems:

Problem 1. Find $\frac{dy}{dx}$. Simplify each answer as much as possible.

a. $y = e^{2\sqrt{x}}$ **b.** $y = \ln(xe^x)$

2. Use calculus to find the value of x that maximizes the following function.

$$f(x) = (8x - 1)e^{-4x}$$

Problems 3 - 5. Evaluate the integrals:

3.
$$\int \frac{e^x}{(1+e^x)^2}\, dx$$

4.
$$\int_{-1}^{1} \frac{e^{2x}-1}{2e^x}\, dx$$

5.
$$\int \frac{e^{2x}}{e^{2x}+1}\, dx$$

Assignment 4. *Natural Growth and Decay, Differential Equations*

Read Section 3.7, 3.8 and 9.3
You should be able to do the following problems:
Section 3.8/Problems 1 - 4, 8 - 10, 13 - 16, Section 9.3/Problems 1 - 2, 11, 16, 19
Hand in the following problems:

Problems 1 and 2. Find the solution of the following differential equations:

$$\textbf{1.}\quad \frac{dy}{dx} = \frac{x}{x+1} \qquad\qquad \textbf{2.}\quad \frac{dy}{dx} = -yx^{-2}$$

3. In 1960 the American scientist W. F. Libby won the Nobel prize for his discovery of carbon dating, a method for determining the age of certain fossils. Carbon dating is based on the fact that nitrogen is converted to radioactive carbon-14 by cosmic radiation in the upper atmosphere. This radioactive carbon is absorbed by plant and animal tissue through the life processes while the plant or animal lives. However, when the plant or animal dies the absorption process stops and the amount of carbon-14 decreases through radioactive decay. The half-life for carbon-14 is 5,750 years. Calculate how long it will take for a 2 gram sample of carbon-14 to decay down to $\sqrt{2}$ grams.

4. Let $x(t)$ be the number of malignant cells in the body of a laboratory rat after t months. Assume that the rate of growth of these cells is given by the equation: $\frac{dx}{dt} = \frac{1}{4}x$

a. Find the general solution of the differential equation.
b. Suppose that the number of cells is 10,000 at $t = 0$. Calculate how many months it will take for the number of cells to double.
c. At the point in time when the number of cells is 20,000, calculate the instantaneous rate of change (in cells per month) of the number of malignant cells with respect to time.

5. The interior of a new car typically has a considerable amount of volatile organic compounds such as benzene compounds and alkanes. However, as time goes on, the mass of these compounds will steadily decrease. Let $x(t)$ be the mass of organic compounds inside a new car after t weeks. Assume that the rate of decrease of the mass of these compounds is given by the differential equation: $\frac{dx}{dt} = -\lambda x$ where λ is a positive constant

a. Find the general solution of the differential equation. Show all work.
b. It has been determined experimentally that $\lambda = \ln \frac{5}{4}$. If a new car starts with 10 grams of organic compounds in the interior, how many weeks will it take for the level of organic compounds to decrease to 8 grams?

Assignment 5. *Derivatives and Integrals Involving Inverse Trigonometric Functions*

Read Section 3.5

You should be able to do the following problems:

Section 3.5/Problems 49 - 60, Section 5.5/Problems 70

Hand in the following problems:

Problems 1 - 3. Find $\frac{dy}{dx}$

1. $$y = \arctan \frac{x}{2}$$

2. $$y = \sin(\arccos x)$$

3. $$y = x \cdot \arctan(2x) - \frac{1}{4} \ln\left(1 + 4x^2\right)$$

4. Let $y = \arcsin(\sqrt{x})$. Find $\frac{dy}{dx}$ at the point $x = \frac{1}{2}$.

5 - 10. Evaluate the integrals:

5. $$\int \frac{1}{\sqrt{1 - 2x^2}}\, dx$$

6. $$\int \frac{x}{\sqrt{1 - 2x^2}}\, dx$$

7. $$\int_2^{2\sqrt{2}} \frac{1}{t\sqrt{2t^2 - 4}}\, dt$$

8. $$\int \frac{e^x}{1 + 4e^{2x}}\, dx$$

9. $$\int \frac{x + 2}{4x^2 + 4}\, dx$$

10. $$\int \frac{4x^2 - 4x + 2}{2x^2 + 1}\, dx$$

Assignment 6. *Integration of Trigonometric Functions*
Read Section 7.2
You should be able to do the following problems:
Section 7.2/Problems 1 - 49, 51 - 52, 61 - 64
Hand in the following problems:

Problems 1 - 5. Perform the indicated integrations:

1. $\displaystyle\int_0^{\pi} \cos^4 \frac{\theta}{2} d\theta$ **2.** $\displaystyle\int \frac{\sin^3 \theta}{\cos^2 \theta}\, d\theta$ **3.** $\displaystyle\int_0^{\pi/4} \sin^2(2\theta)\, d\theta$

4. $\displaystyle\int \cos\theta \cot\theta \sec\theta\, d\theta$ **5.** $\displaystyle\int_0^{\pi/6} \sec^3 \theta \tan \theta\, d\theta$

Assignment 7. *The Hyperbolic and Inverse Hyperbolic Functions*
Read Section 3.11
You should be able to do the following problems:
Section 3.11/Problems 1 - 18, 30 - 45
Hand in the following problems:

1. Find $\frac{dy}{dx}$ for each of the following functions:

a. $y = \ln(\cosh x)$ **b.** $y = \ln(\cosh x + \sinh x)$

Problems 2 - 5. Evaluate the integrals. Make sure you simplify the answers as much as possible.

2. $$\int \sinh \frac{x}{2} \cosh \frac{x}{2}\, dx$$

3. $$\int \sinh^2 x\, dx$$

4. $$\int_{-1}^{1} \sinh^3 x\, dx$$

5. $$\int \operatorname{sech}^3 x \tanh x\, dx$$

Assignment 8. *Trigonometric Substitution*

Read Section 7.3

You should be able to do the following problems:

Section 7.3/Problems 1 - 30, 37 - 38, 41

Hand in the following problems:

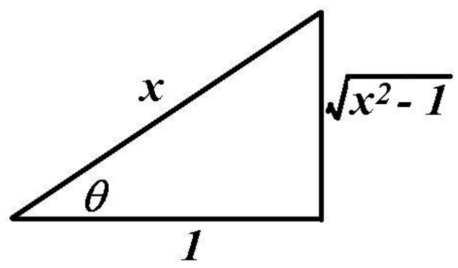
Problems 1 - 3. Perform the indicated integrations:

$$\textbf{1.}\quad \int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}}\,dx \qquad \textbf{2.}\quad \int \frac{1}{x^2\sqrt{1-4x^2}}\,dx \qquad \textbf{3.}\quad \int \frac{1}{\left(\sqrt{\pi^2+x^2}\right)^3}\,dx$$

4. Find the area of the region to the right of the y axis that is bounded by the curves $f(x) = \frac{2}{\sqrt{4-x^2}}$ and $g(x) = \sqrt{4-x^2}$.

5. The triangle given below suggests a trigonometric substitution to use on the following integral. Use this trigonometric substitution to calculate the integral.

$$\int \frac{1}{x^3\sqrt{x^2-1}}\,dx$$



Assignment 9. *Volume*

Read Section 6.2 - 6.3

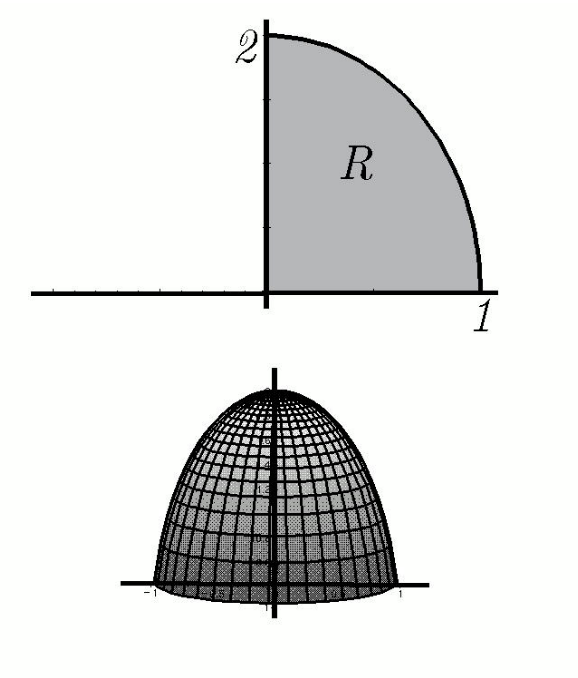
You should be able to do the following problems:

Section 6.2/Problems 1 - 18, 48 - 49, Section 6.3/Problems 3 - 20, 37 - 43

Hand in the following problems:

1. Let Q be the region above the x-axis and below the curve $y = \frac{1}{2-x}$ between $x = 0$ and $x = 1$. Find the volume of the solid generated if Q is revolved around the x-axis.

2. Let $\mathcal{R}$ be the region bounded by the y-axis, the x-axis and the curve $y = 2\sqrt{1-x^2}$. Note that this curve can also be described by the equation $x = \sqrt{1 - \frac{y^2}{4}}$



a. Calculate the volume of the solid of revolution, using the *Disk Method.*

b. Once again, calculate the volume but this time use the *Shell Method.*

3. Let Ω be the region under the curve $y = \frac{1}{\sqrt{x+1}}$ and above the x-axis between $x = 0$ and $x = 1$.

a. Find the volume of the solid of revolution generated if region Ω is revolved around the x-axis.

b. Find the volume of the solid of revolution generated if the region Ω is revolved around the vertical line $x = -1$.

4. Let $\mathcal{D}$ be the region above $y = \sinh x$ and below $y = \cosh x$ between $x = 0$ and $x = 1$.

a. Find the volume of the solid generated if $\mathcal{D}$ is revolved around the x-axis.

b. Find the volume of the solid generated if $\mathcal{D}$ is revolved around the y-axis.

Assignment 10. *Integration by Parts*

Read Section 7.1
You should be able to do the following problems:
Section 7.1/Problems 1 - 46, 51 - 52, 61 - 63
Hand in the following problems:

Problems 1 - 3. Evaluate the integrals:

1.
$$\int \frac{1}{\sqrt{x}} \ln x \, dx$$

2.
$$\int x \arctan x \, dx$$

3.
$$\int_{-\pi}^{\pi} e^x \sin x \; dx$$

4. Let R be the region enclosed by $y = \ln x$, the line $x = e$, and the x-axis. Find the volume of the solid generated when the region R is revolved about the x-axis.

5. Let $\mathcal{D}$ be the region below $y = \cos x$ and above the x-axis for $0 \leq x \leq \frac{\pi}{2}$. Find the volume of the solid generated when the region $\mathcal{D}$ is revolved around the line $x = \frac{\pi}{2}$

Assignment 11. *Arc Length and Surface Area*

Read Section 8.1 - 8.2, 10.1 - 10.2
You should be able to do the following problems:
Section 8.1/Problems 7-20 Section 8.2/Problems 5-16 Section 10.2/Problems 41-44, 61-66
Hand in the following problems:

1. Each of the following curves are described in the form $y = f(x)$. Calculate the arclength of the indicated intervals.

a. $y = \cosh x$ for $0 \leq x \leq \ln 2$.

b. $y = \frac{2x\sqrt{x}}{3}$ for $0 \leq x \leq 15$

2. The following curve is described parametrically. Calculate the arclength.

$$x = \sin t - \cos t \quad \text{and} \quad y = \sin t + \cos t \quad \text{for } \frac{\pi}{4} \leq t \leq \frac{\pi}{2}$$

3. Each of the following curves are described in the form $y = f(x)$. In each case, calculate the area of the surface generated if the curve is revolved around the x-axis.

a. $y = 2\sqrt{x+1}$ for $1 \leq x \leq 10$

b. $y = \sqrt{2x - x^2}$ for $0 \leq x \leq 2$

4. The following curve is described parametrically. Calculate the area of the surface generated if the curve is revolved around the x-axis.

$$x = e^t \quad \text{and} \quad y = \frac{1}{3}e^{3t} \quad \text{for } 0 \leq t \leq \frac{\ln 3}{4}$$

5. Let C be the segment of the curve $y = \cosh x$ for $0 \leq x \leq 1$. Find the surface area of the surface generated if C is revolved around the y-axis.

Assignment 12. *Partial Fractions*

Read Section 7.4
You should be able to do the following problems:
Section 7.4/Problems 1 - 38
Hand in the following problems:

Problems 1 - 5. Perform the integrations:

1. $$\int \frac{1}{x^3 - x}\,dx$$

2. $$\int \frac{1-x}{x^2(1+x)}\,dx$$

3. $$\int \frac{2x^2+4x}{(x+1)^3}\,dx$$

4. $$\int_0^1 \frac{2x^2+8x-1}{(x+2)(2x^2+1)}\,dx$$

5. $$\int \frac{1}{(x+1)^2 x}\,dx$$

Assignment 13. *Numerical Approximation of Integrals*

Read Section 7.7
You should be able to do the following problems:
Section 7.7/Problems 7 - 18 (parts a and c only)
Hand in the following problems:

1. A rod is 1 meter long. If the density of the rod at a point x meters from the left end is $f(x)$ kilograms per meter, then the mass of the rod is given by $\int_0^1 f(x)\,dx$.

Suppose we do not know the formula for $f(x)$, but we have measured the density at different values of x:

$$f(0) = \frac{5}{4} \qquad f\left(\frac{1}{2}\right) = \frac{3}{2} \qquad f(1) = \frac{1}{4}$$

Use the *Trapezoidal Rule* to estimate $\int_0^1 f(x)\,dx$ with $n = 2$ subintervals.

Problems 2 - 4. The following four problems all involve the calculation of the integral:

$$\int_0^{1/4} \arcsin\sqrt{x}\,dx$$

2. Approximate the integral using the Trapezoidal Rule and $n = 4$.

3. Approximate the integral using Simpson's Rule and $n = 4$.

4. Use the methods of antidifferentiaion that you have learned to find the exact value of the integral.

Assignment 14. *Work*

Read Section 6.4

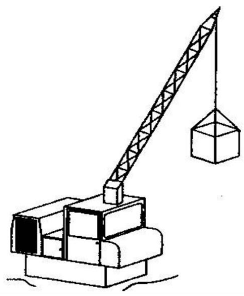
You should be able to do the following problems:

Section 6.4/Problems 7 - 16, 21 - 24, 27, 29

Hand in the following problems:

1. A spring exerts a force of 12 newtons when it is stretched 2 meters beyond its natural length. What is the work done if this spring is stretched from its natural length to this point?

2. A crane is hauling up an object weighing 20 newtons. The top of the crane is 10 meters above the ground. The cable weighs 2 newtons/meter. How much work is done in hauling the object from the ground up to a height of 10 meters?



3. If two oppositely charged electric particles are separated by a distance of x meters, the force of attraction between them is given by:

$$F = \frac{kq_1q_2}{x^2}$$

where k, q_1 and q_2 are constants. If we start with a distance of a meters between the charges, calculate the work done if we move the charges apart until they are a distance of $2a$ meters apart.

4. The pressure P of a gas is related to its volume V by the formula $P = \frac{k}{V}$ where k is a constant. A quantity of gas with an initial volume of 1 cubic meter and a pressure of 2.5 newtons per square meter expands to a volume of 16 cubic meters. Find the work done by the gas.

Assignment 15. *Improper Integrals*

Read Section 7.8

You should be able to do the following problems:

Section 7.8/Problems 1 - 59

Hand in the following problems:

Problems 1 - 2. Determine which of the following integrals converge. If the integral diverges, justify your answer. If the integral converges, find the value that it converges to.

1. a) $\displaystyle\int_0^\infty \frac{1}{(\pi^2+x^2)^{3/2}}\,dx$ b) $\displaystyle\int_2^\infty \frac{3}{(x-1)^4}\,dx$

2. a) $\displaystyle\int_0^9 \frac{dx}{\sqrt{9-x}}$ b) $\displaystyle\int_1^2 \frac{3}{(x-1)^4}\,dx$

3. Let R be the region below $f(x) = 12\,(x+2)^{-2}$ and above $g(x) = 3e^{-3x}$ for $x \geq 0$. The region R is not bounded to the right.

Represent the area of R as an improper integral and then evaluate it.

4. Evaluate the following improper integral:

$$\int_1^\infty \frac{4}{x^2\,(x^2+1)}\,dx$$

5. Let R be the region between the x-axis and the curve $y = f(x)$ for $x \geq 1$. If R is revolved around the x-axis, a solid of revolution is produced that extends from $x = 1$ to infinity.

The volume of this unbounded solid can be easily expressed using the Disk Method to obtain $V = \int_1^\infty \pi(f(x))^2\,dx$, which either converges or diverges depending on $f(x)$. Perform the calculation of this improper integral for each of the following functions:

a. $f(x) = \frac{1}{\sqrt{x}}$ **b.** $f(x) = \frac{1}{e^{x/2}}$

Assignment 16. *L'Hôpital's Rule*

Read Section 4.4

You should be able to do the following problems:

Section 4.4/Problems 7 - 66, 76 - 78

Hand in the following problems:

Find the limits:

1. $$\lim_{x\to\infty} \frac{1+2xe^x}{x+e^{2x}}$$

2. $$\lim_{x\to 2\pi} \frac{\ln(\cos(x))}{(x-2\pi)^2}$$

3. $$\lim_{x\to\infty} x\ln\left(1+\frac{7}{x}\right)$$

4. $$\lim_{x\to 0} \frac{x+e^{-x}-1}{\pi^2x^2-\pi x+\sin(\pi x)}$$

5. $$\lim_{x\to\infty} \left(1-\frac{1}{2x}\right)^{2x}$$

Assignment 17. *Geometric Series*

Read Section 11.1, 11.2
You should be able to do the following problems:
Section 11.2/Problems 17 - 28, 31 - 34, 37, 57 - 61, 63
Hand in the following problems:

1. Determine which of the given geometric series converge. For the series that do converge, determine the exact number or expression that the series converges to.

a.
$$\frac{9}{10} + \frac{9}{100} + \frac{9}{1,000} + \cdots$$

b.
$$1 + \frac{e}{2} + \frac{e^2}{4} + \frac{e^3}{8} + \cdots$$

c.
$$\frac{1}{\pi^2} - \frac{1}{\pi^4} + \frac{1}{\pi^6} - \frac{1}{\pi^8} + \cdots$$

d.
$$\left(1 - \frac{e}{\pi}\right) + \left(1 - \frac{e}{\pi}\right)^2 + \left(1 - \frac{e}{\pi}\right)^3 + \cdots$$

For problems **2 - 4** the series converge for some values of x and diverge for other values of x. Determine the values of x for which the series converge and find the expressions that the series converge to.

2.
$$\sum_{n=1}^{\infty} 2^n x^n = 2x + 4x^2 + 8x^3 + 16x^4 + \cdots$$

3.
$$1 + \frac{x}{2e} + \frac{x^2}{4e^2} + \frac{x^3}{8e^3} + \cdots$$

4.
$$\frac{1}{e^x} + \frac{1}{e^{2x}} + \frac{1}{e^{3x}} + \frac{1}{e^{4x}} + \cdots$$

5. Find an infinite series for arctanh x by using the fact that

$$\operatorname{arctanh} x = \int_0^x \frac{dt}{1-t^2}$$

Use this series to estimate arctanh(0.50) by substituting $x = 0.50$ into your series and adding up the first four nonzero terms.

Assignment 18. *Taylor Series*

Read Section 11.8, 11.9, 11.10, 11.11
You should be able to do the following problems:
Section 11.10/Problems 5 - 29
Hand in the following problems:

Problem 1. Each of the following functions have series of the form $\sum_{n=0}^{\infty} c_n x^n$. Calculate the coefficients c_n and express the power series in summation notation.

a. $f(x) = \dfrac{\pi x}{\pi x + 1}$ **b.** $f(x) = 1 - \ln(e - x)$ **c.** $f(x) = x\, e^x$

Problem 2. Calculate c_0, c_1, c_2, c_3 and c_4 for the power series $\sum_{n=0}^{\infty} c_n x^n$ that represents $f(x) = \tan x$. Use the first two nonzero terms of the series to approximate the tangent of $\frac{1}{4}$ radian. Compare your approximation with the actual value of the tangent of $\frac{1}{4}$ radian as shown on a calculator.

Problem 3. Start with the Maclaurin series for $\cosh x$ and use this to generate an infinite series for the following integral. Express your answer in $\sum$ notation.

$$\int_0^x t^2 \cosh\left(t^2\right)\, dt$$

Problem 4. The function $\ln(x-1)$ can be expressed as a Taylor series of the form $\sum c_n (x-2)^n$. Calculate the formula for c_n

Note: You will have to consider the case of c_0 separately from the case of c_n where $n > 0$.

Assignment 19. *Convergence Tests*

Read Section 11.1 to 11.7
You should be able to do the following problems:
Section 11.3/Problems 1 - 32, Section 11.4/Problems 3 - 32, Section 11.5/Problems 2 - 20, Section 11.6/Problems 2 - 26, 35
Hand in the following problems:

1. For each of the infinite series that you calculated in problem 1 of Assignment 18, determine the values of x for which the series converge.

2. For each of the following infinite series, use one of the convergence tests (Ratio Test, Integral Test) to determine convergence. Justify your conclusion in each case.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$ **b.** $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$ **c.** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n + \sqrt{n}}$ **d.** $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{(2n)!}$

3. If $\sum |a_n|$ converges then $\sum a_n$ is said to be *absolutely convergent*. If $\sum a_n$ converges but $\sum |a_n|$ does not converge then $\sum a_n$ is said to be *conditionally convergent*. For each of the following infinite sums, classify the sum as *divergent*, *conditionally convergent* or *absolutely convergent*.

a. $\displaystyle\sum_{n=0}^{\infty} (-1)^n$ **b.** $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n\pi}{n}$ **c.** $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}}$ **d.** $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^n}{n!}$