Two functions are said to be *inverses* of each other if the functions cancel.

Function	Inverse
Multiply by 2	

Two functions are said to be *inverses* of each other if the functions cancel.

Function	Inverse
Multiply by 2	Divide by 2
Add 7	

Two functions are said to be *inverses* of each other if the functions cancel.

Function	Inverse
Multiply by 2	Divide by 2
Add 7	Subtract 7
Cube the number	

Two functions are said to be *inverses* of each other if the functions cancel.

Function	Inverse
Multiply by 2	Divide by 2
Add 7	Subtract 7
Cube the number	Take a cube root

Two functions are said to be *inverses* of each other if the functions cancel.

Function	Inverse
$f(x) = 2x$	$g(x) = \frac{x}{2}$
$f(x) = x + 7$	$g(x) = x - 7$
$f(x) = x^3$	$g(x) = \sqrt[3]{x} = x^{1/3}$

Two functions are said to be *inverses* of each other if the functions cancel. $g(f(x)) = x$ for all x

Function	Inverse
$f(x) = 2x$	$g(x) = \frac{x}{2}$
$f(x) = x + 7$	$g(x) = x - 7$
$f(x) = x^3$	$g(x) = \sqrt[3]{x} = x^{1/3}$

$$f(x) = 2x \qquad g(x) = \frac{x}{2}$$

$$g(f(x)) = \frac{f(x)}{2} = \frac{2x}{2} = x$$

$$g(f(x)) = x \quad \text{for all } x$$

$$f(x) = x + 7 \qquad\qquad g(x) = x - 7$$

$$g(f(x)) = f(x) - 7 = (x + 7) - 7 = x$$

$$g(f(x)) = x \quad \text{for all } x$$

Formal definition of inverse:

Two functions, f and g are said to be inverses of each other if:

$$f(g(x)) = x \quad \text{for all } x \text{ in the domain of } g$$

and

$$g(f(x)) = x \quad \text{for all } x \text{ in the domain of } f$$

Notation:

If f and g are inverses of each other, then we use the notation $f^{-1}(x)$ to mean $g(x)$

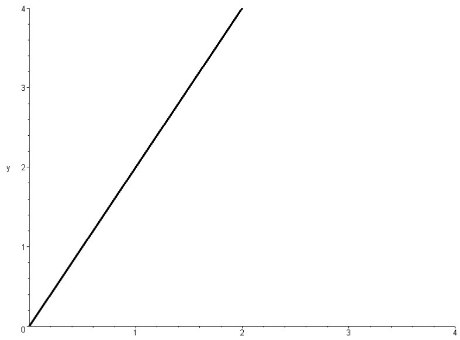
Function	Inverse
$f(x) = 2x$	$f^{-1}(x) = \frac{x}{2}$
$f(x) = x + 7$	$f^{-1}(x) = x - 7$
$f(x) = x^3$	$f^{-1}(x) = \sqrt[3]{x} = x^{1/3}$

Warning

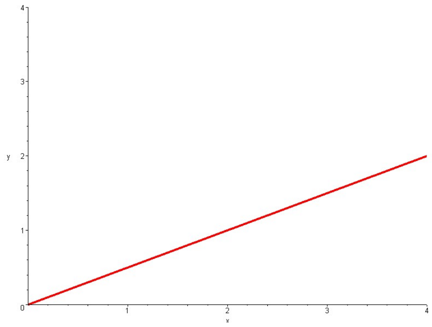
Do not confuse $f^{-1}(x)$ with $(f(x))^{-1} = \frac{1}{f(x)}$

Function	Inverse	Reciprocal
$f(x) = 2x$	$f^{-1}(x) = \frac{x}{2}$	$\frac{1}{f(x)} = \frac{1}{2x}$
$f(x) = x + 7$	$f^{-1}(x) = x - 7$	$\frac{1}{f(x)} = \frac{1}{x+7}$
$f(x) = x^3$	$f^{-1}(x) = \sqrt[3]{x}$	$\frac{1}{f(x)} = \frac{1}{x^3} = x^{-3}$

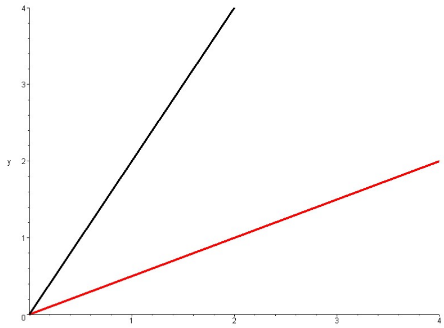
$$f(x) = 2x$$



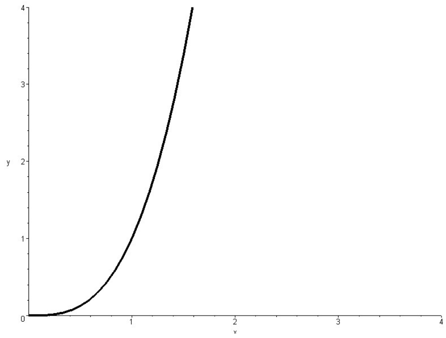
$$f^{-1}(x) = \frac{x}{2}$$



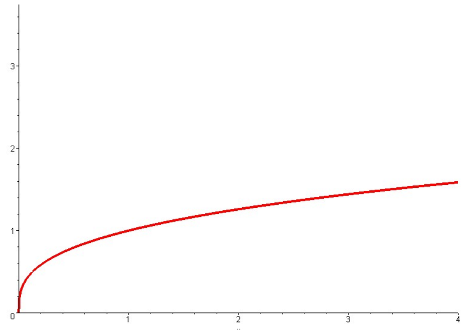
$$f(x) = 2x \qquad\qquad f^{-1}(x) = \frac{x}{2}$$



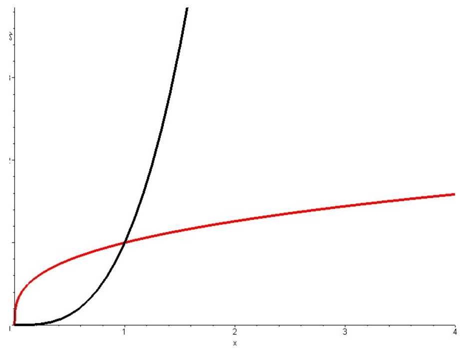
$$f(x) = x^3$$



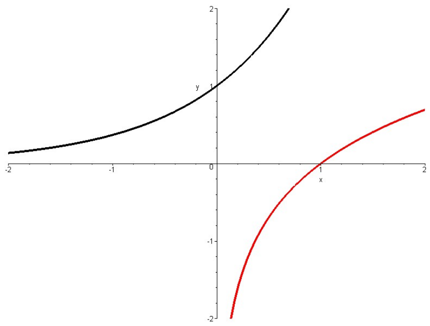
$$f^{-1}(x) = \sqrt[3]{x}$$



$$f(x) = x^3 \qquad\qquad f^{-1}(x) = \sqrt[3]{x}$$



If the black graph is $y = f(x)$ then the red graph is $y = f^{-1}x$



$$y = f^{-1}(x)$$

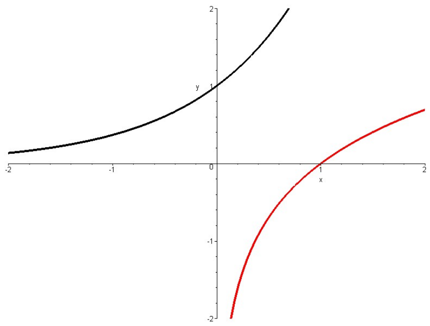
$$f(y) = f\left(f^{-1}(x)\right)$$

$$y = f^{-1}(x)$$

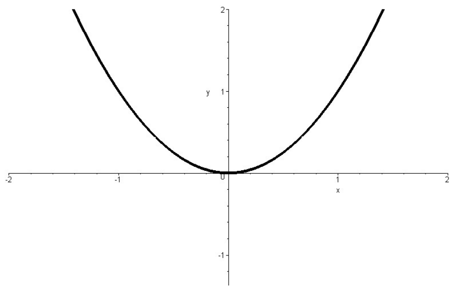
$$f(y) = f\left(f^{-1}(x)\right)$$

$$f(y) = x$$

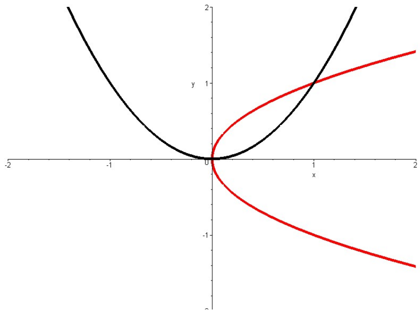
The graph of $y = f^{-1}(x)$ is really the graph of $x = f(y)$



$$f(x) = x^2$$



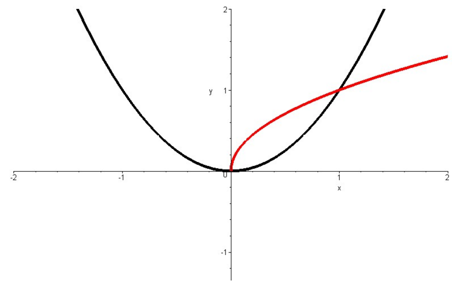
$$y = x^2 \qquad\qquad x = y^2$$



$$y = f(x)$$

If y is a function of x, this is supposed to mean that for every x there is **one unique** y value.

$$f(x) = x^2 \qquad\qquad g(x) = \sqrt{x}$$



$$f(x) = x^2 \qquad\qquad g(x) = \sqrt{x}$$

Will $g(f(x))$ equal x ?

$$f(x) = x^2 \qquad\qquad g(x) = \sqrt{x}$$

Will $g(f(x))$ equal x ?

$$g(f(1)) = g(1) = \sqrt{1} = 1 \qquad \text{Yes}$$

$$g(f(2)) = g(4) = \sqrt{4} = 2 \qquad \text{Yes}$$

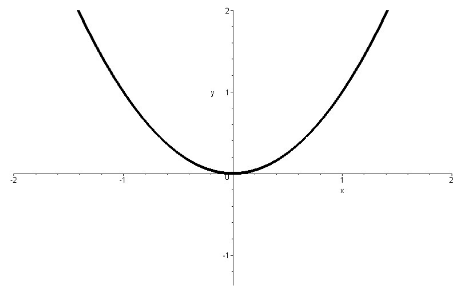
$$g(f(3)) = g(9) = \sqrt{9} = 3 \qquad \text{Yes}$$

$$f(x) = x^2 \qquad\qquad g(x) = \sqrt{x}$$

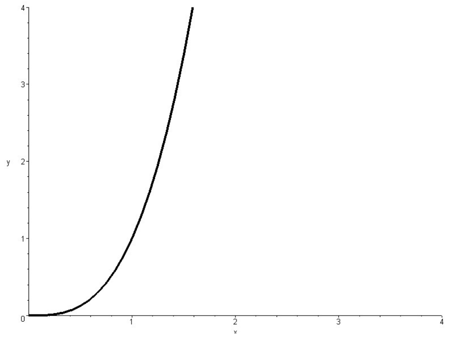
Will $g(f(x))$ equal x ?

$$g(f(-1)) = g(1) = \sqrt{1} = 1 \qquad \text{No}$$
$$g(f(-2)) = g(4) = \sqrt{4} = 2 \qquad \text{No}$$
$$g(f(-3)) = g(9) = \sqrt{9} = 3 \qquad \text{No}$$

If $f(a) = f(b)$ even though $a \neq b$ then the function f is said to be *many-to-one*. Such functions do not have inverses.

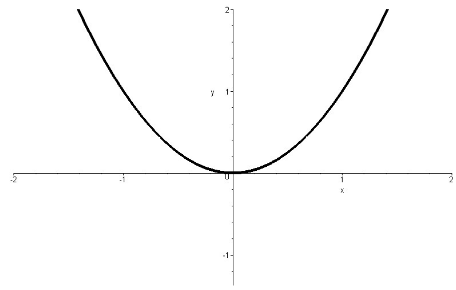


If $a \neq b$ implies that $f(a) \neq f(b)$ then the function f is said to be *one-to-one*. This is the only type of function that will have an inverse.

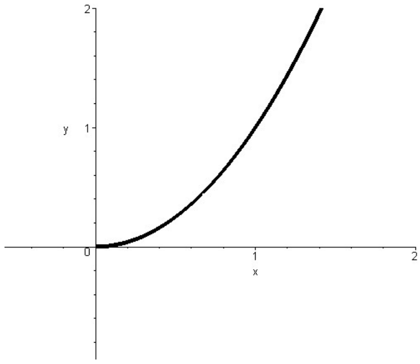


If a function is not *one-to-one*, we can restrict the domain so that it is one-to-one.

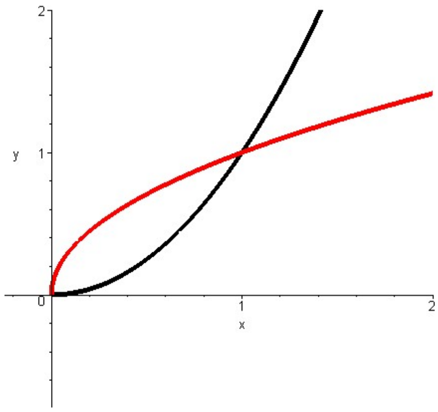
$f(x) = x^2$ with unrestricted domain



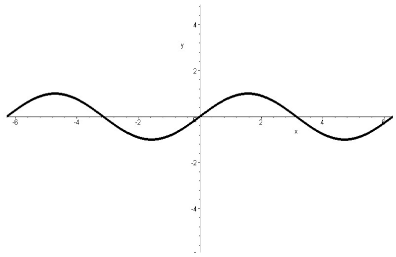
$$f(x) = x^2 \qquad \text{with } x \geq 0$$



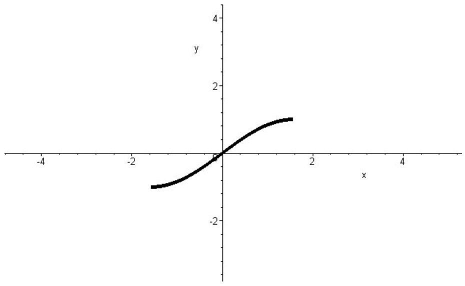
$f(x) = x^2 \quad \text{for } x \geq 0$ $\qquad\qquad$ $f^{-1}(x) = \sqrt{x}$



The sine function is not one-to-one



The domain of the sine function can be restricted so that it is one-to-one



$f(x) = \sin x \qquad\qquad g(x) = \sin^{-1} x$