

First Partial Fractions Decomposition Theorem:

If $P(x)$, $Q_1(x)$ and $Q_2(x)$ are polynomials with

$$\deg(P(x)) < \deg(Q_1(x)Q_2(x))$$

and with $Q_1(x)$ and $Q_2(x)$ relatively prime then there are polynomials $R_1(x)$ and $R_2(x)$ with

$$\deg(R_1(x)) < \deg(Q_1(x)) \quad \text{and} \quad \deg(R_2(x)) < \deg(Q_2(x))$$

such that:

$$\frac{P(x)}{Q_1(x)Q_2(x)} = \frac{R_1(x)}{Q_1(x)} + \frac{R_2(x)}{Q_2(x)}$$

Second Partial Fractions Decomposition Theorem:

If $P(x)$ and $Q(x)$ are polynomials with

$$\deg(P(x)) < \deg((Q(x))^n)$$

then there are polynomials $R_1(x), R_2(x), \dots, R_n(x)$ such that:

$$\frac{P(x)}{(Q(x))^n} = \frac{R_1(x)}{Q(x)} + \frac{R_2(x)}{(Q(x))^2} + \frac{R_3(x)}{(Q(x))^3} + \dots + \frac{R_n(x)}{(Q(x))^n}$$

where each numerator $R_k(x)$ has degree less than $\deg(Q(x))$.

Second Partial Fractions Decomposition Theorem:

If $P(x)$ and $Q(x)$ are polynomials with

$$\deg(P(x)) < \deg((Q(x))^n)$$

then there are polynomials $R_1(x), R_2(x), \dots, R_n(x)$ such that:

$$\frac{P(x)}{(Q(x))^n} = \sum_{k=1}^n \frac{R_k(x)}{(Q(x))^k}$$

where each numerator $R_k(x)$ has degree less than $\deg(Q(x))$.