$$1 + 2 + 3 + 4 + \cdots$$

$$1 + 1 + 1 + 1 + \cdots$$

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$$

$$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots$$

$$a_1 + a_2 + a_3 + a_4 + \cdots$$

$$\sum_{n=1}^{\infty} n = 1 + 2 + 3 + 4 + \cdots$$

$$\sum_{n=1}^{\infty} 1 = 1 + 1 + 1 + 1 + \cdots$$

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$$

$$\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots$$

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + a_4 + \cdots$$

$$\sum_{n=1}^{\infty} a_n = \lim_{N\to\infty} (a_1 + a_2 + a_3 + \cdots + a_N)$$

If the limit exists and equals a finite number L then we say that the infinite sum *converges* to L. If there is no finite limit then we say that the infinite sum *diverges*.

Notation:

If $\sum_{n=1}^{\infty} a_n$ converges, this is abbreviated as:

$$\sum a_n < \infty$$

$$\sum_{n=1}^{\infty} n = 1 + 2 + 3 + 4 + \cdots$$

$$\sum_{n=1}^{\infty} 1 = 1 + 1 + 1 + 1 + \cdots$$

Converge or diverge?

Suppose the series converges.

$$\lim_{N\to\infty} (a_1 + a_2 + a_3 + \cdots + a_{N-1} + a_N) = L$$

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Now, subtract:

$$\lim_{N\to\infty} (a_N) = L - L = 0$$

Theorem: If $\sum a_n$ converges then $\lim_{n \to \infty} a_n = 0$

Theorem: If $\sum a_n$ converges then $\lim_{n\to\infty} a_n = 0$

Equivalently, if a_n is not approaching 0 as $n \to \infty$, then $\sum a_n$ diverges.

$$\sum_{n=1}^{\infty} n = 1 + 2 + 3 + 4 + \cdots$$

$$\sum_{n=1}^{\infty} 1 = 1 + 1 + 1 + 1 + \cdots$$

These sums must diverge

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$$

$$\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots$$

Converge or diverge?

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$$

$$\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots$$

A series $\sum a_n$ is called a *geometric series* if the ratio of each term to the one before is always a constant.