$$\int_0^{1/2} \sin\left(x^2\right)\, dx$$

$$f(x) = \sin\left(x^2\right)$$

$$c_n = \frac{f^{(n)}(0)}{n!}$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

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$$\sin\left(x^2\right) = \sum_{n=0}^{\infty} \frac{(-1)^n \left(x^2\right)^{2n+1}}{(2n+1)!} = x^2 - \frac{\left(x^2\right)^3}{3!} + \cdots$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\sin\left(x^2\right) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!} = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \cdots$$

$$\int_0^{1/2} \sin\left(x^2\right)\, dx = \int_0^{1/2} \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!}\, dx$$

$$\int_0^{1/2} \sin\left(x^2\right)\, dx = \int_0^{1/2} \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!}\, dx$$
$$= \sum_{n=0}^{\infty} \int_0^{1/2} \frac{(-1)^n x^{4n+2}}{(2n+1)!}\, dx$$

$$\begin{aligned}
\int_0^{1/2} \sin\left(x^2\right)\, dx &= \int_0^{1/2} \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!}\, dx \\
&= \sum_{n=0}^{\infty} \int_0^{1/2} \frac{(-1)^n x^{4n+2}}{(2n+1)!}\, dx \\
&= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \int_0^{1/2} x^{4n+2}\, dx
\end{aligned}$$

$$\int_0^{1/2} x^{4n+2}\,dx = \left[\frac{x^{4n+3}}{4n+3}\right]_0^{1/2} = \frac{1}{2^{4n+3}(4n+3)}$$

$$\int_0^{1/2} \sin\left(x^2\right)\, dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!2^{4n+3}(4n+3)}$$

$$\int_0^{1/2} \sin\left(x^2\right)\, dx \approx \sum_{n=0}^{N} \frac{(-1)^n}{(2n+1)!2^{4n+3}(4n+3)}$$

N	Sum
1	0.04148065476
2	0.04148102467
3	0.04148102427
4	0.04148102427

$$\sin(3\pi)$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \cdots$$

$$\sin(3\pi) = \sum_{n=0}^{\infty} \frac{(-1)^n (3\pi)^{2n+1}}{(2n+1)!} = 3\pi - \frac{27\pi^3}{3!} + \cdots$$

$$\sin(3\pi) \approx \sum_{n=0}^{N} \frac{(-1)^n (3\pi)^{2n+1}}{(2n+1)!}$$

N	Sum
3	-821.0051320
4	795.8723560
5	-509.7773770

$$\sin(3\pi) \approx \sum_{n=0}^{N} \frac{(-1)^n (3\pi)^{2n+1}}{(2n+1)!}$$

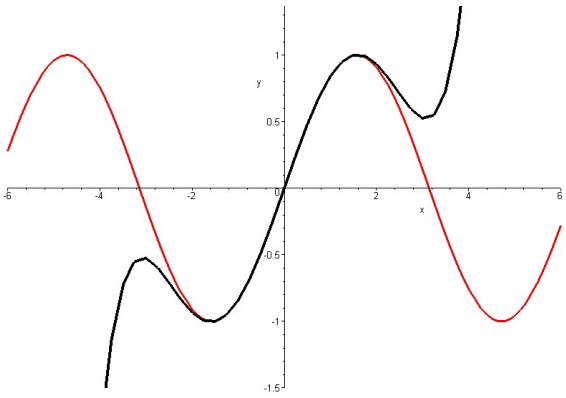
N	Sum
6	233.6599127
7	-80.8014565
8	21.8914987

$$\sin(3\pi) \approx \sum_{n=0}^{N} \frac{(-1)^n (3\pi)^{2n+1}}{(2n+1)!}$$

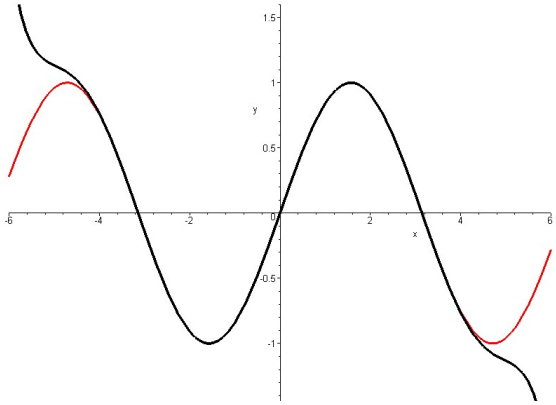
N	Sum
9	-4.78057612
10	0.860341602
11	-0.1299007648

N	Sum
12	0.0166987416
13	-0.00185099088
14	0.000178204554

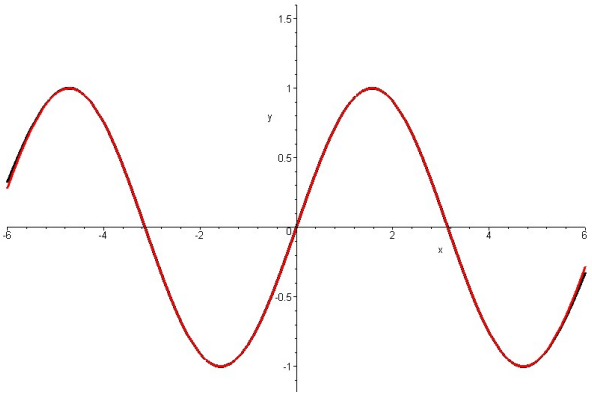
$$\sum_{n=0}^{2} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!}$$



$$\sum_{n=0}^{5} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$



$$\sum_{n=0}^{7} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$



A Maclaurin series:

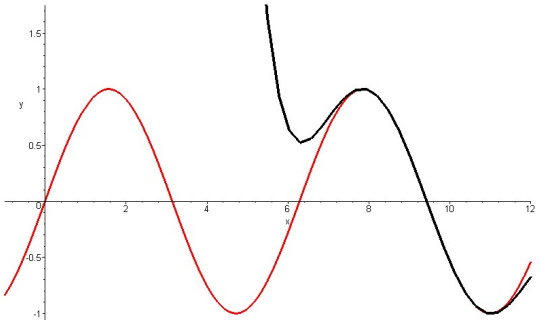
$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + \cdots$$

is a special case of a Taylor series:

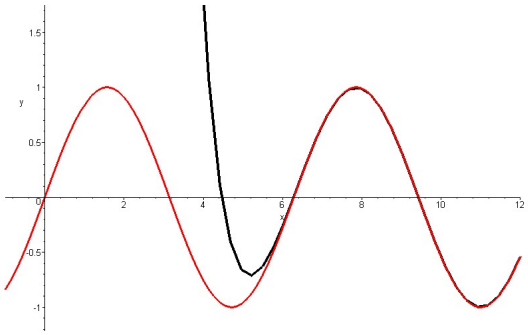
$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1 (x-a) + c_2 (x-a)^2 + \cdots$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x - 3\pi)^{2n+1}}{(2n+1)!}$$

$$\sum_{n=0}^{2} \frac{(-1)^{n+1}(x-3\pi)^{2n+1}}{(2n+1)!}$$



$$\sum_{n=0}^{4} \frac{(-1)^{n+1}(x-3\pi)^{2n+1}}{(2n+1)!}$$



Calculate $\sin 9$

$$\sin 9 \approx \sum_{n=0}^{N} \frac{(-1)^{n+1}(9-3\pi)^{2n+1}}{(2n+1)!}$$

N	Sum
1	0.4120037335
2	0.4121189802
3	0.4121184851