Exam 2

1. *(16 points)* Suppose $w = f(x, y, z)$ where x, y and z are given by:

$$x = \rho \cos\theta \sin\phi \qquad y = \rho \sin\theta \sin\phi \qquad z = \rho \cos\phi$$

You are not given the formula for how w depends on x, y and z, but you are given the following partial derivatives:

$$\frac{\partial w}{\partial x} = 3 \qquad \frac{\partial w}{\partial y} = 2 \qquad \frac{\partial w}{\partial z} = 1$$

Find $\frac{\partial w}{\partial \theta}$ when $\rho = 2$, $\theta = \frac{\pi}{2}$ and $\phi = \frac{\pi}{2}$

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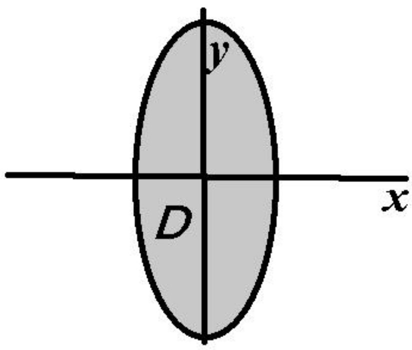
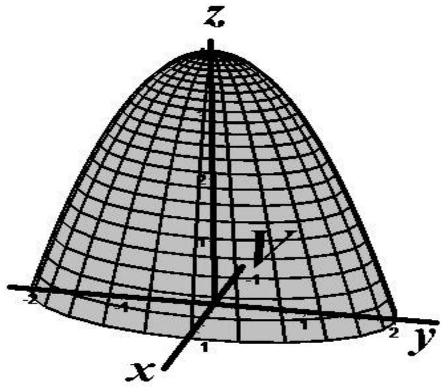
2. *(16 points)* Let $\mathcal{D}$ be the region in the xy plane inside the ellipse described by the equation $x^2 + \frac{y^2}{4} = 1$. Let $\mathcal{V}$ be the three dimensional region that is above $\mathcal{D}$ but below the surface $z = 4 - 4x^2 - y^2$





Express the volume of $\mathcal{V}$ as a double integral in the $\iint \; dy\,dx$ order of integration.

3. *(24 points)* Again, let $z = 4 - 4x^2 - y^2$.

a) Find a vector $\vec{\mathbf{n}}$ that is perpendicular to the surface at $\left(\frac{1}{2},\ 1,\ 2\right)$.

b) Find the equation of the plane tangent to this surface at $\left(\frac{1}{2},\ 1,\ 2\right)$.

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4. *(24 points)* For each of the following double integrals, reverse the order of integration. There are no antiderivatives to calculate here.

a) $$\int_{-2}^{0}\int_{0}^{y+2} f(x,y)\,dx\,dy$$

b) $$\int_{0}^{1}\int_{3}^{4-x^2} f(x,y)\,dy$$

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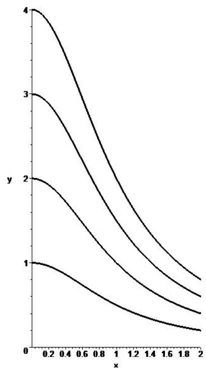
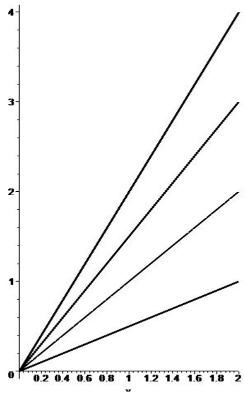
5. Which of the following graphs show some of the level sets of the function $f(x, y) = \frac{y}{1+x^2}$ for $x \geq 0$ and $y \geq 0$

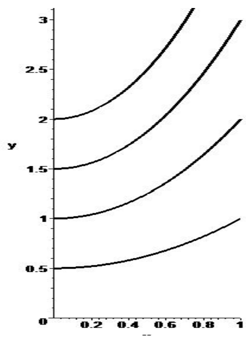
Graph 1



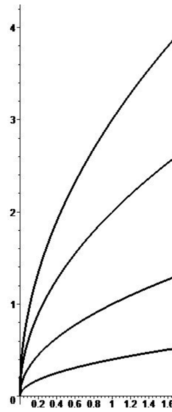
Graph 2



Graph 3



Graph 4



6. For $f(x,\ y,\ z) = 6x + y + 2z$, calculate the directional derivative $D_{\vec{\mathbf{u}}}\, f$ where $\vec{\mathbf{u}} = \frac{1}{3}\langle 1,\ 2,\ 2\rangle$

a) 1 **b)** 2 **c)** 3 **d)** 4 **e)** 6

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7. Let $g(x,\ y,\ z) = 2x + 2y + z$. The value of the directional derivative $D_{\vec{\mathbf{v}}}\, g$ depends on the direction of $\vec{\mathbf{v}}$. Which of the following is the largest possible value of $D_{\vec{\mathbf{v}}}\, g$?

a) 3 **b)** 4 **c)** 6 **d)** 8 **e)** 9

8. Calculate the following double integral:

$$\int_1^4 \int_0^2 \frac{2y}{\sqrt{x}}\, dy\, dx$$

a) 1 **b)** 2 **c)** 4 **d)** 8 **e)** 16