MA 243 Calculus III **Exam 4 (Practice)**

Problems 1 - 12 are multiple choice questions. Show all work on the rest of the problems.

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1. Let $f(x,y) = 2x + 2y - x^2 - y^2$. The point $(1,\ 1,\ 2)$ lies on the surface. Which of the following is the correct classification of this point?

a) A maximum point **b)** A minimum point **c)** A saddle point **d)** None of these

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2. Which of the following is the vector equation of the line that passes through the points $(1,1,1)$ and $(1,0,2)$?

a) $\langle x,y,z\rangle = \langle 1,1,1\rangle + t\langle 1,0,2\rangle$ **b)** $\langle x,y,z\rangle = \langle 1,1,1\rangle + t\langle 0,-1,1\rangle$
c) $\langle x,y,z\rangle = t\langle 0,-1,1\rangle$ **d)** $\langle x,y,z\rangle = \langle 1,0,2\rangle + t\langle 1,1,1\rangle$

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3. Which one of the following equations represents a plane that is perpendicular to the xz-plane?

a) $y = z$ **b)** $x = 1 - y$ **c)** $y = 1 - z$ **d)** $z = 1 - x$

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4. Which vector is tangent to the curve $\vec{\mathbf{r}}(t) = \langle t, \cos t, \sin t\rangle$ at $t = \frac{\pi}{2}$?

a) $\langle \frac{\pi}{2}, 0, 1\rangle$ **b)** $\langle 1,-1,0\rangle$ **c)** $\langle 0,-1,0\rangle$ **d)** $\langle 1,0,0\rangle$

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5. If $z = x^2e^{3y} + x^3 + y$ find $\frac{\partial^2 z}{\partial x \partial y}$

a) $2xe^{3y}$ **b)** $6xe^{3y}$ **c)** $2e^{3y}$ **d)** $2xe^{3y} + 3x^2 + 1$

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6. Which of the following double integrals equals $\int_0^\infty \int_0^y f(x,y)\,dx\,dy$?

a) $\int_0^y \int_0^\infty f(x,y)\ dy\,dx$ **b)** $\int_0^\infty \int_0^x f(x,y)\ dy\,dx$
c) $\int_0^x \int_y^\infty f(x,y)\ dy\,dx$ **d)** $\int_0^\infty \int_x^\infty f(x,y)\ dy\,dx$

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7. A triple integral over a region Q is given below in rectangular coordinates:

$$\iiint_Q z\,dV = \int_{-1}^{1}\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} z\,dz\,dy\,dx$$

The region Q is a portion of a sphere of radius 1. If we convert $\iiint_Q z\,dV$ to spherical coordinates, which of the following triple integrals will we get?

a) $\int_0^{\pi/2}\int_0^{\pi}\int_0^1 \rho\cos\phi\,d\rho\,d\theta\,d\phi$ **b)** $\int_0^{\pi}\int_0^{2\pi}\int_0^1 \rho\cos\phi\,d\rho\,d\theta\,d\phi$
c) $\int_0^{\pi/2}\int_0^{\pi}\int_0^1 \rho^3\cos\phi\sin\phi\,d\rho\,d\theta\,d\phi$ **d)** $\int_0^{\pi}\int_0^{2\pi}\int_0^1 \rho^3\cos\phi\sin\phi\,d\rho\,d\theta\,d\phi$

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8. If $(\rho,\theta,\phi) = \left(4, \frac{\pi}{4}, \frac{\pi}{6}\right)$ are the spherical coordinates of a point and (r,θ,z) is the same point in cylindrical coordinates, calculate r

a) 2 **b)** $2\sqrt{2}$ **c)** $2\sqrt{3}$ **d)** 4

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9. Calculate the double integral: $$\int_0^2\int_1^2 \left(\frac{x}{y}\right)^2 dy\,dx$$

a) $\frac{1}{6}$ **b)** $\frac{1}{3}$ **c)** $\frac{4}{3}$ **d)** 3

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10. Let $f(x,y) = x^2 + 4y^2$. If (x,y) changes from $(1,1)$ to $(1.5,\ 1.25)$, which of the following is the value of the differential df ?

a) 2 **b)** 2.5 **c)** 3 **d)** 3.5

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11. Let T be the region below $z = 12 - 4x - 2y$ for $x \geq 0$, $y \geq 0$ and $z \geq 0$.
Which of the following triple integrals represents the volume of T ?

a) $\displaystyle\int_0^6 \int_0^{6-x} \int_0^{12-4x-2y} 1\, dz\, dy\, dx$ **b)** $\displaystyle\int_0^3 \int_0^{6-2x} \int_0^{12-4x-2y} 1\, dz\, dy\, dx$

c) $\displaystyle\int_0^6 \int_0^3 \int_0^{12-4x-2y} 1\, dz\, dy\, dx$ **d)** $\displaystyle\int_0^6 \int_0^3 \int_0^{12} 1\, dz\, dy\, dx$

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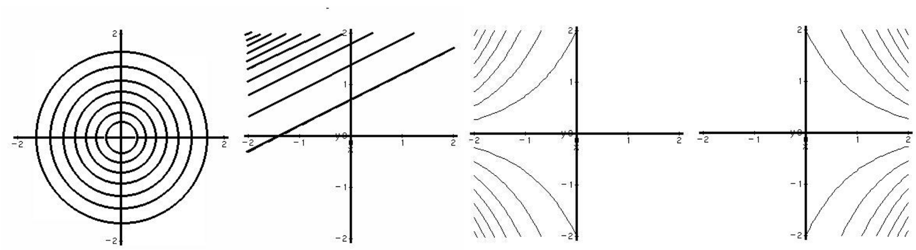
12. Which of the following shows the level sets of $f(x, y) = e^{y-x}$?

a) **b)** **c)** **d)**



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13. Let T be the region above the paraboloid $z = 1 - x^2 - y^2$ and below the paraboloid $z = 3 - 3x^2 - 3y^2$. Find the volume of T. Show all work.

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14. Convert to polar coordinates and evaluate:

$$\int_{-1}^{1} \int_0^{\sqrt{1-x^2}} \frac{1}{\sqrt{1 + x^2 + y^2}}\, dy\, dx$$

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15. Calculate the z-coordinate of the centroid of the region that is inside the cone $z = 4\sqrt{x^2 + y^2}$ but below the plane $z = 4$.

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16. Let Ω be the surface around the side of the cone $z = 4\sqrt{x^2 + y^2}$ but below the plane $z = 4$. Calculate the surface area of Ω.

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17. Find the maximum volume possible for a box sitting on the xy plane inscribed under the paraboloid $z = 4 - x^2 - y^2$