1. Let $f(x, y) = 2x + 2y - x^2 - y^2$. The point $(1,\ 1,\ 2)$ lies on the surface. Which of the following is the correct classification of this point?

a) A maximum point
b) A minimum point
c) A saddle point
d) None of these

2. Which of the following is the vector equation of the line that passes through the points $(1, 1, 1)$ and $(1, 0, 2)$?

a) $\langle x, y, z\rangle = \langle 1, 1, 1\rangle + t\langle 1, 0, 2\rangle$ **b)** $\langle x, y, z\rangle = \langle 1, 1, 1\rangle + t\langle 0, -1, 1\rangle$

c) $\langle x, y, z\rangle = t\langle 0, -1, 1\rangle$ **d)** $\langle x, y, z\rangle = \langle 1, 0, 2\rangle + t\langle 1, 1, 1\rangle$

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3. Which one of the following equations represents a plane that is perpendicular to the xz-plane?

a) $y = z$ **b)** $x = 1 - y$ **c)** $y = 1 - z$ **d)** $z = 1 - x$

4. Which vector is tangent to the curve $\vec{\mathbf{r}}(t) = \langle t,\, \cos t,\, \sin t\rangle$ at $t = \frac{\pi}{2}$?

a) $\langle \frac{\pi}{2}, 0, 1\rangle$ **b)** $\langle 1, -1, 0\rangle$ **c)** $\langle 0, -1, 0\rangle$ **d)** $\langle 1, 0, 0\rangle$

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5. If $z = x^2 e^{3y} + x^3 + y$ find $\frac{\partial^2 z}{\partial x \partial y}$

a) $2xe^{3y}$ **b)** $6xe^{3y}$ **c)** $2e^{3y}$ **d)** $2xe^{3y} + 3x^2 + 1$

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6. Which of the following double integrals equals $\int_0^\infty \int_0^y f(x,y)\,dx\,dy$?

a) $\int_0^y \int_0^\infty f(x,y)\;dy\,dx$

b) $\int_0^\infty \int_0^x f(x,y)\;dy\,dx$

c) $\int_0^x \int_y^\infty f(x,y)\;dy\,dx$

d) $\int_0^\infty \int_x^\infty f(x,y)\;dy\,dx$

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7. A triple integral over a region Q is given below in rectangular coordinates:

$$\iiint_Q z\,dV = \int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{1-x^2-y^2}} z\,dz\,dy\,dx$$

The region Q is a portion of a sphere of radius 1. If we convert $\iiint_Q z\,dV$ to spherical coordinates, which of the following triple integrals will we get?

a) $\int_0^{\pi/2}\int_0^{\pi}\int_0^{1} \rho\cos\phi\,d\rho\,d\theta\,d\phi$ **b)** $\int_0^{\pi}\int_0^{2\pi}\int_0^{1} \rho\cos\phi\,d\rho\,d\theta\,d\phi$

c) $\int_0^{\pi/2}\int_0^{\pi}\int_0^{1} \rho^3\cos\phi\sin\phi\,d\rho\,d\theta\,d\phi$ **d)** $\int_0^{\pi}\int_0^{2\pi}\int_0^{1} \rho^3\cos\phi\sin\phi\,d\rho\,d\theta\,d\phi$

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8. If $(\rho, \theta, \phi) = \left(4, \frac{\pi}{4}, \frac{\pi}{6}\right)$ are the spherical coordinates of a point and (r, θ, z) is the same point in cylindrical coordinates, calculate r

a) 2 **b)** $2\sqrt{2}$ **c)** $2\sqrt{3}$ **d)** 4

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9. Calculate the double integral: $\int_0^2 \int_1^2 \left(\frac{x}{y}\right)^2 \, dy \, dx$

a) $\frac{1}{6}$ **b)** $\frac{1}{3}$ **c)** $\frac{4}{3}$ **d)** 3

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10. Let $f(x, y) = x^2 + 4y^2$. If (x, y) changes from $(1, 1)$ to $(1.5,\ 1.25)$, which of the following is the value of the differential df ?

a) 2 **b)** 2.5 **c)** 3 **d)** 3.5

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11. Let T be the region below $z = 12 - 4x - 2y$ for $x \geq 0$, $y \geq 0$ and $z \geq 0$. Which of the following triple integrals represents the volume of T ?

a) $\displaystyle\int_0^6 \int_0^{6-x} \int_0^{12-4x-2y} 1\, dz\, dy\, dx$

b) $\displaystyle\int_0^3 \int_0^{6-2x} \int_0^{12-4x-2y} 1\, dz\, dy\, dx$

c) $\displaystyle\int_0^6 \int_0^3 \int_0^{12-4x-2y} 1\, dz\, dy\, dx$

d) $\displaystyle\int_0^6 \int_0^3 \int_0^{12} 1\, dz\, dy\, dx$

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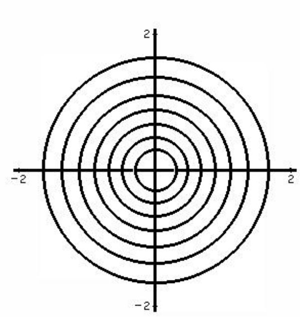
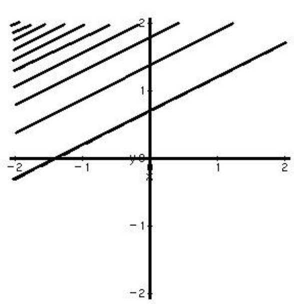
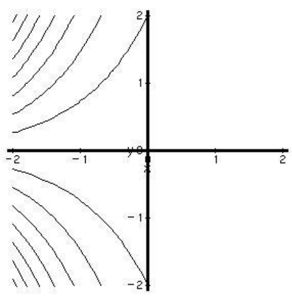
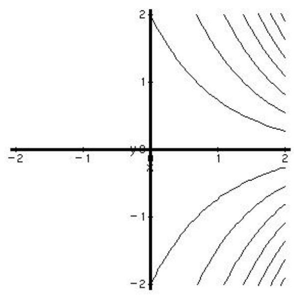
12. Which of the following shows the level sets of $f(x, y) = e^{y-x}$?

a) **b)** **c)** **d)**









13. Let T be the region above the paraboloid $z = 1 - x^2 - y^2$ and below the paraboloid $z = 3 - 3x^2 - 3y^2$. Find the volume of T. Show all work.

14. Convert to polar coordinates and evaluate:

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} \frac{1}{\sqrt{1+x^2+y^2}}\,dy\,dx$$

15. Calculate the z-coordinate of the centroid of the region that is inside the cone $z = 4\sqrt{x^2 + y^2}$ but below the plane $z = 4$.

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16. Let Ω be the surface around the side of the cone $z = 4\sqrt{x^2 + y^2}$ but below the plane $z = 4$. Calculate the surface area of Ω.

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17. Find the maximum volume possible for a box sitting on the xy plane inscribed under the paraboloid $z = 4 - x^2 - y^2$