Embry-Riddle Aeronautical University **MA 243** *Calculus III*

Sample Exam I

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1. Suppose a particle is moving along a three dimensional path and that its coordinates after t seconds are given by the parametric equations:

$$\vec{\mathbf{r}}(t) = \left\langle \frac{3}{2}t^2,\ \frac{3}{2}t^2,\ (\sqrt{2})t^3 \right\rangle$$

a) Find the velocity and acceleration vectors **b)** At $t = 1$, find $|\text{proj}_{\vec{\mathbf{v}}}\,\vec{\mathbf{a}}|$

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2. Once again, consider the curve described in problem 1. Find the length of the curve between $t = 0$ and $t = 1$.

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3. Let $\mathcal{L}$ be the line that passes through the points $(1, 1, 2)$ and $(2, 2, 4)$.

a) Find the equation of this line.

b) Let $\vec{\mathbf{k}} = \langle 0, 0, 1\rangle$. Find the length of the projection of $\vec{\mathbf{k}}$ in the direction of the line $\mathcal{L}$.

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4. In two dimensions, the equation $y = x + 1$ would describe a straight line. However, if we consider all points (x, y, z) in three dimensions for which the equation $y = x + 1$ is true, we get a *surface*. What surface is it? Either draw an accurate sketch of this particular surface or, if your artistic skills aren't very good, give a complete verbal description of what this surface looks like.

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5. If the lengths of the side of a triangle are x, y and z and $\theta = \frac{\pi}{3}$ is the angle opposite the side of length z, then these variables are related by the equation: $z^2 = x^2 + y^2 - xy$. Calculate $\frac{\partial z}{\partial x}$ at $x = 1$, $y = 1$ and $z = 1$.

The remaining questions on this test are multiple choice questions.

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6. The equation $x^2 + y^2 + z^2 = 2y$ describes a sphere. What is the radius of this sphere?

a) 1 **b)** 2 **c)** $\sqrt{2}$ **d)** $\sqrt{2y}$

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7. The line $\langle x, y, z\rangle = \langle 1+t,\ 1-t,\ t\rangle$ is perpendicular to only one of the following vectors. Which one?

a) $\langle 1, -1, -1\rangle$ **b)** $\langle 1, 1, 0\rangle$ **c)** $\langle 1, 0, 1\rangle$ **d)** $\langle 1, 1, 1\rangle$

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8. Which of the following equations would describe the plane that contains the point $(1, 1, 1)$ and also contains every point on the y-axis ?

a) $x = z$ **b)** $z = y$ **c)** $x + y + z = 0$ **d)** $x + y + z = 3$

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9. The position of a particle after t seconds is given by:

$$\vec{\mathbf{r}}(t) = \langle 0,\ \cos(2t), \sin(2t)\rangle$$

Find the speed of the particle

a) 0 **b)** 1 **c)** $\sqrt{2}$ **d)** 2

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10. Which of the following equations would describe a *cone*?

a) $z = x^2 + y^2$ **b)** $z = \sqrt{1 - x^2 - y^2}$ **c)** $z = x^2 - y^2$ **d)** $z = \sqrt{x^2 + y^2}$

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11. Find the dot product of $\vec{\mathbf{u}} = \left\langle 0,\ \frac{1}{2},\ \frac{1}{3}\right\rangle$ and $\vec{\mathbf{v}} = \langle 1,\ -4,\ 6\rangle$.

a) $\langle 0, -2,\ 2\rangle$ **b)** 0 **c)** 1 **d)** 4

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12. Once again, let $\vec{\mathbf{u}} = \left\langle 0,\ \frac{1}{2},\ \frac{1}{3}\right\rangle$ and $\vec{\mathbf{v}} = \langle 1,\ -4,\ 6\rangle$. Find the angle (in radians) between these two vectors.

a) 0 **b)** $\frac{\pi}{4}$ **c)** $\frac{\pi}{2}$ **d)** π

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13. Find a vector perpendicular to $\vec{\mathbf{u}} = \langle 1, 1, 0\rangle$ and $\vec{\mathbf{v}} = \langle 1, 1, 1\rangle$

a) $\langle 0, 1, -1\rangle$ **b)** $\langle 1, 0, 1\rangle$ **c)** $\langle 1, -1, 1\rangle$ **d)** $\langle 1, -1, 0\rangle$
