Sample Exam I

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1. Suppose a particle is moving along a three dimensional path and that its coordinates after t seconds are given by the parametric equations:

$$\vec{\mathbf{r}}(t) = \left\langle \frac{3}{2}t^2,\ \frac{3}{2}t^2,\ (\sqrt{2})t^3 \right\rangle$$

a) Find the velocity and acceleration vectors
b) At $t = 1$, find $|\mathrm{proj}_{\vec{\mathbf{v}}}\,\vec{\mathbf{a}}|$

$$\vec{\mathbf{r}}(t) = \left\langle \frac{3}{2}t^2,\ \frac{3}{2}t^2,\ (\sqrt{2})t^3 \right\rangle$$

$$\vec{\mathbf{v}} = \left\langle 3t,\ 3t,\ 3\sqrt{2}t^2 \right\rangle$$

2. Once again, consider the curve described in problem 1. Find the length of the curve between $t = 0$ and $t = 1$.

3. Let $\mathcal{L}$ be the line that passes through the points $(1,1,2)$ and $(2,2,4)$.

a) Find the equation of this line.

b) Let $\vec{\mathbf{k}} = \langle 0,0,1 \rangle$. Find the length of the projection of $\vec{\mathbf{k}}$ in the direction of the line $\mathcal{L}$.

4. In two dimensions, the equation $y = x + 1$ would describe a straight line. However, if we consider all points (x, y, z) in three dimensions for which the equation $y = x+1$ is true, we get a *surface*. What surface is it? Either draw an accurate sketch of this particular surface or, if your artistic skills aren't very good, give a complete verbal description of what this surface looks like.

5. The equation $x^2 + y^2 + z^2 = 2y$ describes a sphere. What is the radius of this sphere?

a) 1 **b)** 2 **c)** $\sqrt{2}$ **d)** $\sqrt{2y}$

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6. The line $\langle x, y, z \rangle = \langle 1 + t,\ 1 - t,\ t \rangle$ is perpendicular to only one of the following vectors. Which one?

a) $\langle 1, -1, -1 \rangle$ **b)** $\langle 1, 1, 0 \rangle$
c) $\langle 1, 0, 1 \rangle$ **d)** $\langle 1, 1, 1 \rangle$

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7. Which of the following equations would describe the plane that contains the point $(1, 1, 1)$ and also contains every point on the y-axis ?

a) $x = z$ **b)** $z = y$

c) $x + y + z = 0$ **d)** $x + y + z = 3$

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8. The position of a particle after t seconds is given by:

$$\vec{\mathbf{r}}(t) = \left\langle 2\sin(\pi t),\ \sqrt{2}\cos(\pi t), \sqrt{2}\cos(\pi t) \right\rangle$$

Find the speed of the particle

a) 2 **b)** π **c)** $\sqrt{2\pi}$ **d)** 2π

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9. Which of the following equations would describe a *cone*?

a) $z = x^2 + y^2$ **b)** $z = \sqrt{1 - x^2 - y^2}$

c) $z = x^2 - y^2$ **d)** $z = \sqrt{x^2 + y^2}$

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10. Find the dot product of $\vec{\mathbf{u}} = \left\langle 0,\ \frac{1}{2},\ \frac{1}{3} \right\rangle$ and $\vec{\mathbf{v}} = \langle 1,\ -4,\ 6 \rangle$.

a) $\langle 0, -2,\ 2 \rangle$ **b)** 0

c) 1 **d)** 4

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11. Again let $\vec{\mathbf{u}} = \left\langle 0,\ \frac{1}{2},\ \frac{1}{3} \right\rangle$ and $\vec{\mathbf{v}} = \langle 1,\ -4,\ 6 \rangle$. Find the angle (in radians) between these two vectors.

a) 0 **b)** $\frac{\pi}{4}$ **c)** $\frac{\pi}{2}$ **d)** π

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12. Find a vector perpendicular to $\vec{\mathbf{u}} = \langle 1, 1, 0 \rangle$ and $\vec{\mathbf{v}} = \langle 1, 1, 1 \rangle$

a) $\langle 0, 1, -1 \rangle$

b) $\langle 1, 0, 1 \rangle$

c) $\langle 1, -1, 1 \rangle$

d) $\langle 1, -1, 0 \rangle$