MA 243 *Calculus III* **Spring 2025** **Exam I - Dr. E. Jacobs**

Name :_____ Score : _____

Show all work on problems 1 - 4 and write your solutions on the blank pages provided. The pages will be stapled together at the end of the exam. Express all answers in simplest form. Exact answers only, please - no decimal approximations.

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1. *(24 points)* Let $\mathbf{A} = (0,\ 0,\ 0)$, $\mathbf{B} = (1,\ 2,\ -1)$ and $\mathbf{C} = (1,\ 2,\ 5)$

a) Let $\vec{\mathbf{u}}$ be the vector from point **A** to point **B**. Let $\vec{\mathbf{v}}$ be the vector from point **A** to point **C**. Calculate $|\text{proj}_{\vec{\mathbf{u}}}\,\vec{\mathbf{v}}|$.

b) Let $\mathcal{T}$ be the triangle with vertices **A**, **B** and **C**. Use the cross product to find the area of this triangle.

c) Find the equation of the plane that contains all points of triangle $\mathcal{T}$.

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2. *(20 points)* Let C be the segment of the curve $\vec{\mathbf{r}} = \langle \cos t,\ \sin t,\ t \rangle$ for $0 \leq t \leq 2\pi$. Find the arc length of C. Show all work.

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3. *(18 points)* Suppose the position of a particle at time t is given by the equation:

$$\vec{\mathbf{r}} = \langle x,\ y,\ z \rangle = \langle \cos t,\ \sin t,\ t \rangle$$

a) Find the velocity vector $\vec{\mathbf{v}}$ and the acceleration vector $\vec{\mathbf{a}}$ when $t = 2\pi$

b) At $t = 2\pi$, calculate the dot product $\vec{\mathbf{v}} \bullet \vec{\mathbf{a}}$ and the cross product $\vec{\mathbf{v}} \times \vec{\mathbf{a}}$

c) The curve described by the equation $\vec{\mathbf{r}} = \langle \cos t,\ \sin t,\ t \rangle$ passes through the point $(1,\ 0,\ 2\pi)$. Find the equation of the line tangent to the curve at this point.

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4. *(18 points)* Let $z = x^2 y + \frac{x}{y}$. Calculate the following partial derivatives:

$$\frac{\partial^2 z}{\partial x^2} \qquad \frac{\partial^2 z}{\partial y^2} \qquad \frac{\partial^2 z}{\partial x \partial y}$$

Problems 5 - 8. *(20 points)* The multiple choice questions are given on the reverse side of this page. Circle the correct choices.

The following questions are multiple choice. For each question, circle the correct answer directly on this page.

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5. Find the center of the sphere described by the following equation:

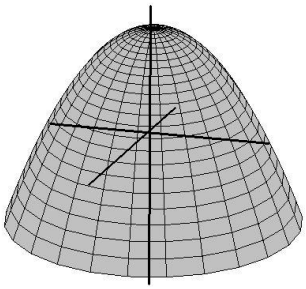
$$x^2 + y^2 + z^2 = 2x - 2y$$

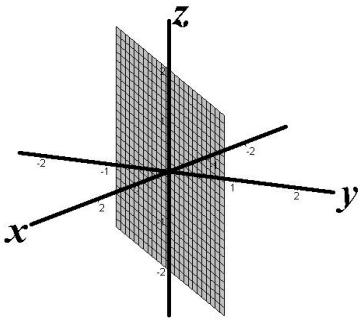
a) $(0,\ 0,\ 0)$ **b)** $(1,\ 0,\ 0)$ **c)** $(1,\ -1,\ 0)$

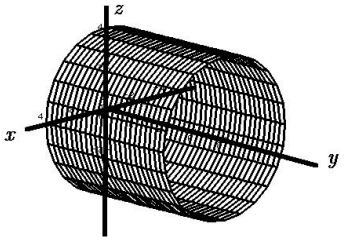
d) $(-1,\ 1,\ 0)$ **e)** $(0,\ 1,\ 1)$

Problems 6 - 8 all refer to the following graphs:

Graph 1 Graph 2 Graph 3







6. Which of the following equations best describes Graph 1?

a) $z = \sqrt{x^2 + y^2}$ **b)** $x^2 + y^2 = 1$ **c)** $y = x$

d) $z = 1 - x^2 - y^2$ **e)** $x^2 + z^2 = 1$ **f)** $z = y$

7. Which of the following equations best describes Graph 2?

a) $z = \sqrt{x^2 + y^2}$ **b)** $x^2 + y^2 = 1$ **c)** $y = x$

d) $z = 1 - x^2 - y^2$ **e)** $x^2 + z^2 = 1$ **f)** $z = y$

8. Which of the following equations best describes Graph 3?

a) $z = \sqrt{x^2 + y^2}$ **b)** $x^2 + y^2 = 1$ **c)** $y = x$

d) $z = 1 - x^2 - y^2$ **e)** $x^2 + z^2 = 1$ **f)** $z = y$