Embry-Riddle Aeronautical University **Dr. E. Jacobs**
MA 243 *Calculus III* **Sample Exam**

Exam II

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1. The volume of a cylinder is given by the formula $V = \pi r^2 h$. Suppose the radius and the height are both growing with time. In particular, suppose that the radius is growing at 2 meters/second and the height is growing at 3 meters/second. Use the *Chain Rule for Partial Derivatives* to calculate $\frac{dV}{dt}$ (the rate at which the volume is growing) when the radius is $r = 1$ meter and the height is $h = 10$ meters.

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2. Let $f(x,y) = 2 + 2x - y$

a) Draw some of the level sets of $f(x,y)$ on an x-y axis.

b) Let $\vec{\mathbf{v}} = \frac{1}{\sqrt{5}}\langle 1, 2\rangle$. Calculate the directional derivative $D_{\vec{\mathbf{v}}} f$

c) Find a unit vector $\vec{\mathbf{u}}$ that makes the directional derivative $D_{\vec{\mathbf{u}}} f$ as large as possible.

d) Find the largest possible value of the directional derivative of this function f.

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3. Let T be the triangle in the xy plane with vertices $(0,0)$, $(1,1)$ and $(1,0)$. Calculate the integral: $\iint_T e^{-x^2}\, dA$

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4. Let G be the region in the xy plane that is bounded by the curves $y = 2x - 2x^2$ and $y = x^2 - x$. Calculate the integral $\iint_G 1\, dA$

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5. The following equation describes a paraboloid:

$$(x-2)^2 + (y-2)^2 + z = 4$$

Find the equation of the plane that is tangent to this paraboloid at $(x,\ y,\ z) = (3,\ 3,\ 2)$

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6. Let $\mathcal{V}$ be the three dimensional region that is above the xy plane but bounded from above by the paraboloid described in problem 5. Express the volume of $\mathcal{V}$ as a double integral. Do not calculate any antiderivatives. I am only interested in the correct limits of integration.

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7. Reverse the order of integration:

$$\int_0^1 \int_{e^y}^{e} f(x,y)\, dx\, dy$$

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8. Reverse the order of integration:

$$\int_0^\infty \int_{-\sqrt{x}}^{\sqrt{x}} f(x,y)\, dy\, dx$$
