MA 243 Calculus III Fall 2022

Name :__

Score : _____

Show all work. Write your solutions on the blank pages provided. The pages will be stapled together at the end of the exam.

1. (20 points) Let R be the region in the xy-plane that is *inside* a circle of radius 3 centered around the origin. Calculate the following double integral by converting to polar coordinates first.

$$\iint_R \frac{1}{\left(3+x^2+y^2\right)^2} \, dA$$

2. (20 points) Let \mathcal{D} be the triangle in the xy plane with vertices (1, 0, 0), (1, 1, 0) and (0, 0, 0). Find the surface area of the portion of the surface $z = 1 + \frac{1}{2}x^2$ that lies directly over \mathcal{D} .

3. (20 points) Let \mathcal{D} be the same triangular region described in problem 2. Let \mathcal{T} be the three dimensional region that is directly over \mathcal{D} but below the surface z = 2x - 2y. The volume of T can be expressed as a triple integral $\iiint_T 1 \, dV$. In each of the triple integrals below, you are given the lower limits of the triple integral and you must fill in the correct upper limits. You are not required to calculate any antiderivatives for this problem.

$$\iiint_T 1 \, dV = \int_0^? \int_0^? \int_0^? 1 \, dz \, dy \, dx = \int_0^? \int_0^? \int_{y+z/2}^? 1 \, dx \, dy \, dz$$

4. (20 points) Let S be the sector in the xy plane that is bounded by y = 0, y = x and $y = \sqrt{2 - x^2}$. The area of this region $\frac{\pi}{4}$. Write the x-coordinate of the centroid as a double integral and then calculate this integral.

5. (20 points) Let Q be the quarter sphere of radius 2 that is directly below $z = \sqrt{4 - x^2 - y^2}$ for $y \ge 0$. Calculate $\iint_Q x \, dV$ by first converting it to spherical coordinates.

Problems 3.



Problem 4.



Problem 5.

