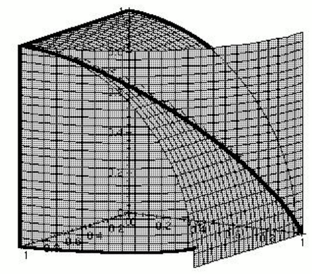
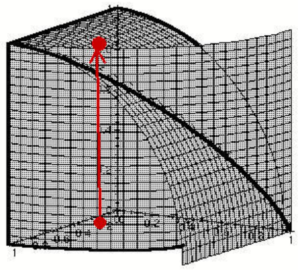
Problem 1. Let Ω be the region in the first octant that is inside both the cylinders $x^2 + y^2 = 1$ and $y^2 + z^2 = 1$.

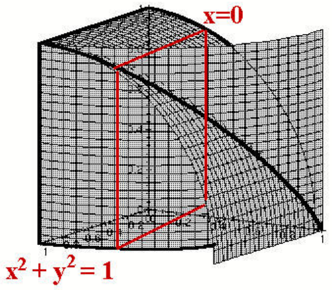


Use a triple integral to evaluate the volume of Ω. I recommend the $\iiint dz\, dx\, dy$ order of integration.

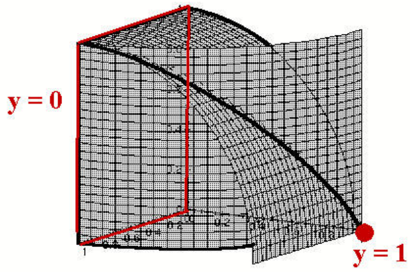
$$\int\int\int_0^{\sqrt{1-y^2}} 1\,dz\,dx\,dy$$



$$\int \int_0^{\sqrt{1-y^2}} \int_0^{\sqrt{1-y^2}} 1 \, dz \, dx \, dy$$



$$\int_0^1 \int_0^{\sqrt{1-y^2}} \int_0^{\sqrt{1-y^2}} 1\,dz\,dx\,dy$$

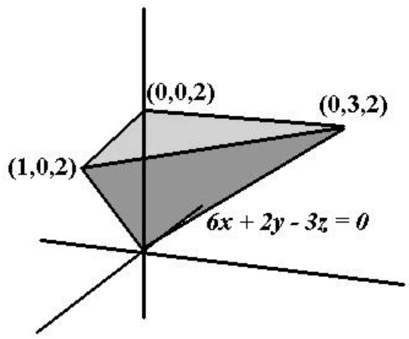


$$
\begin{aligned}
\mathrm{Vol}(\Omega) &= \int_0^1 \int_0^{\sqrt{1-y^2}} \int_0^{\sqrt{1-y^2}} dz\, dx\, dy \\
&= \int_0^1 \left(1 - y^2\right)\, dy \\
&= \frac{2}{3}
\end{aligned}
$$

Problem 3. Let T be the tetrahedron with the following vertices:

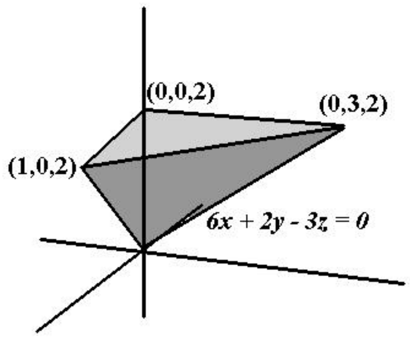
$(0,0,0)$ $(1,0,2)$ $(0,3,2)$ $(0,0,2)$.

$$\text{Vol}(T) = \int_0^1 \int_0^{3-3x} \int_?^? 1\, dz dy dx$$



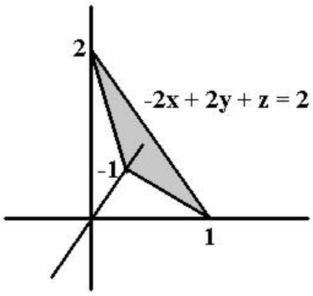
Let T be the tetrahedron with the following vertices:
$(0,0,0) \qquad (1,0,2) \qquad (0,3,2) \qquad (0,0,2).$

$$\text{Vol}(T) = \int_0^2 \int_?^? \int_0^{(3z-2y)/6} 1 \, dx dy dz$$

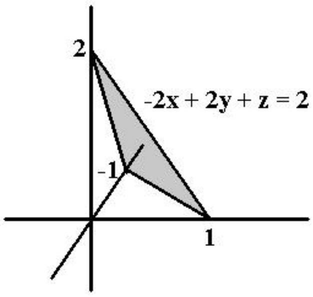


Problem 5.

R is the triangle with vertices $(-1,\ 0,\ 0)$, $(0,\ 1,\ 0)$ and $(0,\ 0,\ 0)$. Let T be the region directly above R but below the plane $-2x + 2y + z = 2$.



Problem 5



$$\int_{-1}^{?}\int_{0}^{?}\int_{0}^{?} 1\,dz\,dy\,dx = \int_{0}^{?}\int_{0}^{?}\int_{y-1+z/2}^{?} 1\,dx\,dy\,dz$$