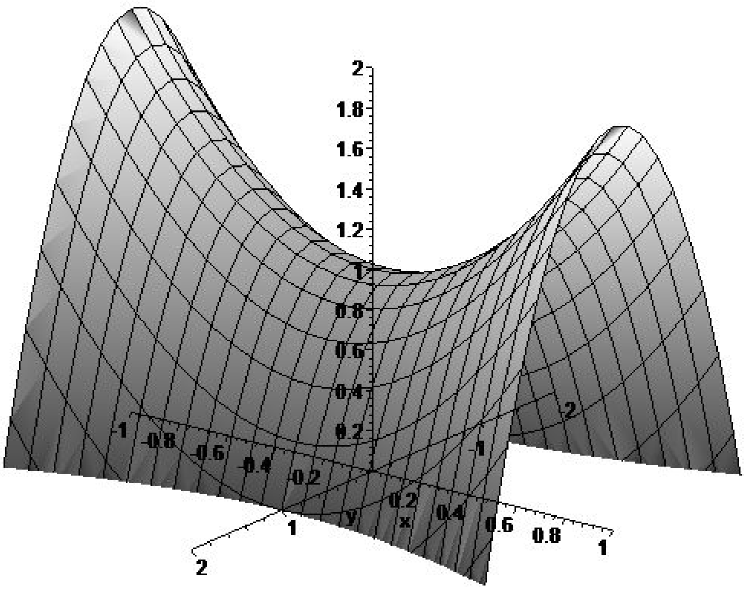
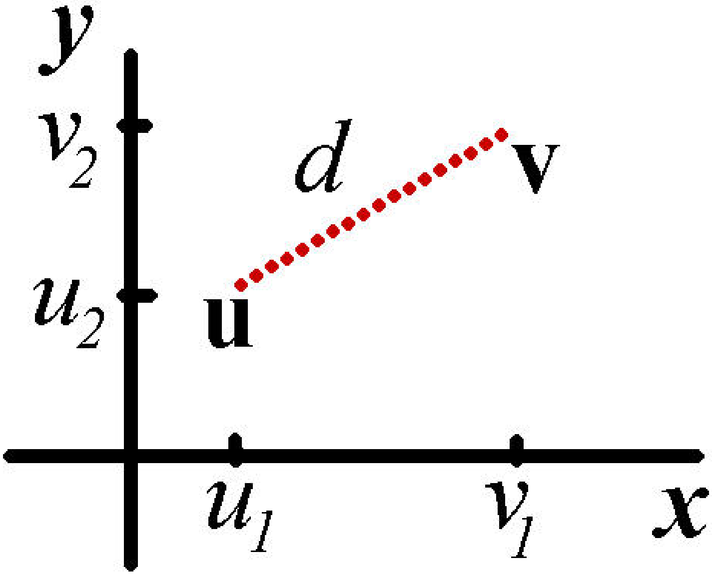
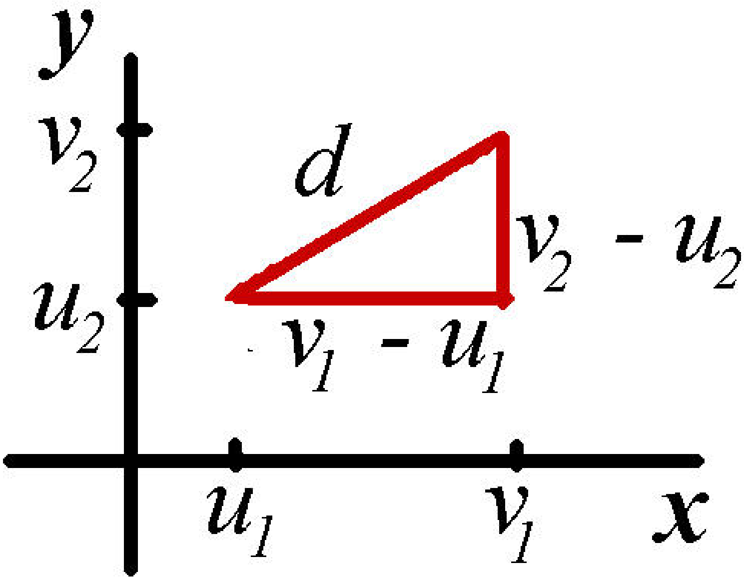
More on Surfaces
Instructor: Elliott Jacobs



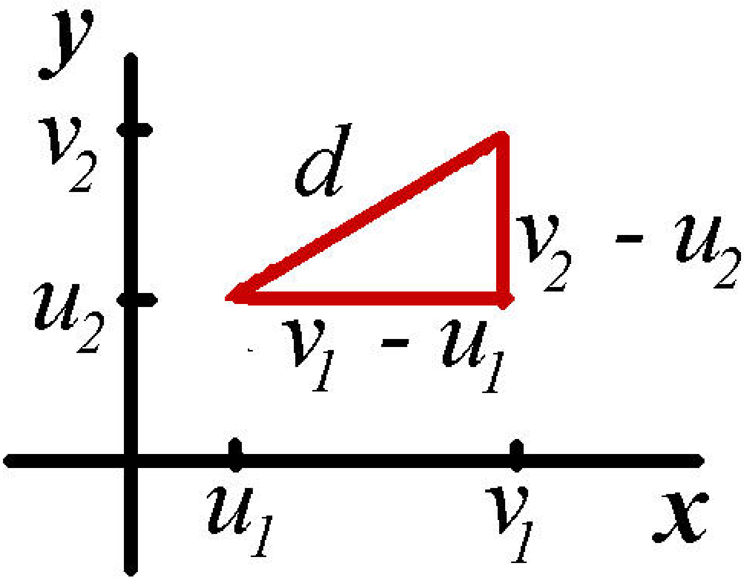
d = distance between $\mathbf{u} = (u_1,\ u_2)$ and $\mathbf{v} = (v_1,\ v_2)$



$$d^2 = (v_1 - u_1)^2 + (v_2 - u_2)^2$$

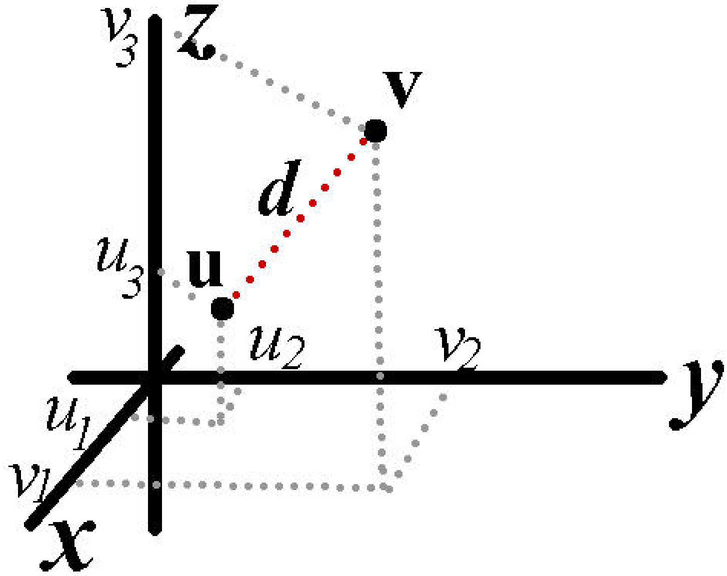


$$d = \sqrt{(v_1 - u_1)^2 + (v_2 - u_2)^2}$$

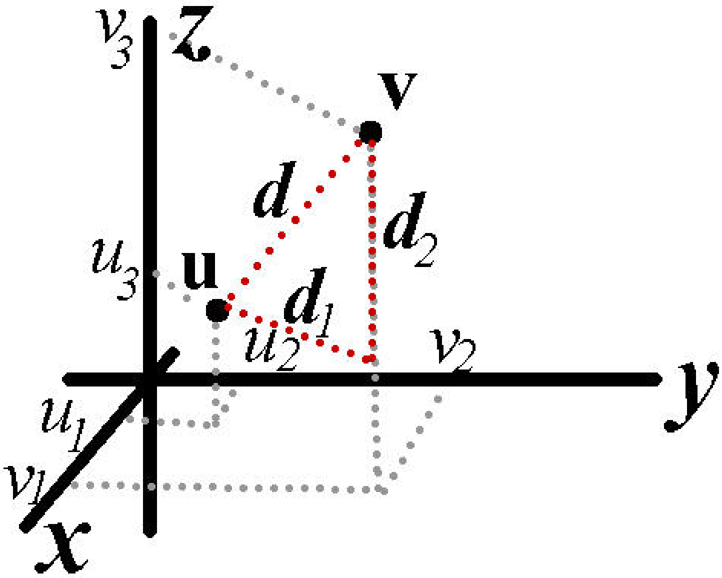


$\mathbf{u} = (u_1,\ u_2,\ u_3)$ $\mathbf{v} = (v_1,\ v_2,\ v_3)$

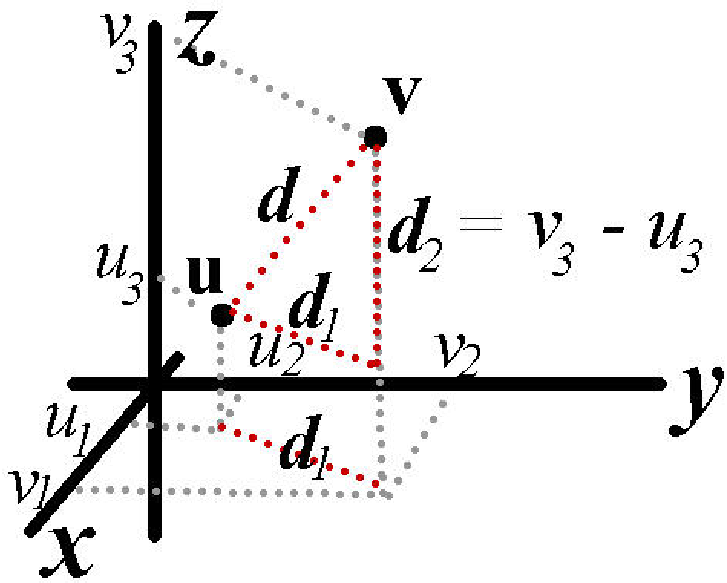
d = distance between $\mathbf{u}$ and $\mathbf{v}$



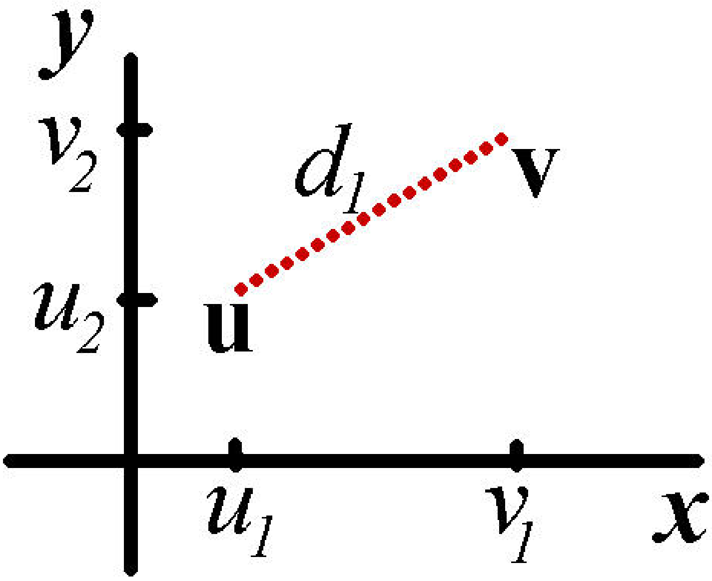
$$d^2 = d_1^2 + d_2^2$$



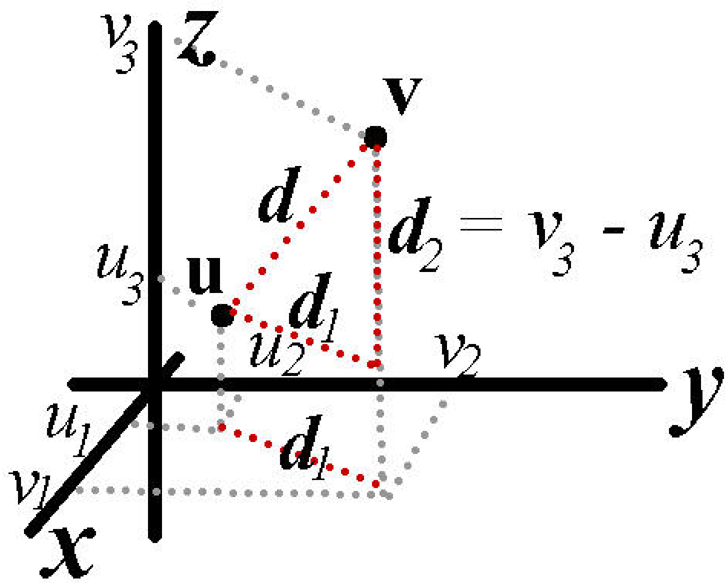
$$d^2 = d_1^2 + d_2^2 = d_1^2 + (v_3 - u_3)^2$$



$$d_1^2 = (v_1 - u_1)^2 + (v_2 - u_2)^2$$

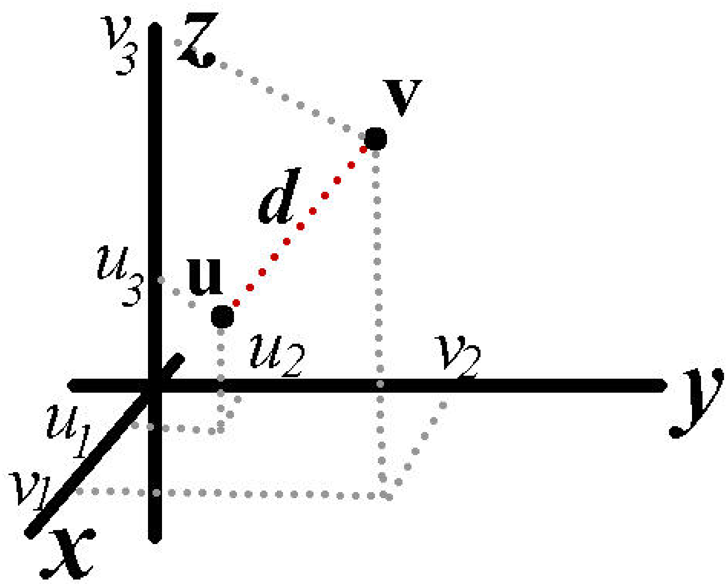


$$d^2 = d_1^2 + d_2^2 = (v_1 - u_1)^2 + (v_2 - u_2)^2 + (v_3 - u_3)^2$$



$$\mathbf{u} = (u_1,\ u_2,\ u_3) \qquad\qquad \mathbf{v} = (v_1,\ v_2,\ v_3)$$

$$d = \sqrt{(v_1 - u_1)^2 + (v_2 - u_2)^2 + (v_3 - u_3)^2}$$



$$\mathbf{u} = (u_1,\ u_2,\ u_3) \qquad \mathbf{v} = (v_1,\ v_2,\ v_3)$$

$$\begin{aligned} d &= \sqrt{(v_1 - u_1)^2 + (v_2 - u_2)^2 + (v_3 - u_3)^2} \\ &= \sqrt{\sum_{i=1}^{3} (v_i - u_i)^2} \end{aligned}$$

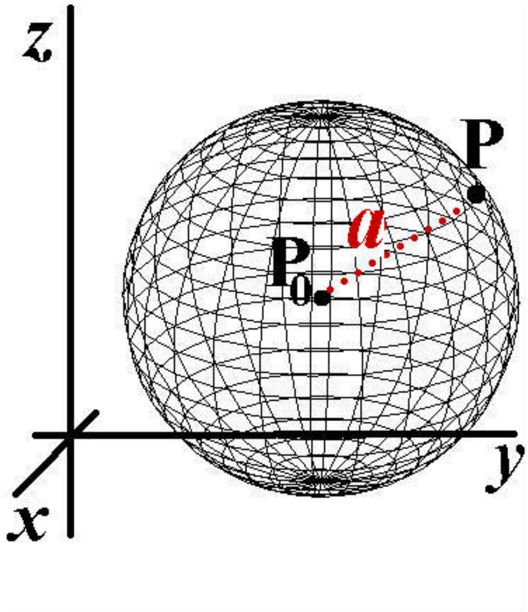
$$\mathbf{u} = (u_1,\ u_2,\ u_3,\ u_4) \qquad \mathbf{v} = (v_1,\ v_2,\ v_3,\ v_4)$$

$$\begin{aligned} d &= \sqrt{(v_1 - u_1)^2 + (v_2 - u_2)^2 + (v_3 - u_3)^2 + (v_4 - u_4)^2} \\ &= \sqrt{\sum_{i=1}^{4} (v_i - u_i)^2} \end{aligned}$$

$$\mathbf{u} = (u_1,\ u_2,\ \cdots,\ u_n) \qquad \mathbf{v} = (v_1,\ v_2,\ \cdots,\ v_n)$$

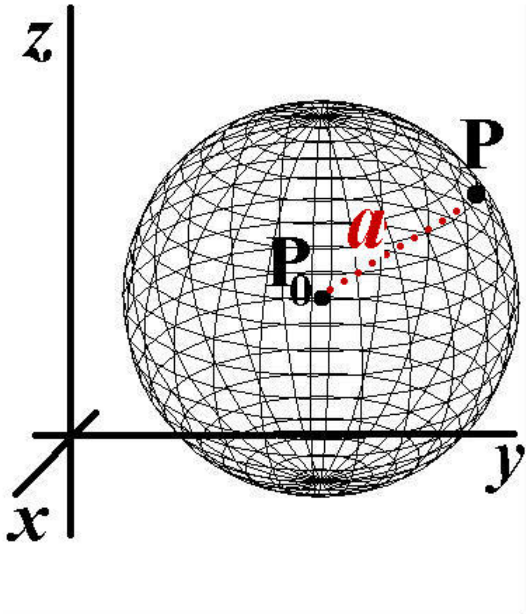
$$d = \sqrt{\sum_{i=1}^{n} (v_i - u_i)^2}$$

Let $\mathbf{P}_0 = (x_0,\ y_0,\ z_0)$ be the center of a sphere of radius a
Let $\mathbf{P} = (x,\ y,\ z)$ be *any* point on the surface of this sphere



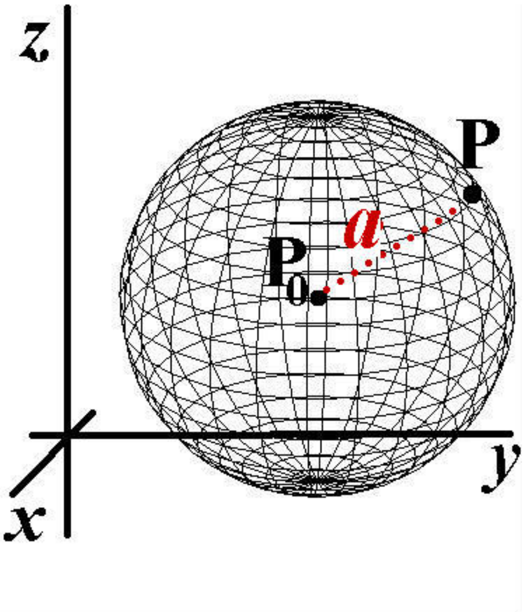
Let $\mathbf{P}_0 = (x_0,\ y_0,\ z_0)$ be the center of a sphere of radius a
Let $\mathbf{P} = (x,\ y,\ z)$ be *any* point on the surface of this sphere

$$\text{Dist between } \mathbf{P} \text{ and } \mathbf{P}_0 = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$$



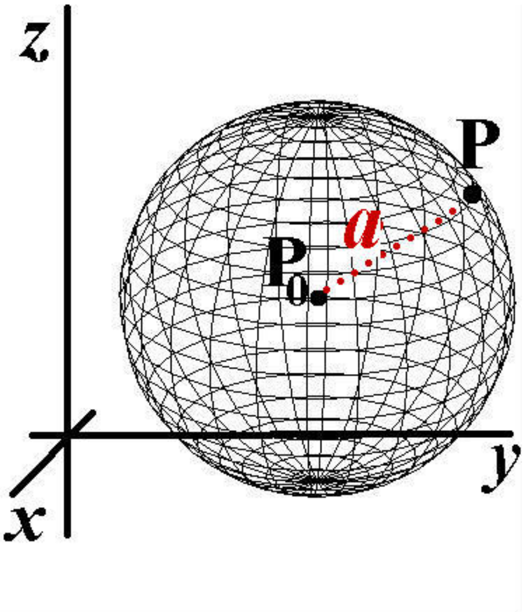
Let $\mathbf{P}_0 = (x_0,\ y_0,\ z_0)$ be the center of a sphere of radius a
Let $\mathbf{P} = (x,\ y,\ z)$ be *any* point on the surface of this sphere

$$\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}=a$$



Let $\mathbf{P}_0 = (x_0,\ y_0,\ z_0)$ be the center of a sphere of radius a
Let $\mathbf{P} = (x,\ y,\ z)$ be *any* point on the surface of this sphere

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2$$



Sketch the sphere described by the equation:

$$(x-1)^2 + (y+2)^2 + z^2 = 4$$

Sketch the sphere described by the equation:

$$(x-1)^2 + (y+2)^2 + z^2 = 4$$

Rewrite as:

$$(x-1)^2 + (y-(-2))^2 + (z-0)^2 = 2^2$$

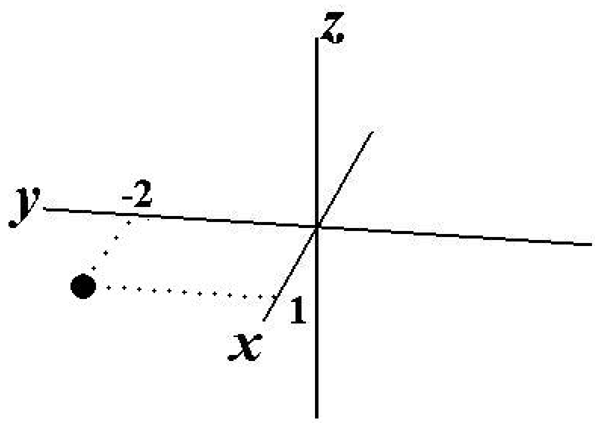
and compare with:

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = a^2$$

The radius is $a = 2$ and the center is $(1,\ -2,\ 0)$

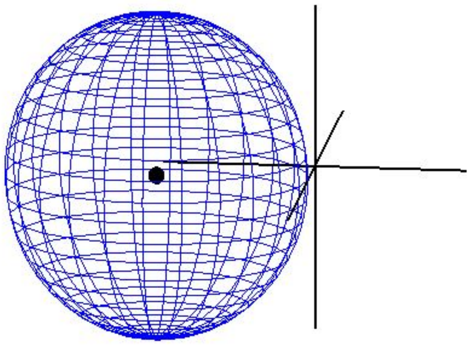
$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2$$

The radius is $a = 2$ and the center is $(1,\ -2,\ 0)$ First draw the center at the correct location:



$$(x-1)^2 + (y-(-2))^2 + (z-0)^2 = 2^2$$

The radius is $a = 2$ and the center is $(1,\ -2,\ 0)$
Next draw the sphere of radius 2 around this center:



Sketch the surface described by the equation:

$$x^2 + y^2 + z^2 = 4z$$

Sketch the surface described by the equation:

$$x^2 + y^2 + z^2 = 4z$$

We must first write the equation in the form:

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2$$

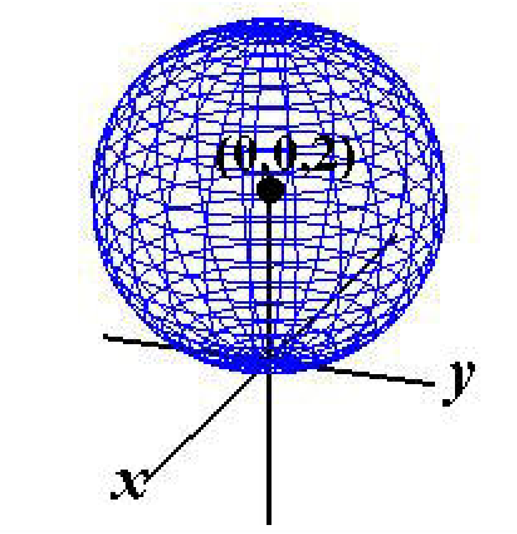
$$x^2 + y^2 + z^2 = 4z$$
$$x^2 + y^2 + z^2 - 4z = 0$$
$$x^2 + y^2 + z^2 - 4z + 4 = 4$$
$$x^2 + y^2 + (z - 2)^2 = 2^2$$

This is a sphere of radius 2 centered around $(0,\ 0,\ 2)$

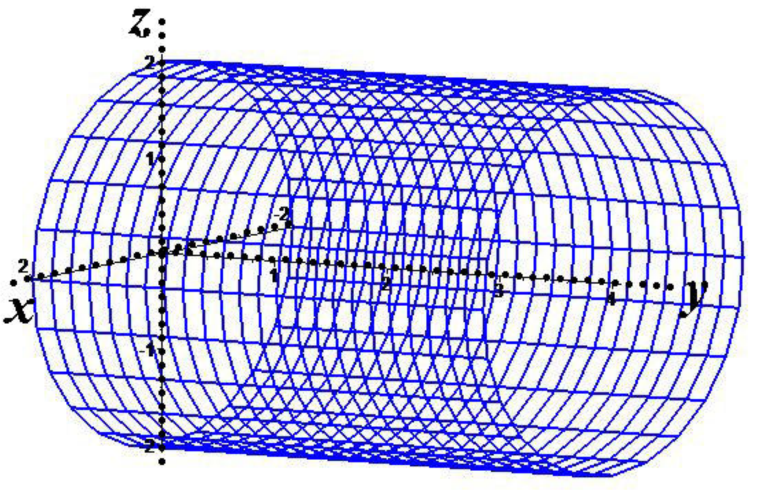
$$x^2 + y^2 + z^2 = 4z$$

$$x^2 + y^2 + (z - 2)^2 = 2^2$$

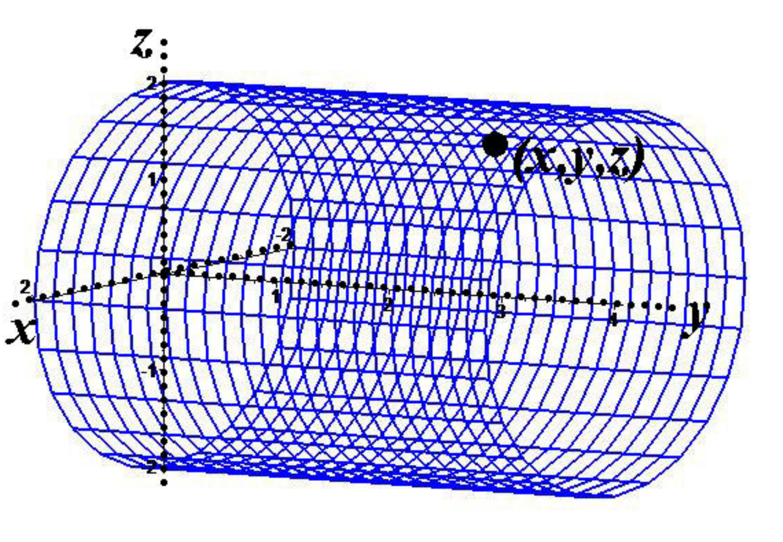
This is a sphere of radius 2 centered around (0, 0, 2)



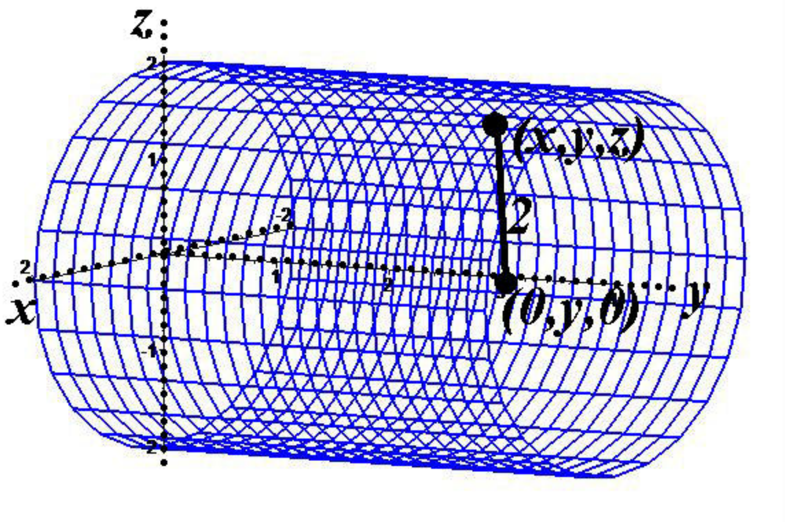
Find the equation of a cylinder of radius 2 centered around the y-axis.



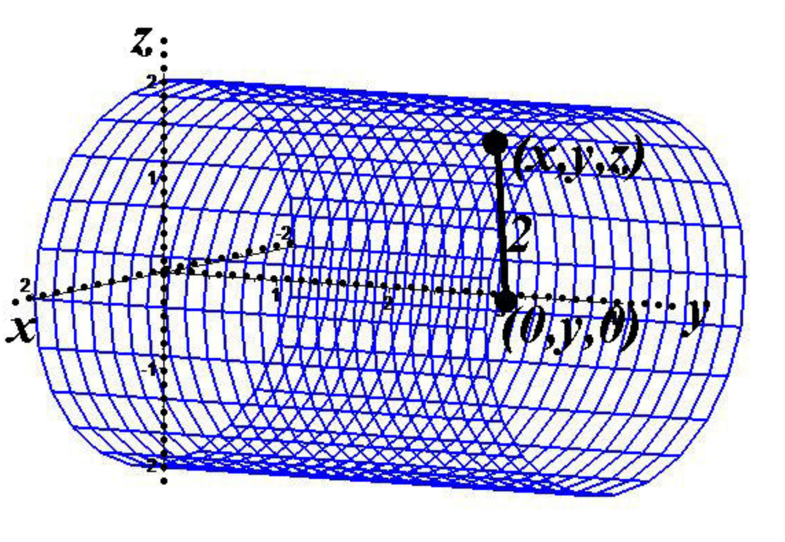
Let $(x,\ y,\ z)$ be any point on the cylinder.



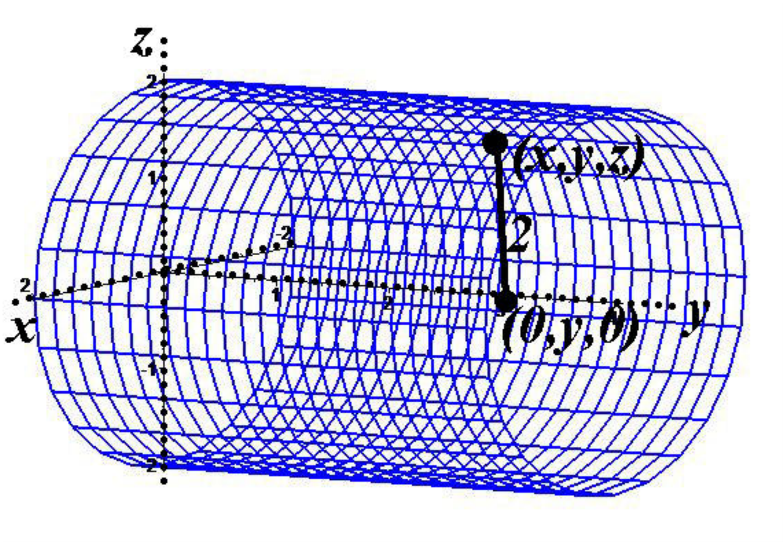
Drop a perpendicular from (x, y, z) to the y-axis.



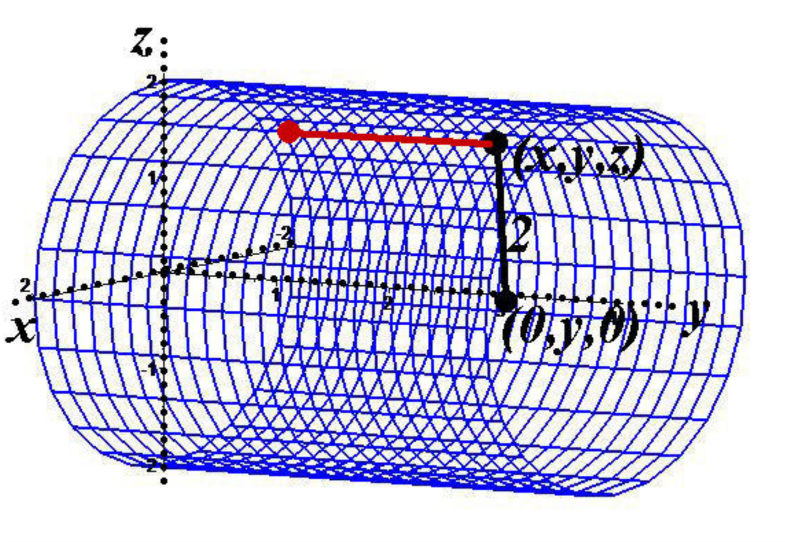
$$\sqrt{(x-0)^2+(y-y)^2+(z-0)^2}=2$$



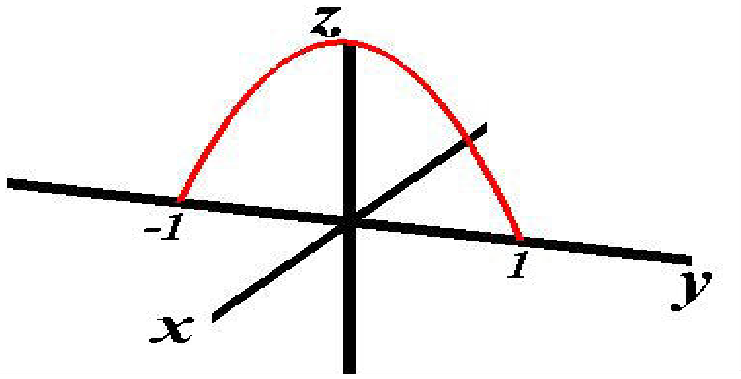
$$x^2 + z^2 = 4$$



$$x^2 + z^2 = 4$$



$$z = 1 - y^2$$



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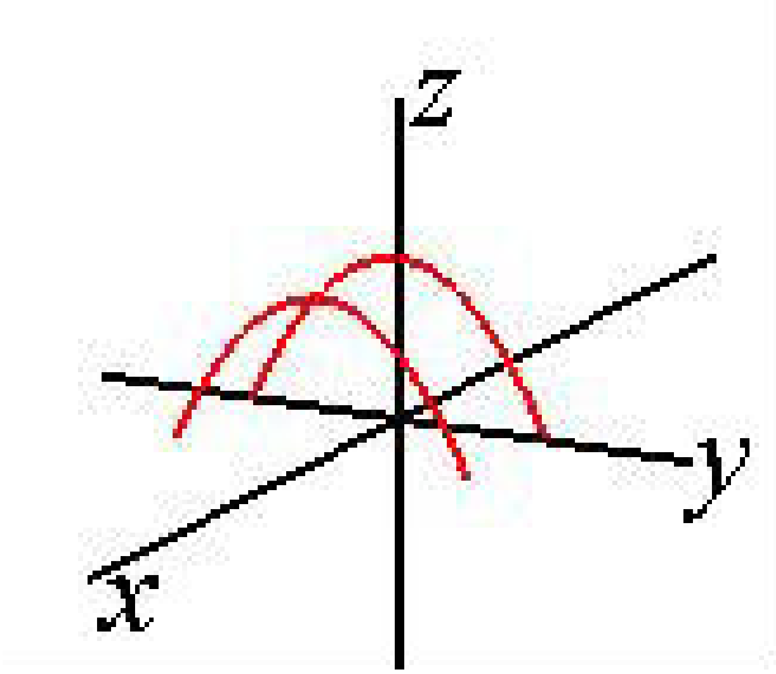

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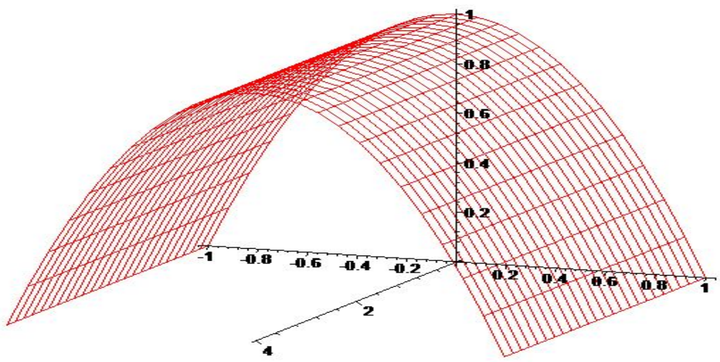
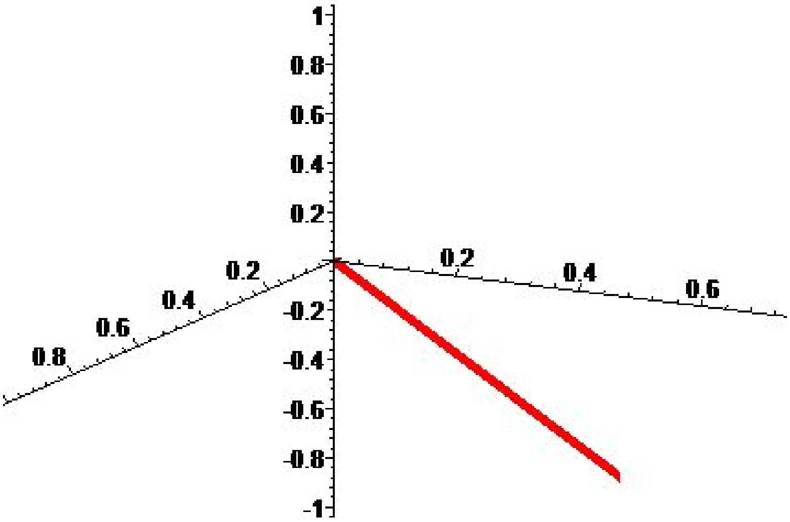


$$y = x$$



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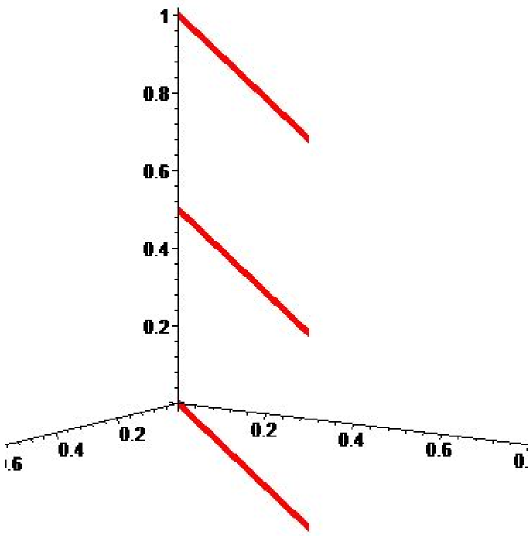

$$y = x$$