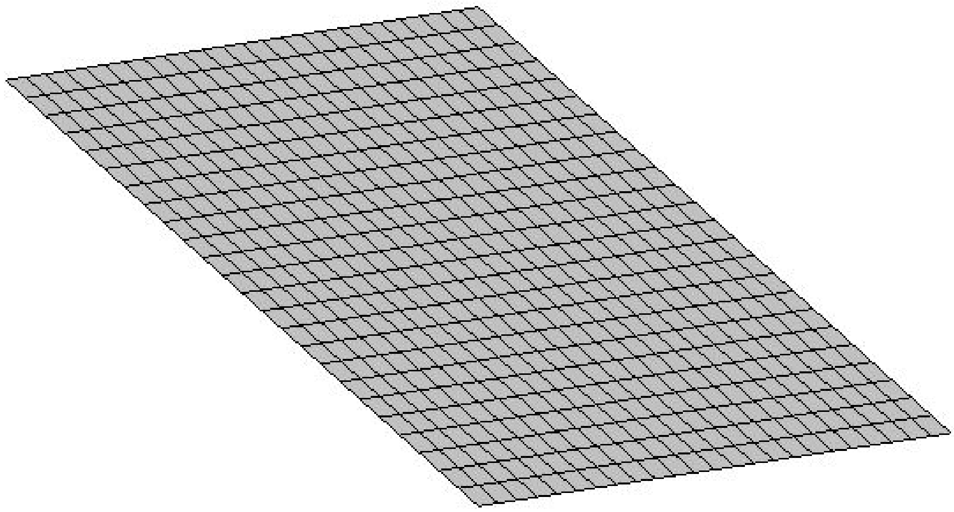
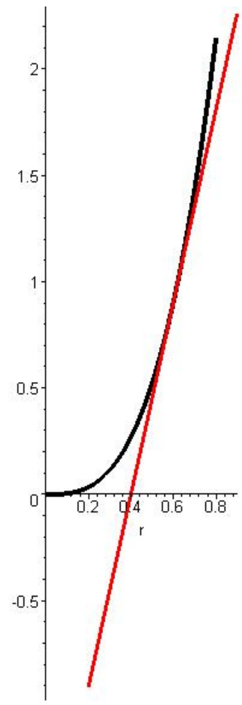
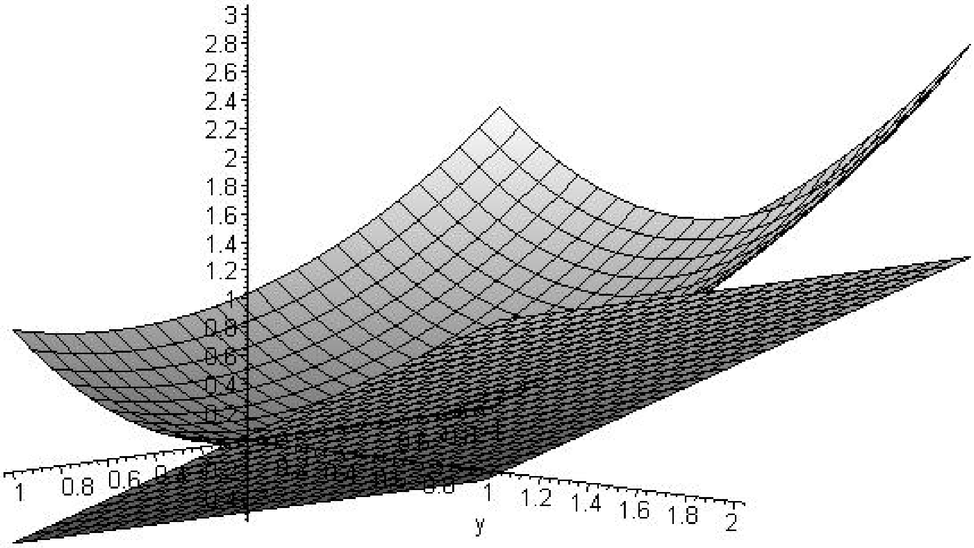
The Equation of a Plane

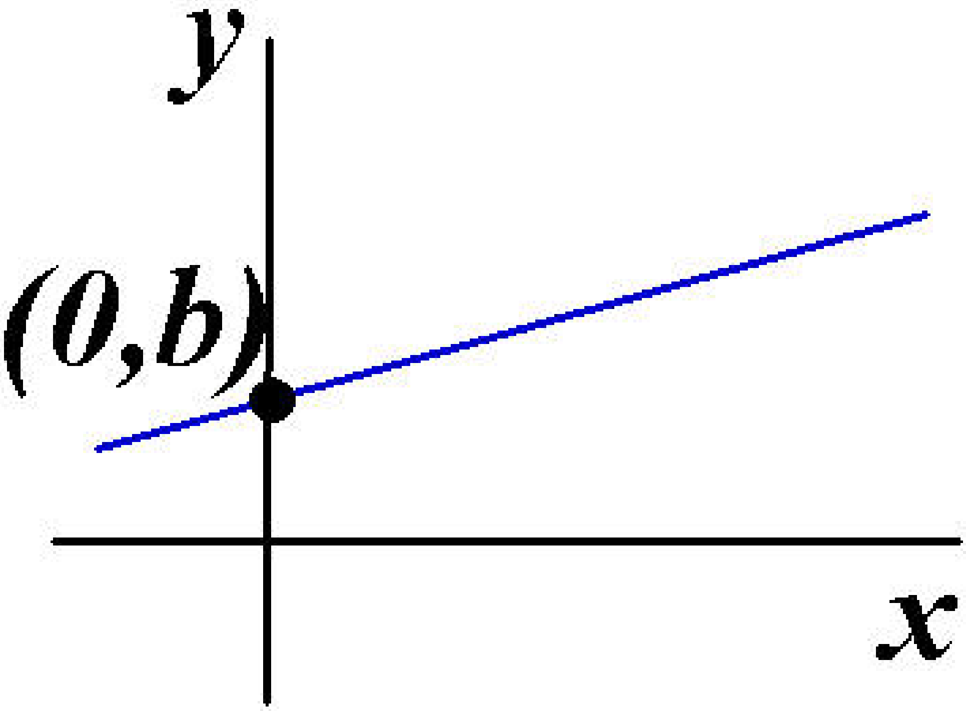




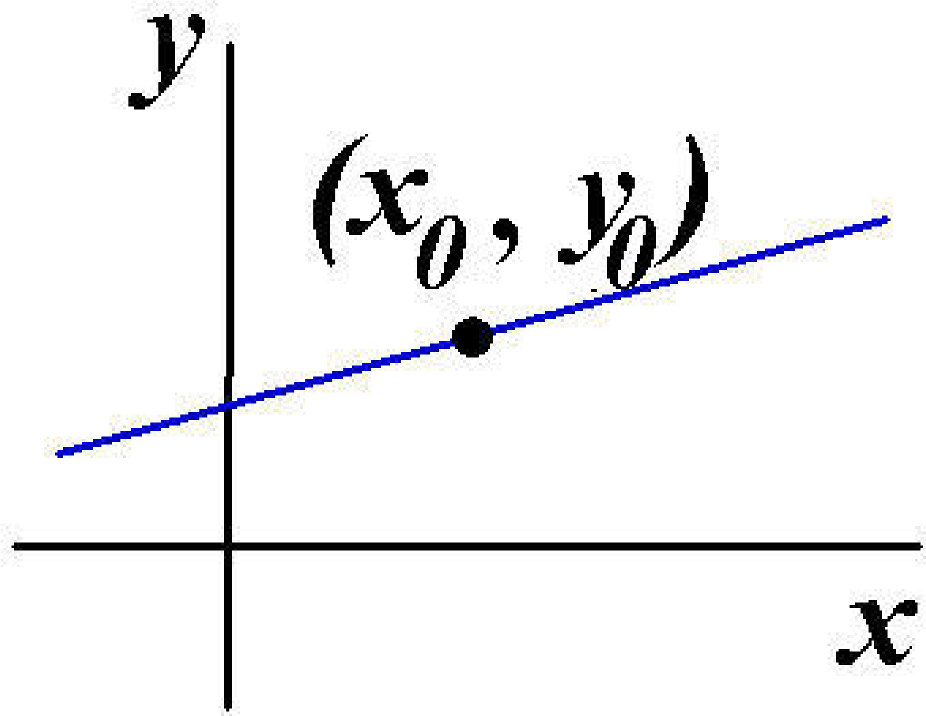
2
1.5
1
0.5
0
-0.5
0.2
0.4
0.6
0.8
r

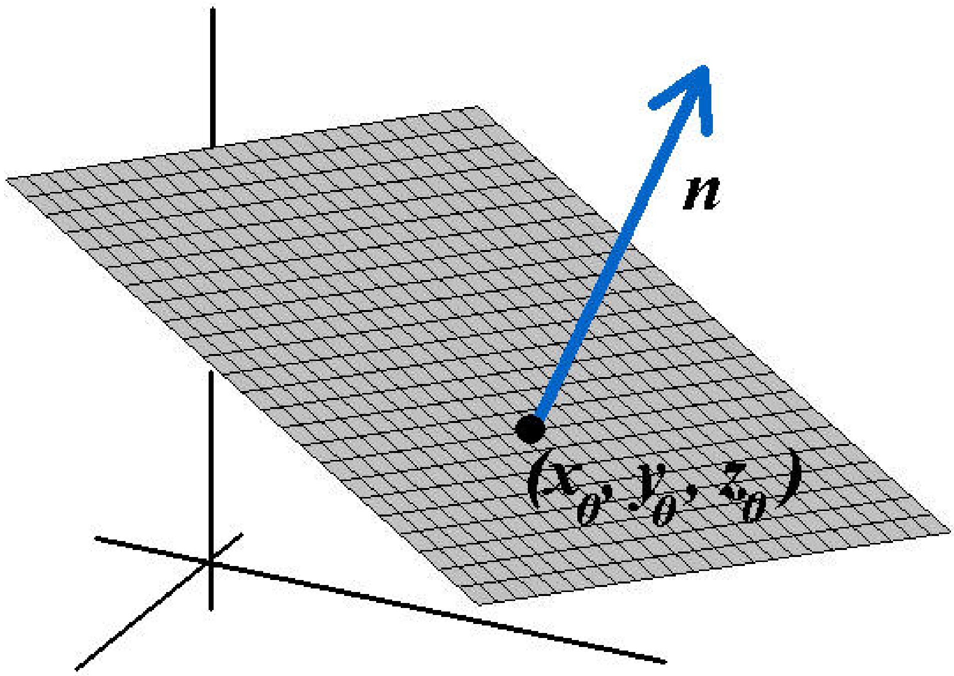

3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
1
0.8
0.6
1
1.2
1.4
1.6
1.8
2
y

$$y = mx + b$$

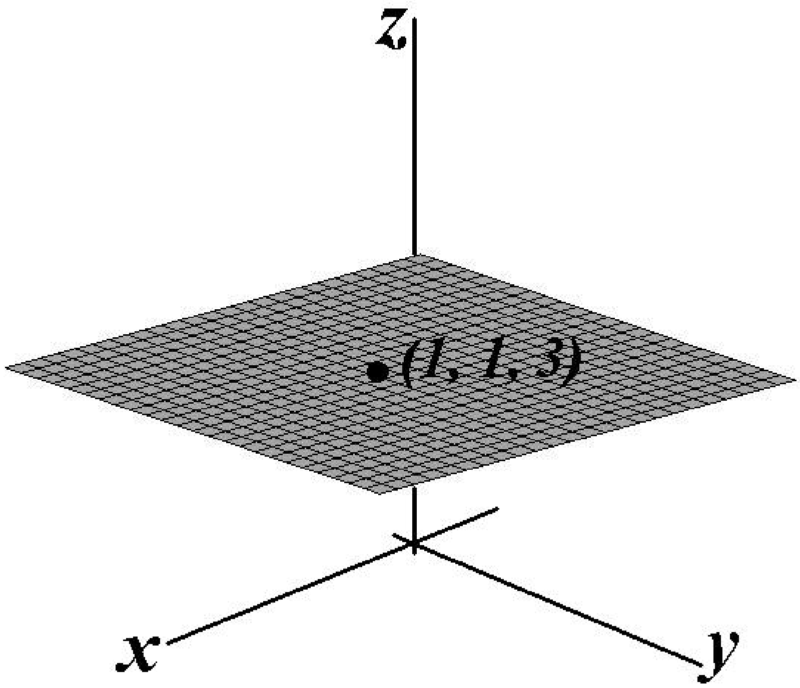


$$y - y_0 = m(x - x_0)$$

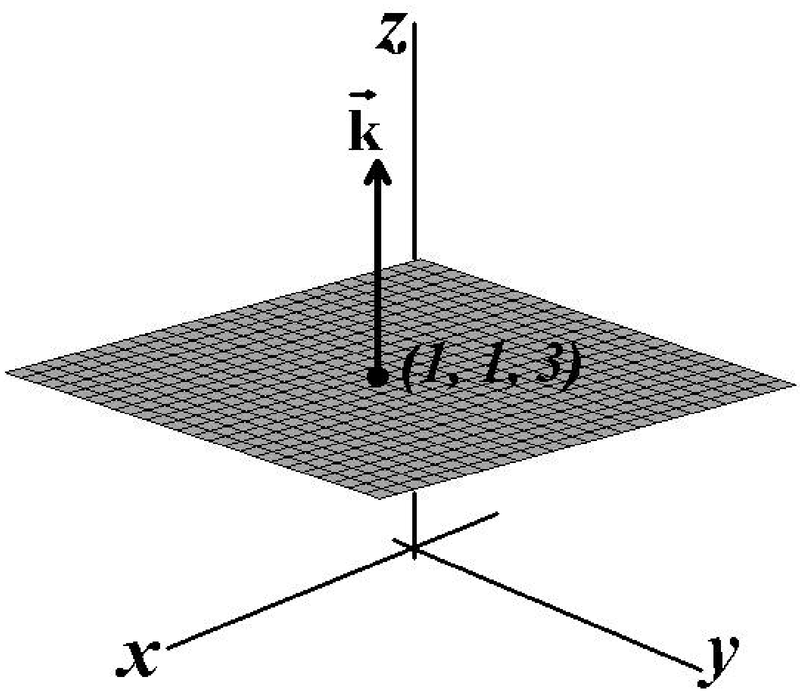



n
(x0, y0, z0)

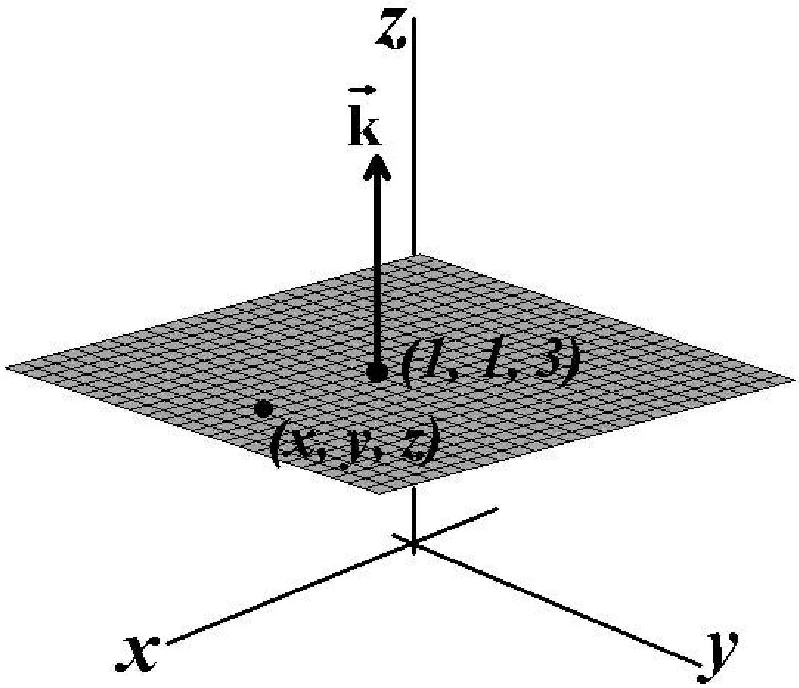
Find the equation of the horizontal plane containing $(1,\ 1,\ 3)$



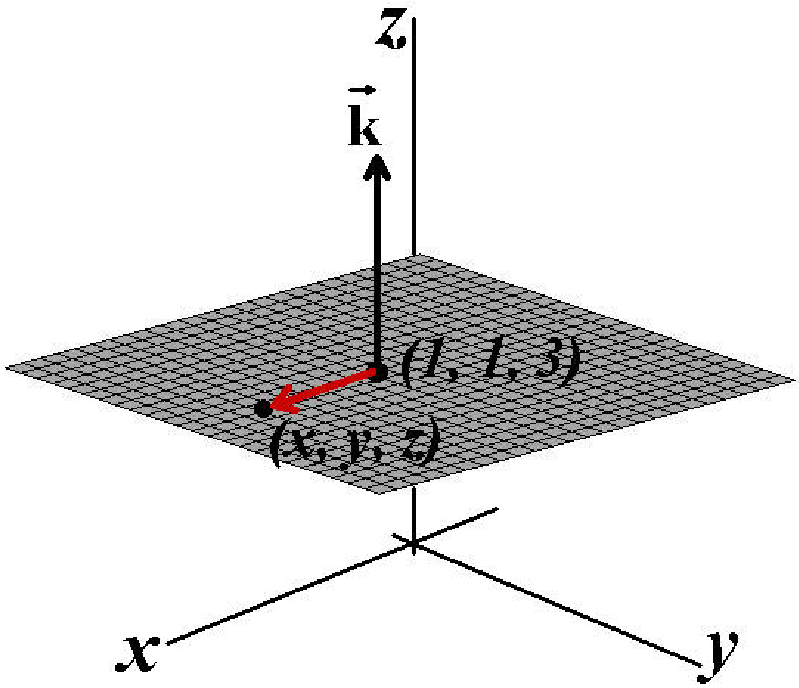
Vector perpendicular to the plane is $\vec{\mathbf{k}} = \langle 0,\ 0,\ 1 \rangle$



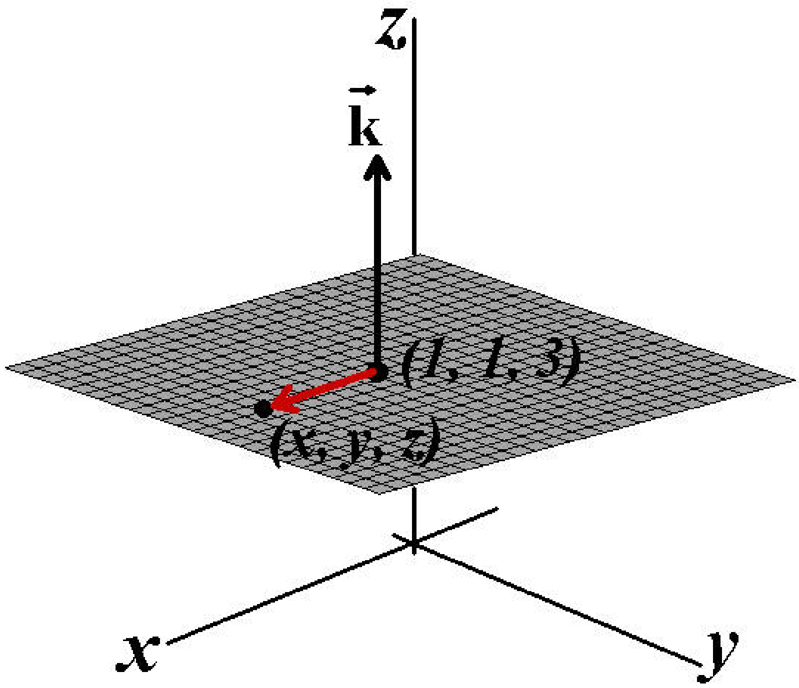
Let $(x,\ y,\ z)$ be any point on this plane



$\langle x,\ y,\ z\rangle - \langle 1,\ 1,\ 3\rangle = \langle x-1,\ y-1,\ z-3\rangle$ lies along the plane



$\vec{\mathbf{k}}$ must be perpendicular to $\langle x-1,\ y-1,\ z-3\rangle$



$$\vec{\mathbf{k}} \bullet \langle x-1,\ y-1,\ z-3\rangle = 0$$

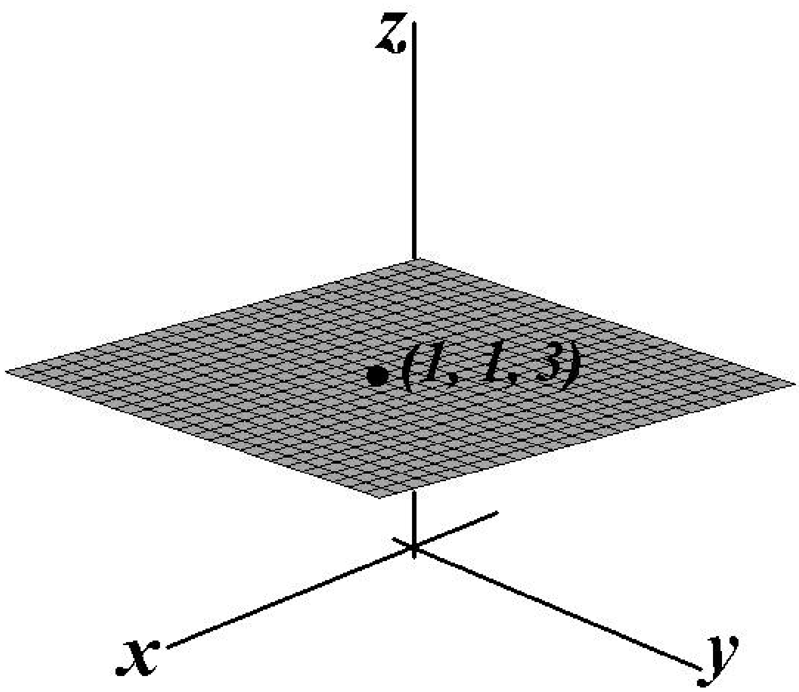
$$\langle 0,\ 0,\ 1\rangle \bullet \langle x-1,\ y-1,\ z-3\rangle = 0$$

$$0 \cdot (x-1) + 0 \cdot (y-1) + 1 \cdot (z-3) = 0$$

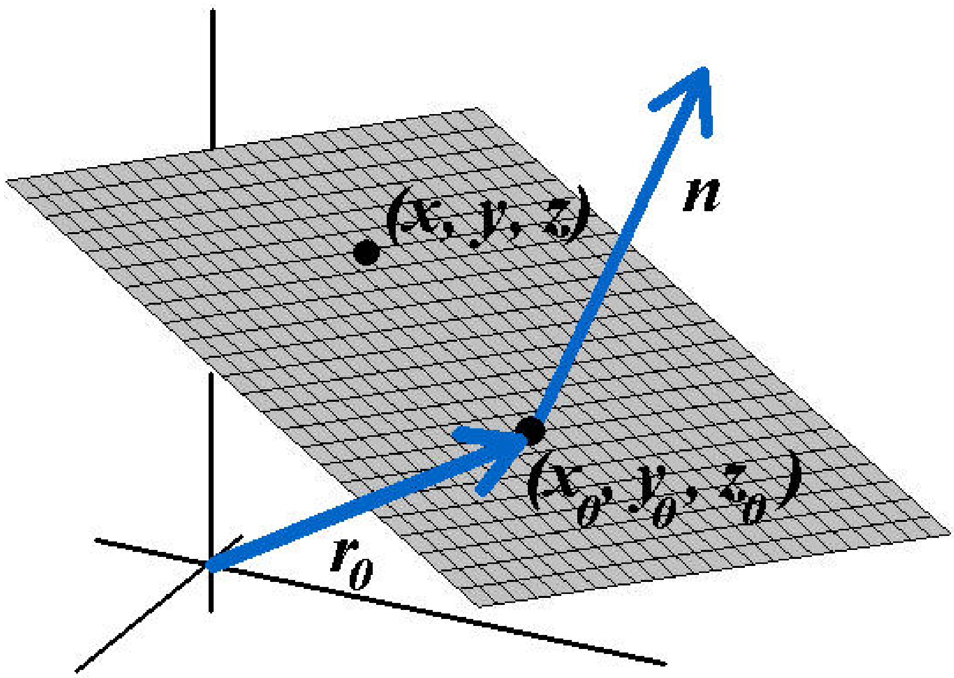
$$z - 3 = 0$$

$$z = 3$$

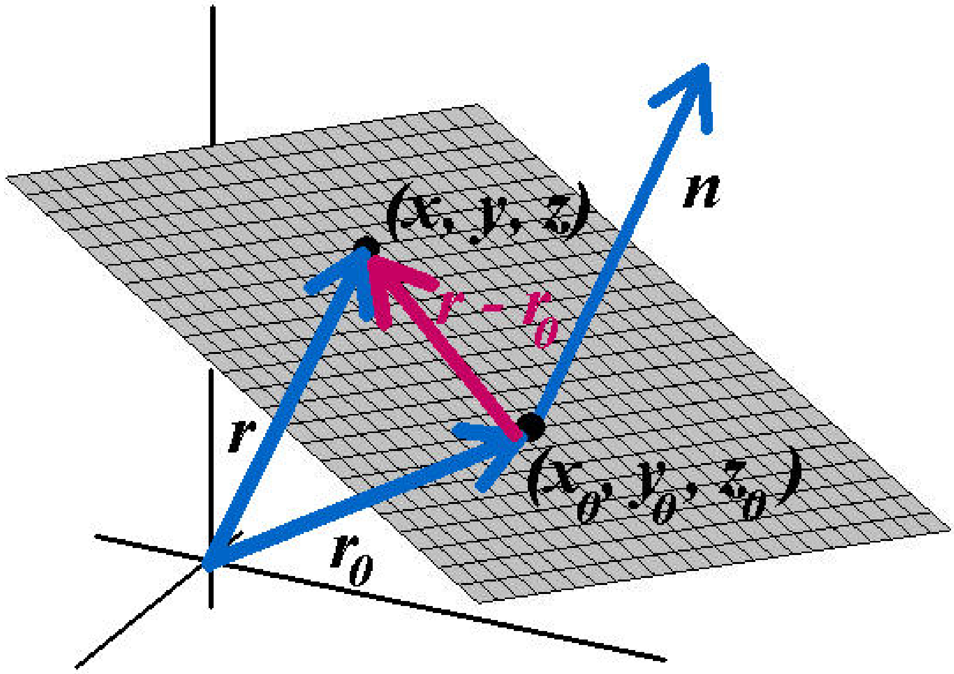
The equation of this horizontal plane is $z = 3$



The General Case



Find the equation relating x, y and z



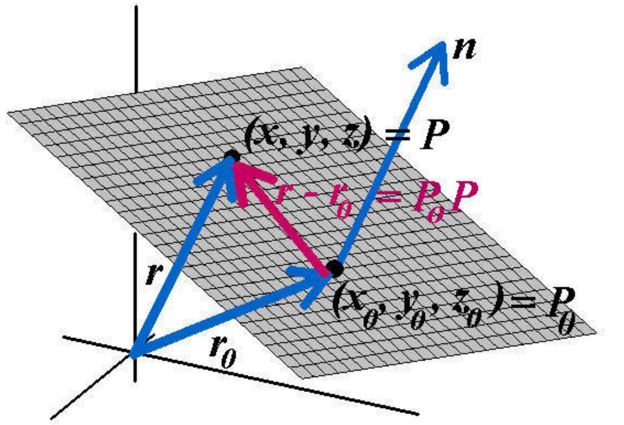
$$\vec{\mathbf{n}} \bullet (\vec{\mathbf{r}} - \vec{\mathbf{r}}_0) = 0$$

Alternative Notation

Let P be the point on the plane that vector $\vec{\mathbf{r}}$ points to
Let P_0 be the point on the plane that vector $\vec{\mathbf{r}}_0$ points to
P_0P is the vector connecting these two points

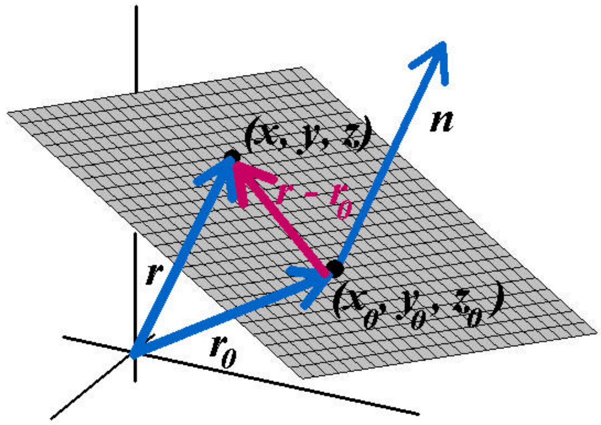
$$P_0P = \vec{\mathbf{r}} - \vec{\mathbf{r}}_0$$

$$\vec{\mathbf{n}} \bullet (\vec{\mathbf{r}} - \vec{\mathbf{r}}_0) = 0 \qquad \vec{\mathbf{n}} \bullet P_0P = 0$$



$$\vec{\mathbf{n}} \bullet (\vec{\mathbf{r}} - \vec{\mathbf{r}}_0) = 0$$

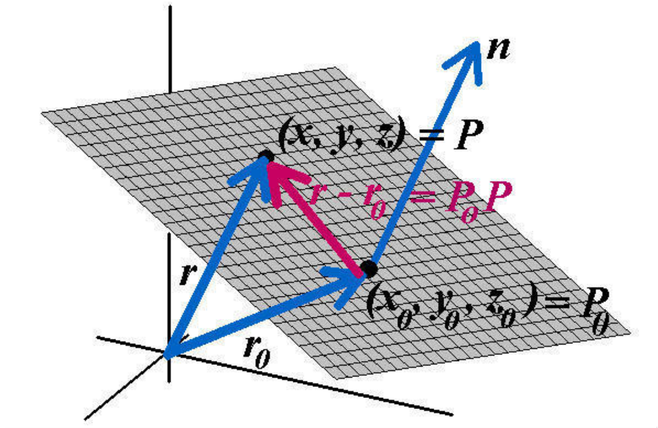
$$\langle a,\ b,\ c\rangle \bullet \langle x - x_0,\ y - y_0,\ z - z_0\rangle = 0$$



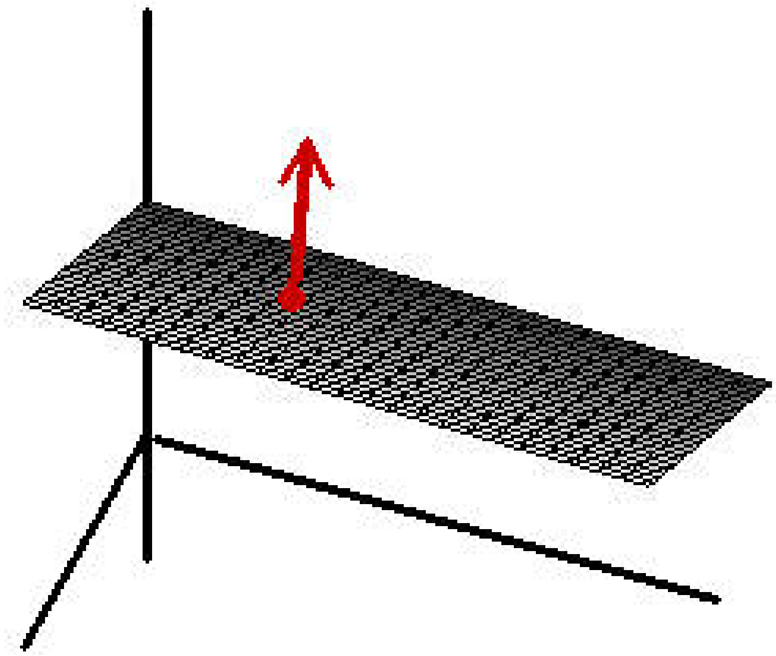
$$\vec{\mathbf{n}} \bullet (\vec{\mathbf{r}} - \vec{\mathbf{r}}_0) = 0$$

$$\langle a,\ b,\ c\rangle \bullet \langle x - x_0,\ y - y_0,\ z - z_0\rangle = 0$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$



Find the equation of the plane containing $(1,\ 1,\ 3)$ and perpendicular to $\vec{\mathbf{n}} = \langle -1,\ 0,\ 1\rangle$



Find the equation of the plane containing (1, 1, 3) and perpendicular to $\vec{\mathbf{n}} = \langle -1,\ 0,\ 1\rangle$

$$\vec{\mathbf{n}} \bullet (\vec{\mathbf{r}} - \vec{\mathbf{r}}_0) = 0$$

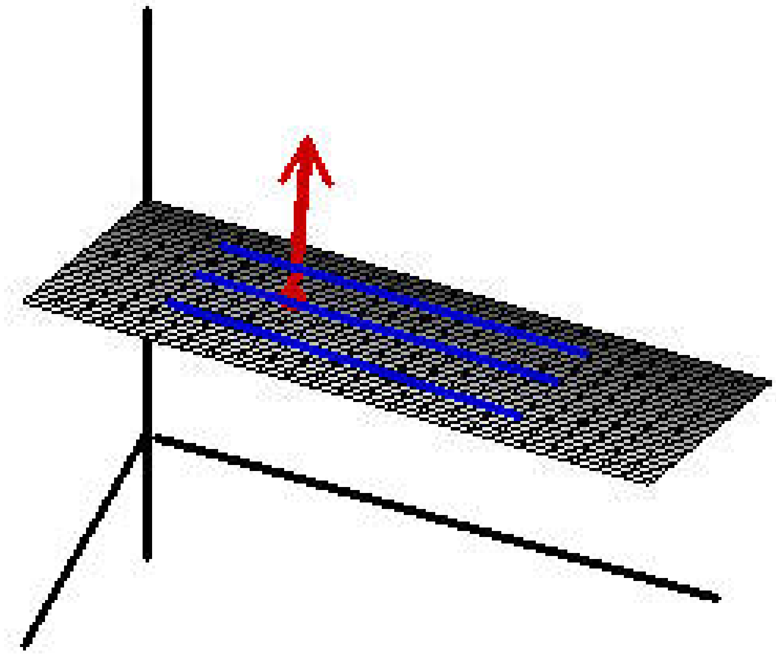
$$\langle -1,\ 0,\ 1\rangle \bullet \langle x-1,\ y-1,\ z-3\rangle = 0$$

$$(-1)(x-1) + (0)(y-1) + (1)(z-3) = 0$$

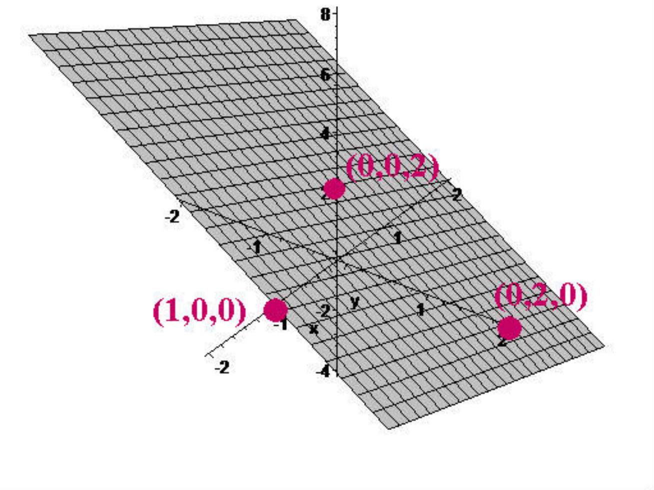
$$-x + 1 + z - 3 = 0$$

$$z = x + 2$$

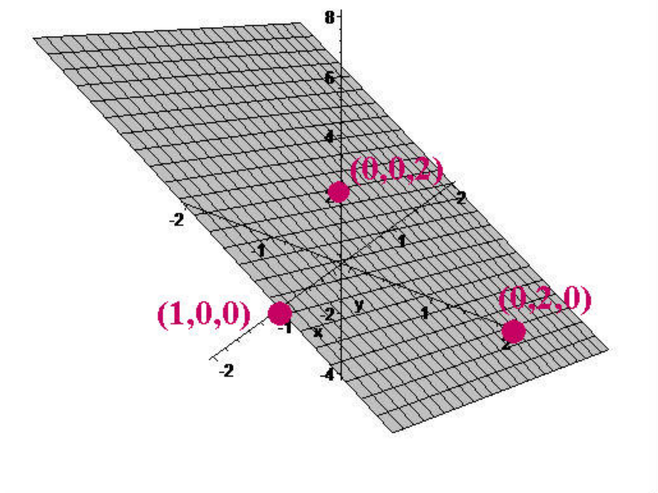
$$z = x + 2$$



Find the equation of the plane that passes through the points $(1,\ 0,\ 0)$, $(0,\ 2,\ 0)$ and $(0,\ 0,\ 2)$



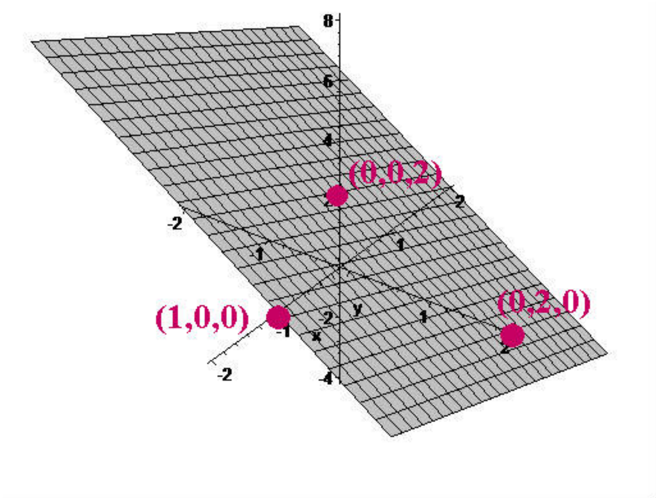
$$\vec{\mathbf{n}} \bullet (\vec{\mathbf{r}} - \vec{\mathbf{r}}_0) = 0$$



$$\vec{\mathbf{n}} \bullet (\vec{\mathbf{r}} - \vec{\mathbf{r}}_0) = 0$$

$$\vec{\mathbf{n}} \bullet (\langle x, y, z \rangle - \langle 1, 0, 0 \rangle) = 0$$

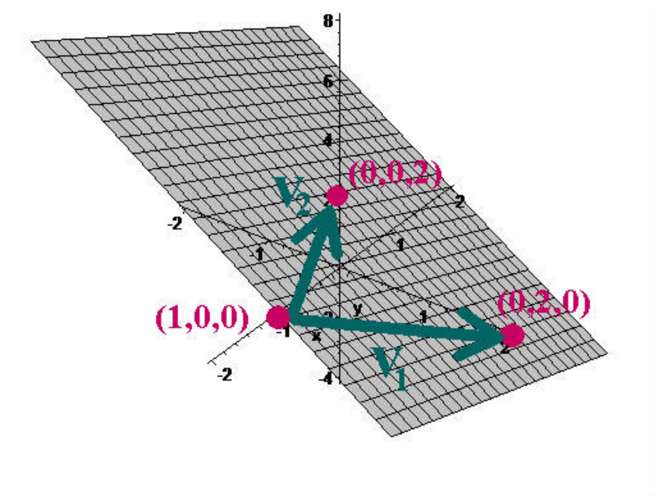
$$\vec{\mathbf{n}} \bullet \langle x - 1,\ \ y,\ \ z \rangle = 0$$



$$\vec{\mathbf{n}} \bullet (\vec{\mathbf{r}} - \vec{\mathbf{r}}_0) = 0$$

$$\vec{\mathbf{n}} \bullet (\langle x, y, z\rangle - \langle 1, 0, 0\rangle) = 0$$

$$\vec{\mathbf{n}} \bullet \langle x-1,\ y,\ z\rangle = 0$$



$$\vec{\mathbf{v}}_1 = \langle 0,2,0\rangle - \langle 1,0,0\rangle = \langle -1,2,0\rangle$$

$$\vec{\mathbf{v}}_2 = \langle 0,0,2\rangle - \langle 1,0,0\rangle = \langle -1,0,2\rangle$$

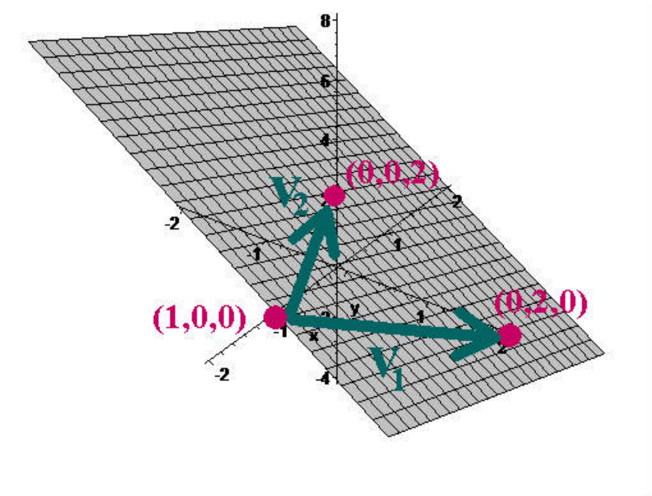
$$\vec{\mathbf{v}}_1 \times \vec{\mathbf{v}}_2 = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ -1 & 2 & 0 \\ -1 & 0 & 2 \end{vmatrix} = \langle 4,\ 2,\ 2\rangle = 2\,\langle 2,\ 1,\ 1\rangle$$

$$\text{Let } \vec{\mathbf{n}} = \langle 2,\ 1,\ 1\rangle$$

$$\vec{\mathbf{n}} \bullet (\vec{\mathbf{r}} - \vec{\mathbf{r}}_0) = 0$$

$$\vec{\mathbf{n}} \bullet \langle x - 1,\ y,\ z \rangle = 0$$

$$\langle 2,\ 1,\ 1 \rangle \bullet \langle x - 1,\ y,\ z \rangle = 0$$

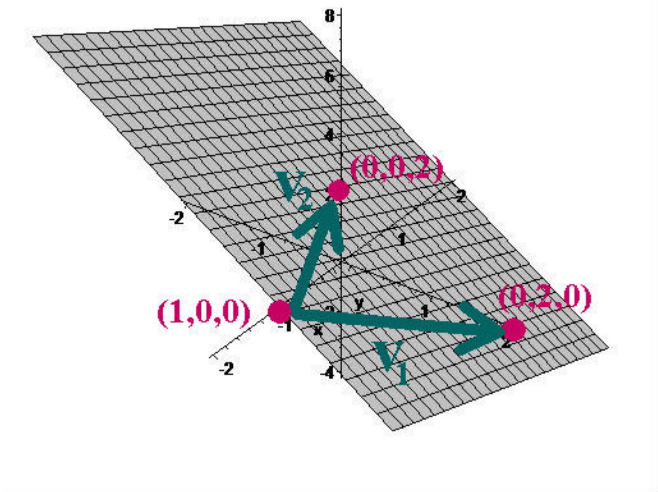


$$\vec{\mathbf{n}} \bullet (\vec{\mathbf{r}} - \vec{\mathbf{r}}_0) = 0$$

$$\langle 2,\ 1,\ 1\rangle \bullet \langle x-1,\ y,\ z\rangle = 0$$

$$2(x-1) + 1y + 1z = 0$$

$$2x + y + z = 2$$

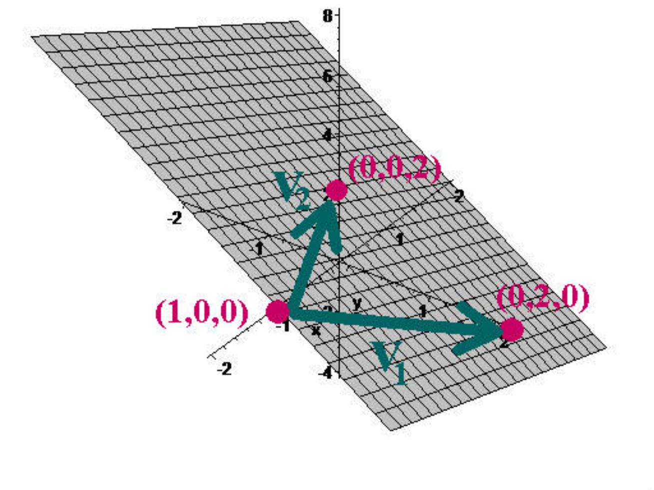


$$\vec{\mathbf{n}} \bullet (\vec{\mathbf{r}} - \vec{\mathbf{r}}_0) = 0 \qquad \text{with } \vec{\mathbf{r}}_0 = \langle 0,\ 0,\ 2 \rangle$$

$$\langle 2,\ 1,\ 1 \rangle \bullet \langle x - 0,\ y - 0,\ z - 2 \rangle = 0$$

$$2(x - 0) + 1(y - 0) + 1(z - 2) = 0$$

$$2x + y + z = 2$$

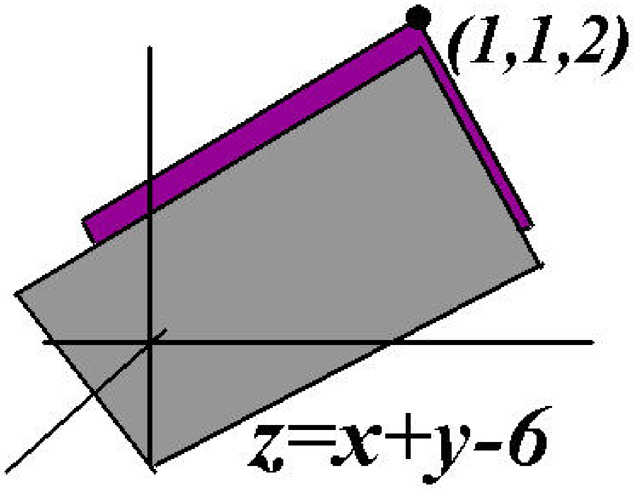


Let $\vec{\mathbf{n}} = \langle a,\ b,\ c \rangle$

The equation $\vec{\mathbf{n}} \bullet (\vec{\mathbf{r}} - \vec{\mathbf{r}}_0) = 0$ can always be written in the form:

$$ax + by + cz = d$$

Find the equation of the plane that passes through (1, 1, 2) and is parallel to the plane $z = x + y - 6$

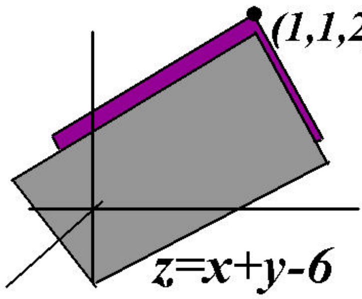


Find the equation of the plane that passes through (1, 1, 2) and is parallel to the plane $z = x + y - 6$

$$\vec{\mathbf{n}} \bullet (\vec{\mathbf{r}} - \vec{\mathbf{r}}_0) = 0$$

$$\vec{\mathbf{n}} \bullet (\langle x, y, z\rangle - \langle 1, 1, 2\rangle) = 0$$

$$\vec{\mathbf{n}} \bullet \langle x - 1,\ y - 1,\ z - 2\rangle = 0$$



Find the equation of the plane that passes through $(1,\ 1,\ 2)$ and is parallel to the plane $z = x + y - 6$

$z = x + y - 6$ can also be written in the form $ax + by + cz = d$

$$x + y - z = 6$$

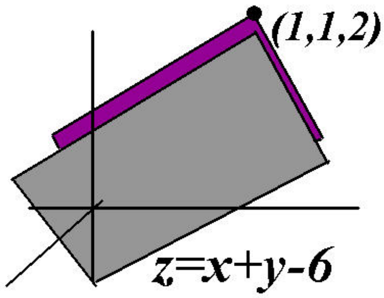
$$(1)x + (1)y + (-1)z = 6$$

$$\vec{\mathbf{n}} = \langle 1,\ 1,\ -1 \rangle$$

Find the equation of the plane that passes through $(1,\ 1,\ 2)$ and is parallel to the plane $z = x + y - 6$

$$\vec{\mathbf{n}} \bullet (\langle x, y, z\rangle - \langle 1, 1, 2\rangle) = 0$$

$$\langle 1,\ 1,\ -1\rangle \bullet \langle x-1,\ y-1,\ z-2\rangle = 0$$



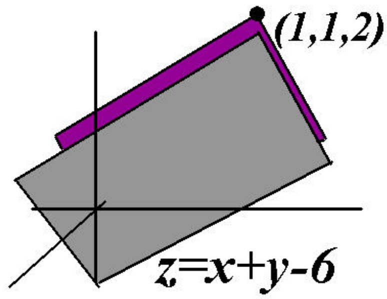
$$\vec{\mathbf{n}} \bullet (\vec{\mathbf{r}} - \vec{\mathbf{r}}_0) = 0$$

$$\langle 1,\ 1,\ -1 \rangle \bullet \langle x-1,\ y-1,\ z-2 \rangle = 0$$

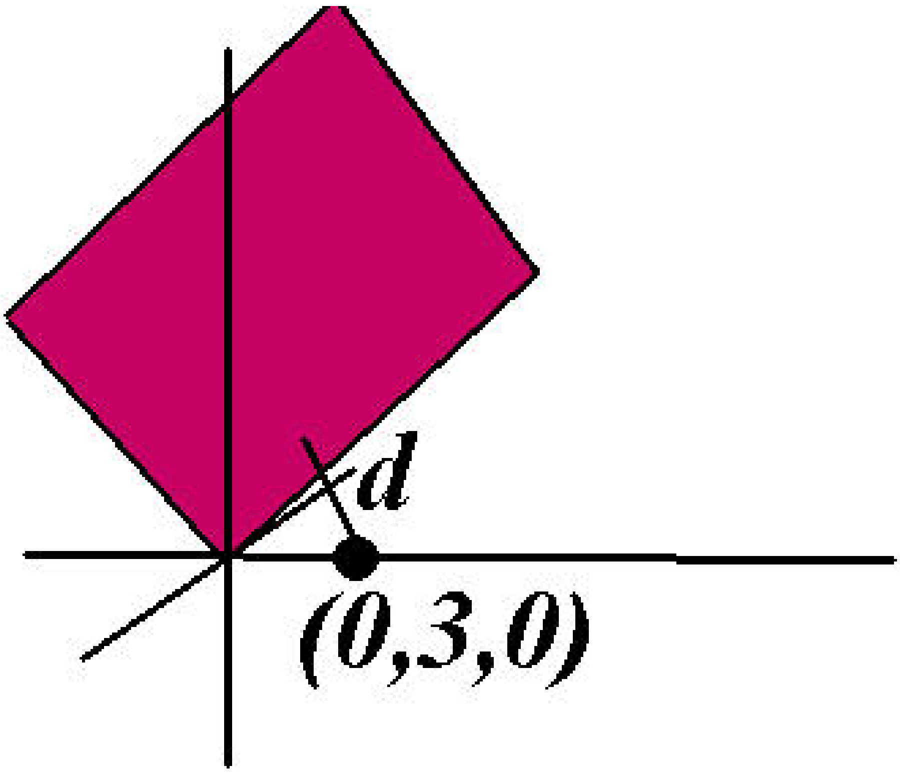
$$1(x-1) + 1(y-1) - 1(z-2) = 0$$

$$x + y - z = 0$$

$$z = x + y$$



Find the distance from the point $(0,\ 3,\ 0)$ to the plane $z = x+y$



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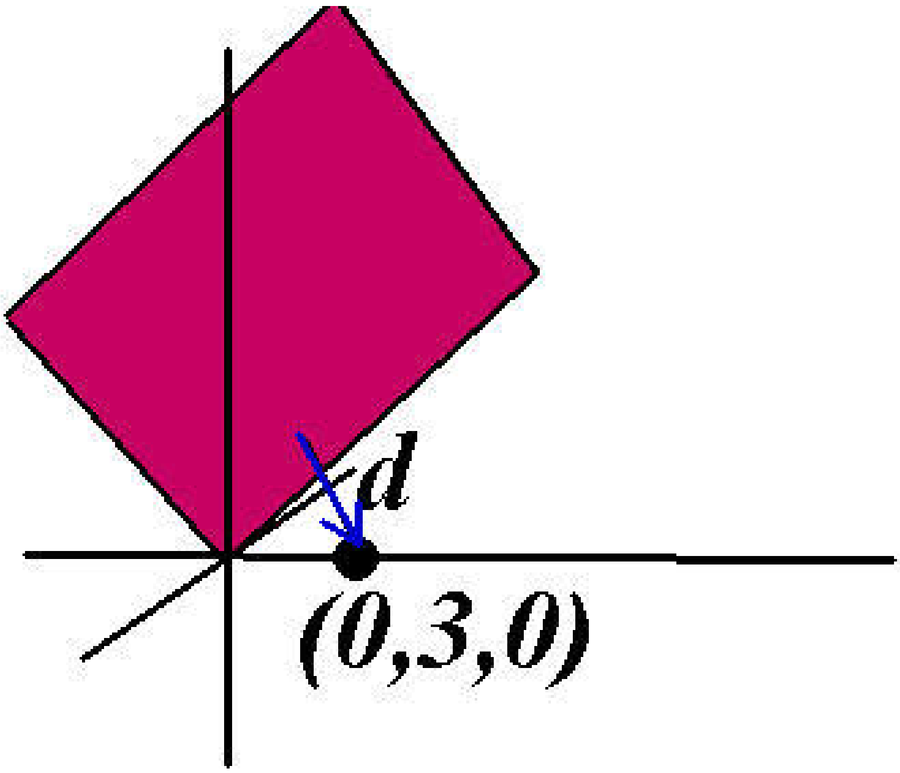

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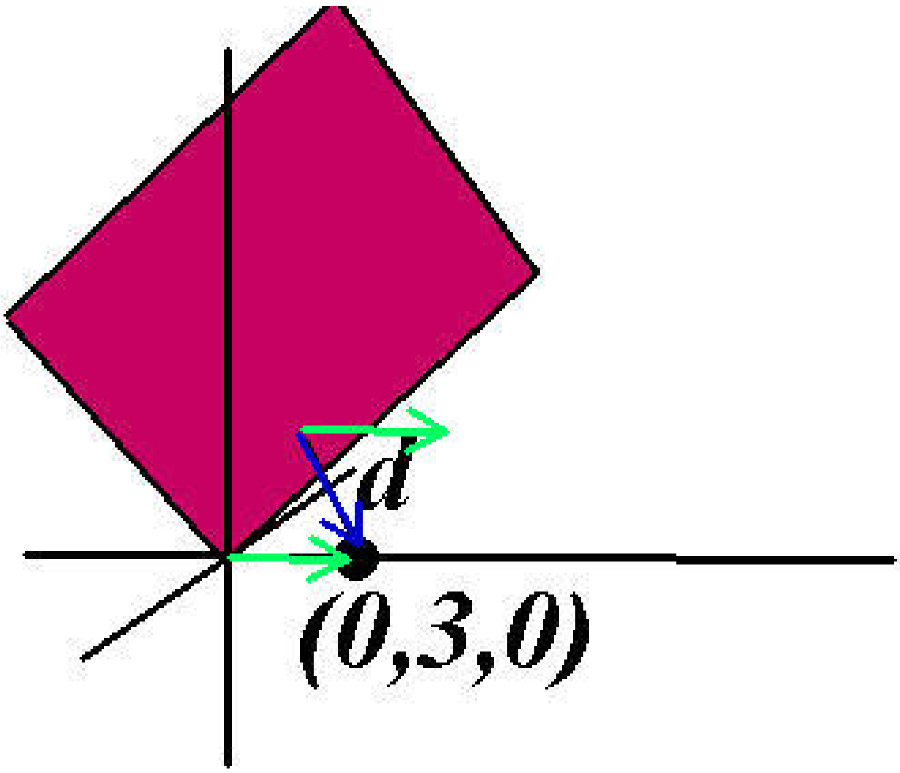

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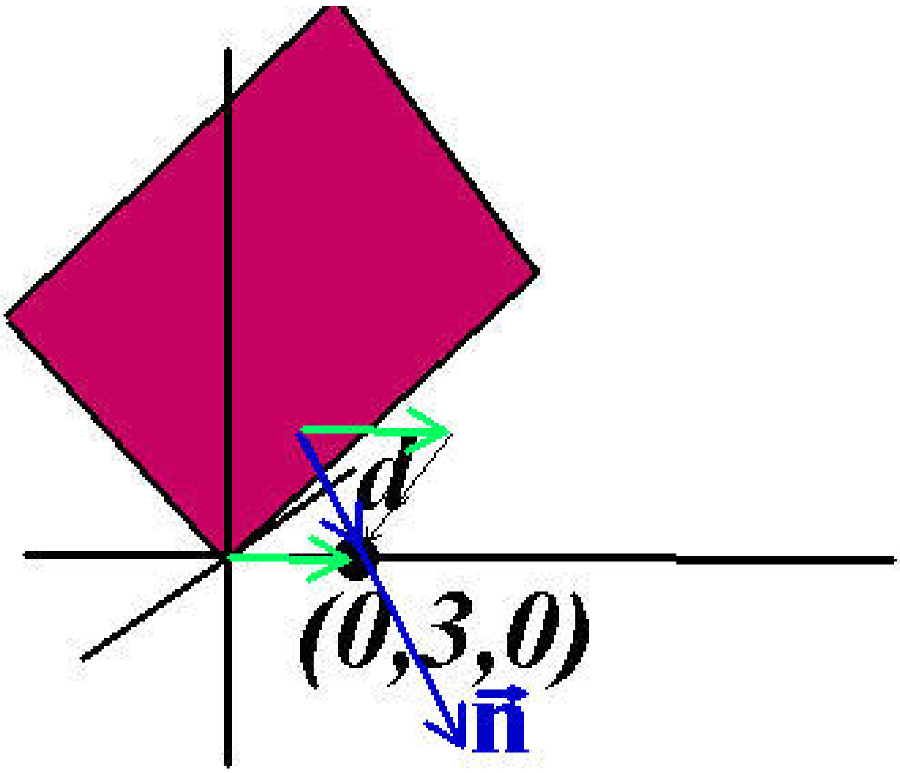
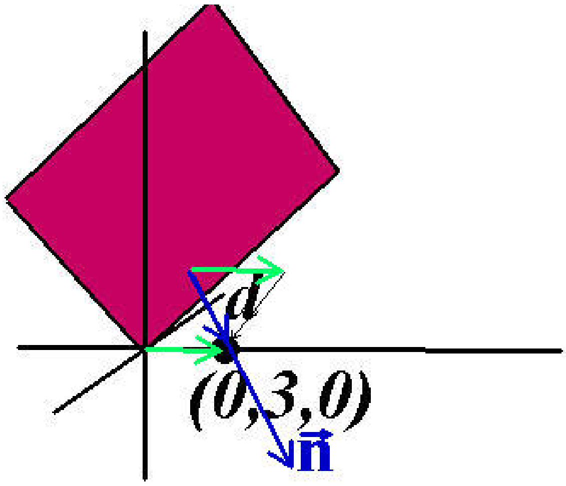


$$d = \left|\text{proj}_{\vec{\mathbf{n}}}\, \langle 0,\ 3,\ 0 \rangle\right|$$



$z = x + y$ is equivalent to $(1)x + (1)y + (-1)z = 0$

$$\vec{\mathbf{n}} = \langle 1,\ 1,\ -1 \rangle$$

$$\begin{aligned} d &= \left|\operatorname{proj}_{\vec{\mathbf{n}}} \langle 0,\ 3,\ 0 \rangle\right| \\ &= \langle 0,\ 3,\ 0 \rangle \bullet \frac{1}{|\vec{\mathbf{n}}|}\,\vec{\mathbf{n}} \\ &= \langle 0,\ 3,\ 0 \rangle \bullet \frac{1}{\sqrt{3}} \langle 1,\ 1,\ -1 \rangle \\ &= \frac{3}{\sqrt{3}} = \sqrt{3} \end{aligned}$$

We can look at this problem as a minimization problem

$$d = \sqrt{x^2 + (y-3)^2 + z^2}$$