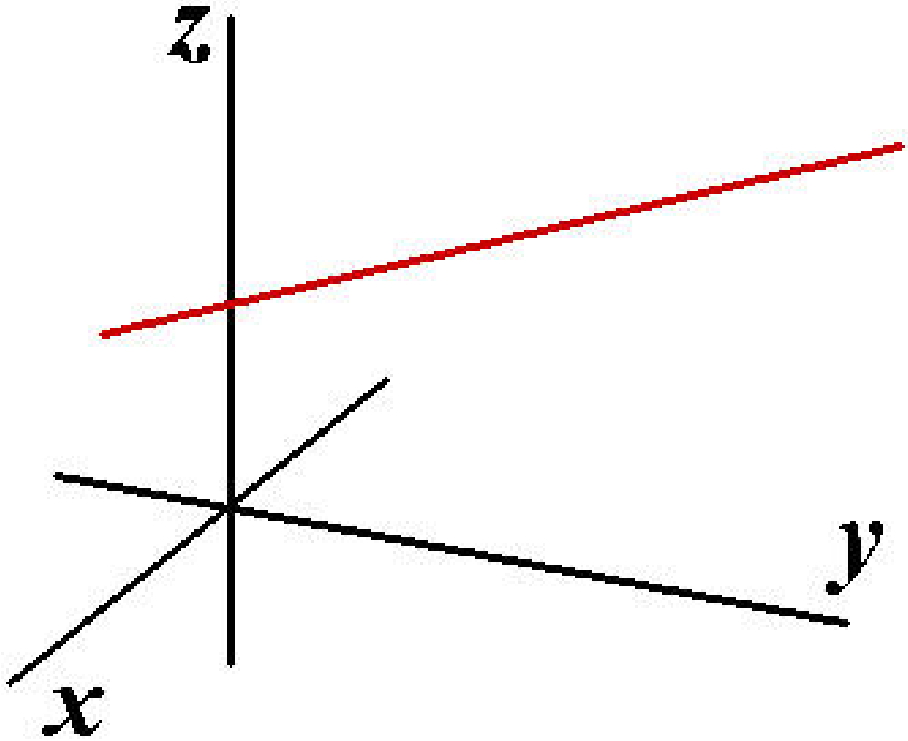
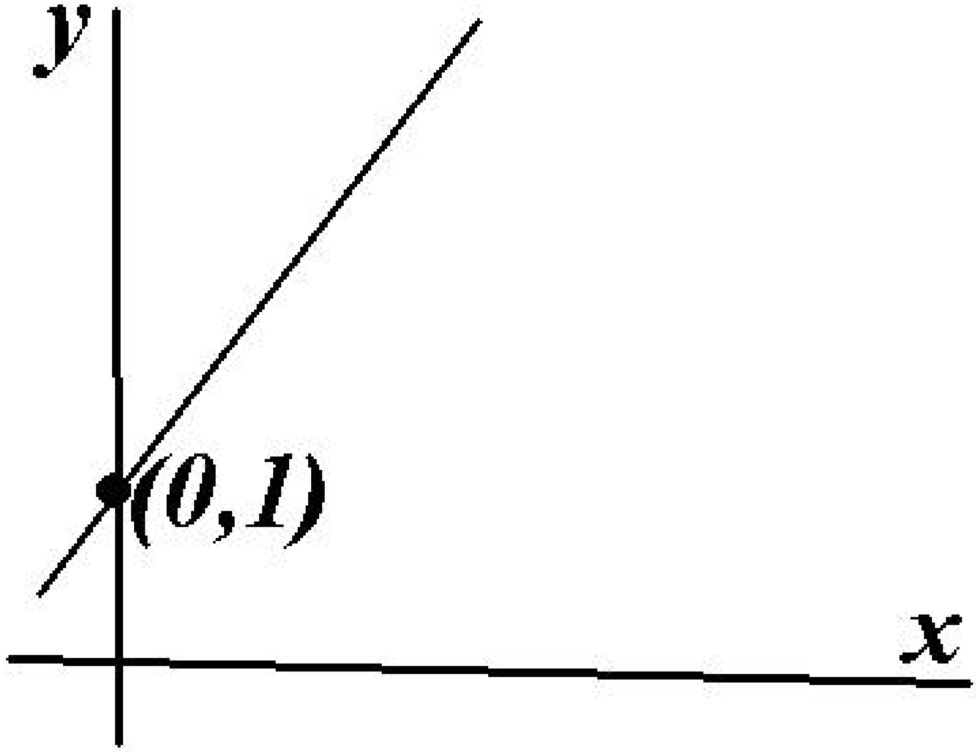
The Equation of a Line in Higher Dimension



$$y = mx + b$$

$$y = 2x + 1$$



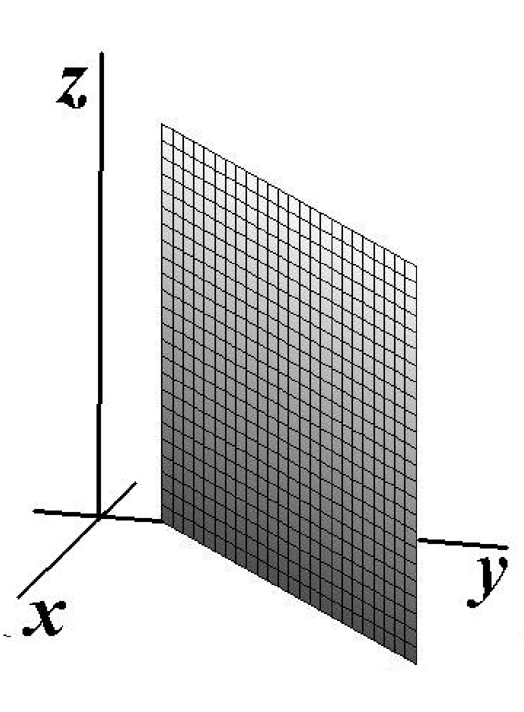
$$y = 2x + 1$$

$$-2x + 1y + 0z = 1$$

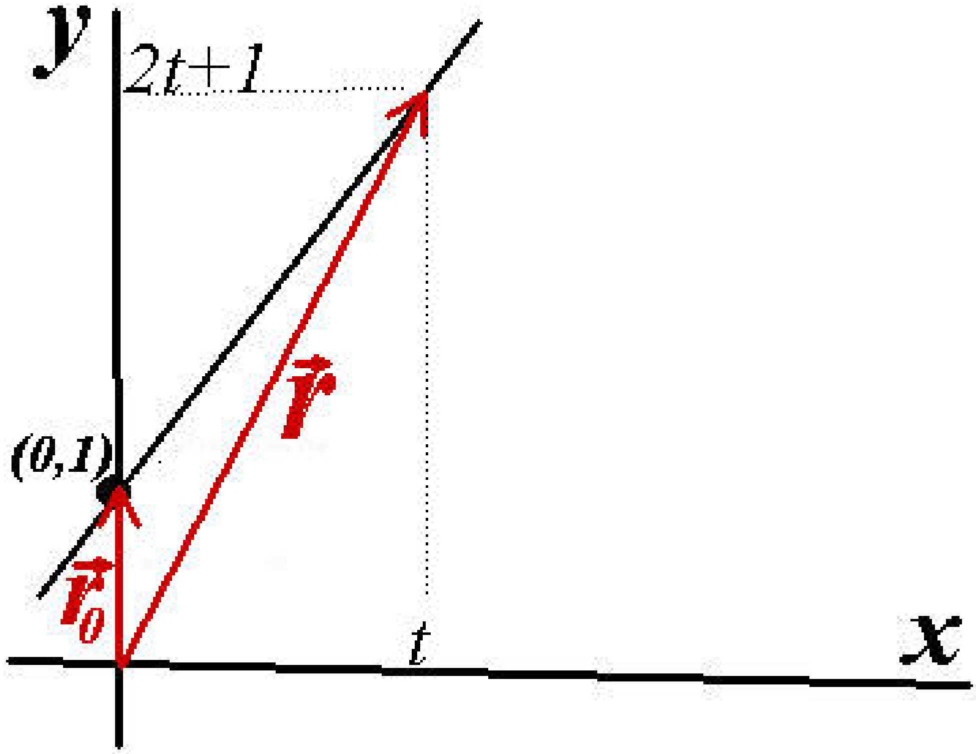
In the form:

$$ax + by + cz = d$$

$$y = 2x + 1$$

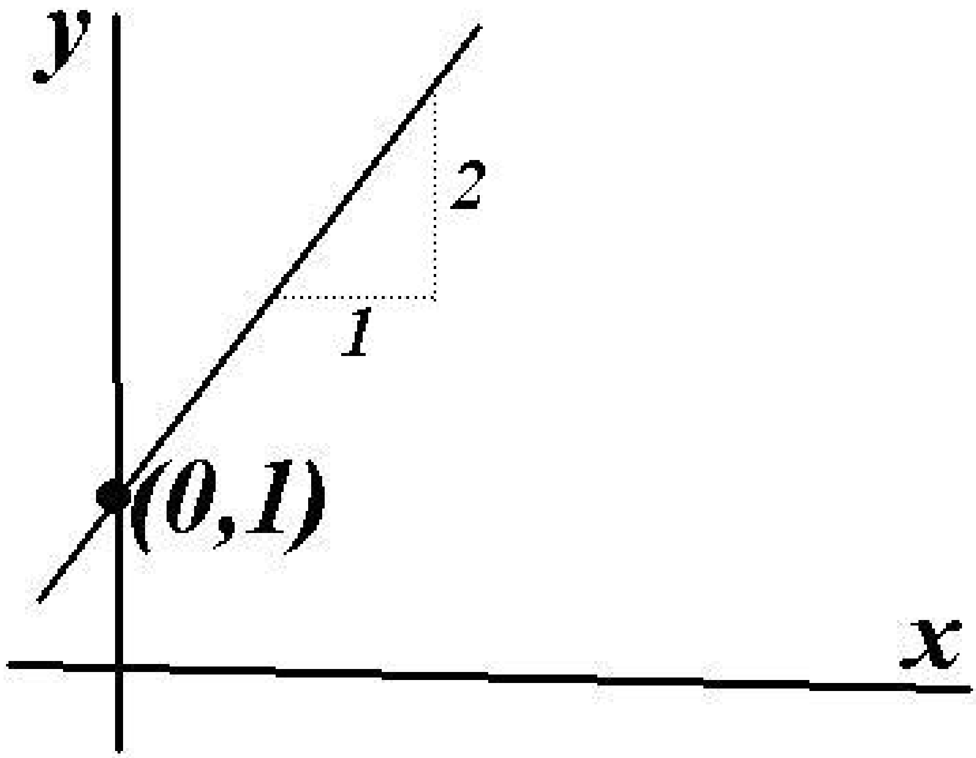


$\vec{\mathbf{r}}_0 = \langle 0,\ 1\rangle \qquad \vec{\mathbf{r}} = \langle x,\ y\rangle = \langle t,\ 2t+1\rangle$

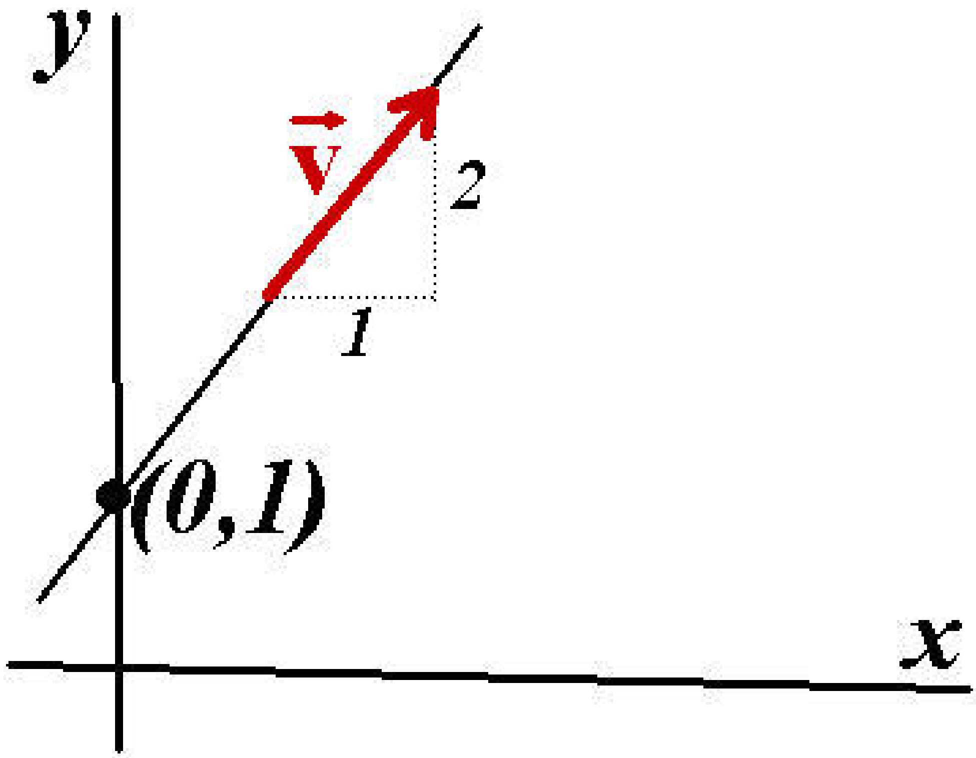


$$
\begin{aligned}
\vec{\mathbf{r}} &= \langle t,\ 2t+1\rangle \\
&= \langle 0,\ 1\rangle + \langle t,\ 2t\rangle \\
&= \langle 0,\ 1\rangle + t\langle 1,\ 2\rangle \\
&= \vec{\mathbf{r}}_0 + t\vec{\mathbf{v}}
\end{aligned}
$$

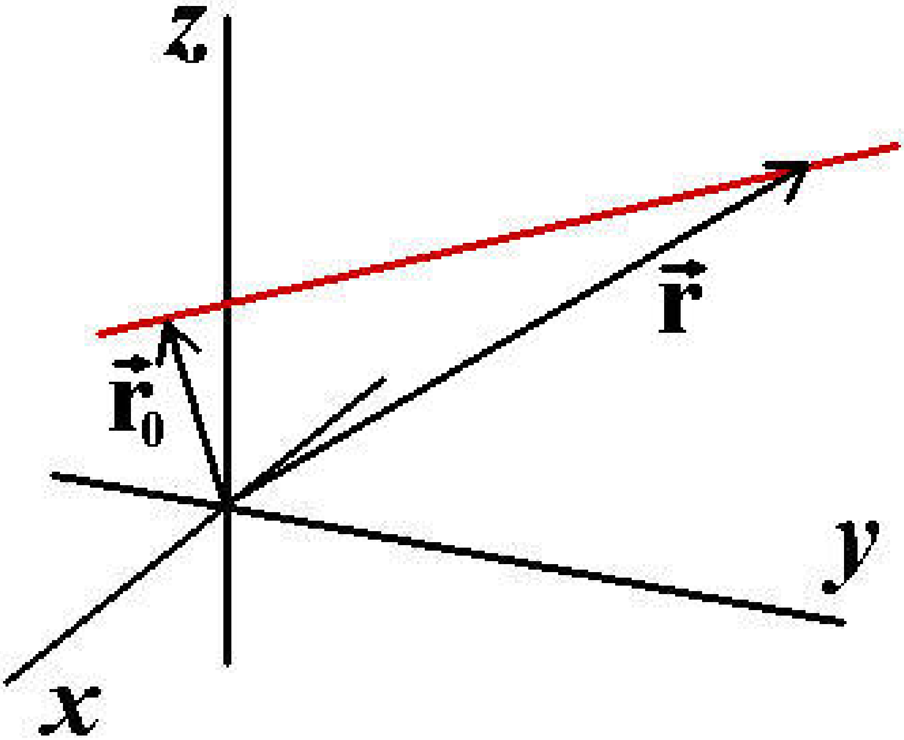
$$m = 2$$



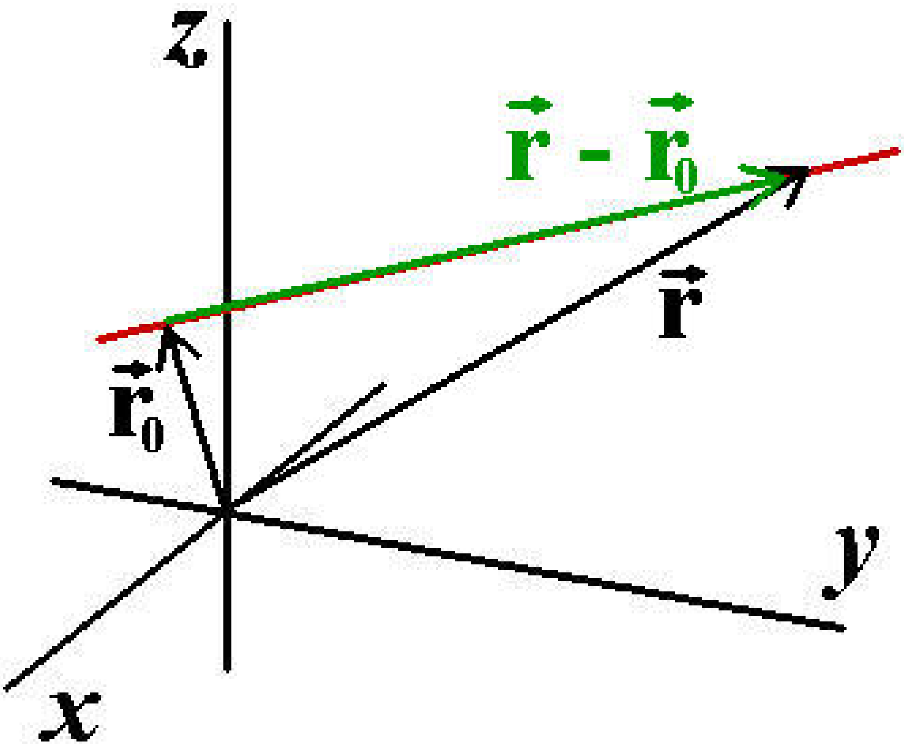
$$\vec{\mathbf{v}} = \langle 1,\ 2 \rangle$$



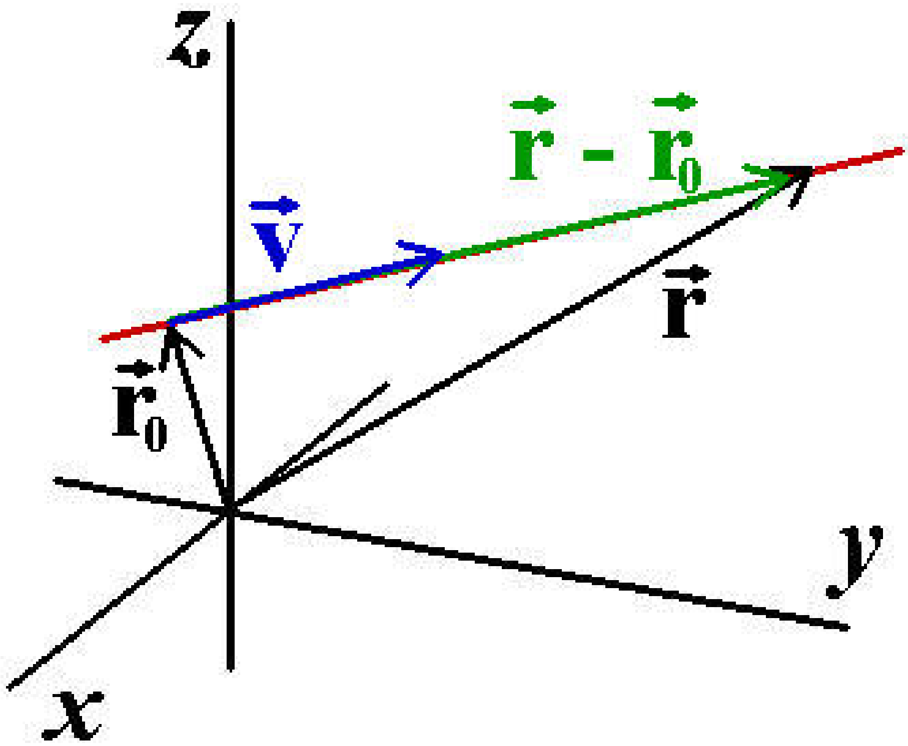
$\vec{\mathbf{r}}_0 = \langle x_0,\ y_0,\ z_0 \rangle$ $\qquad$ $\vec{\mathbf{r}} = \langle x,\ y,\ z \rangle$



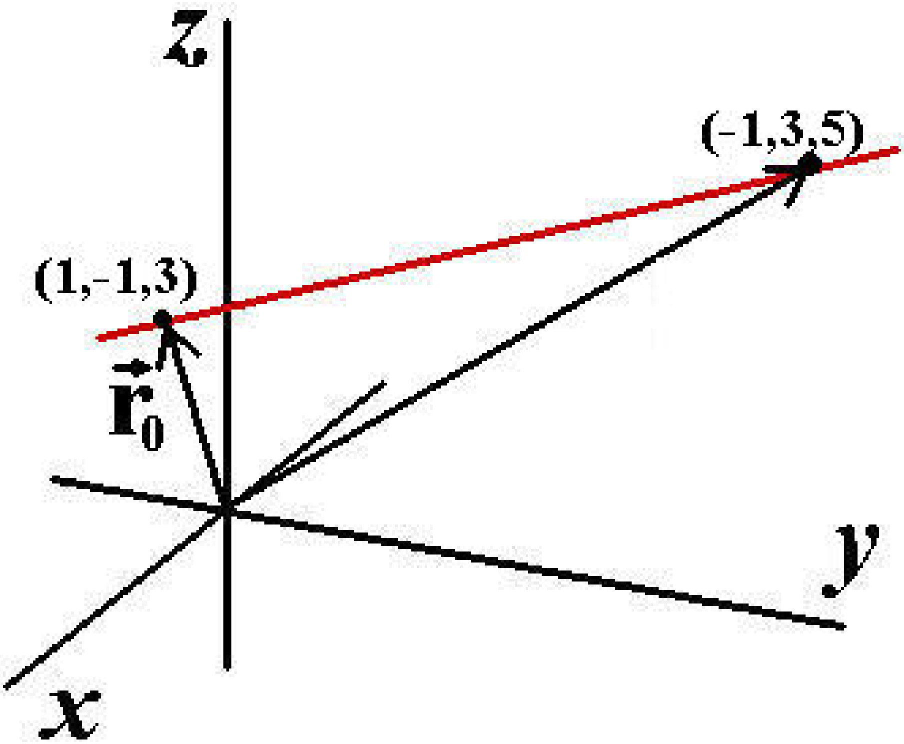
$\vec{\mathbf{r}}_0 = \langle x_0,\ y_0,\ z_0 \rangle \qquad \vec{\mathbf{r}} = \langle x,\ y,\ z \rangle$



$$\vec{\mathbf{r}} - \vec{\mathbf{r}}_0 = t\vec{\mathbf{v}} \quad \text{so} \quad \vec{\mathbf{r}} = \vec{\mathbf{r}}_0 + t\vec{\mathbf{v}}$$

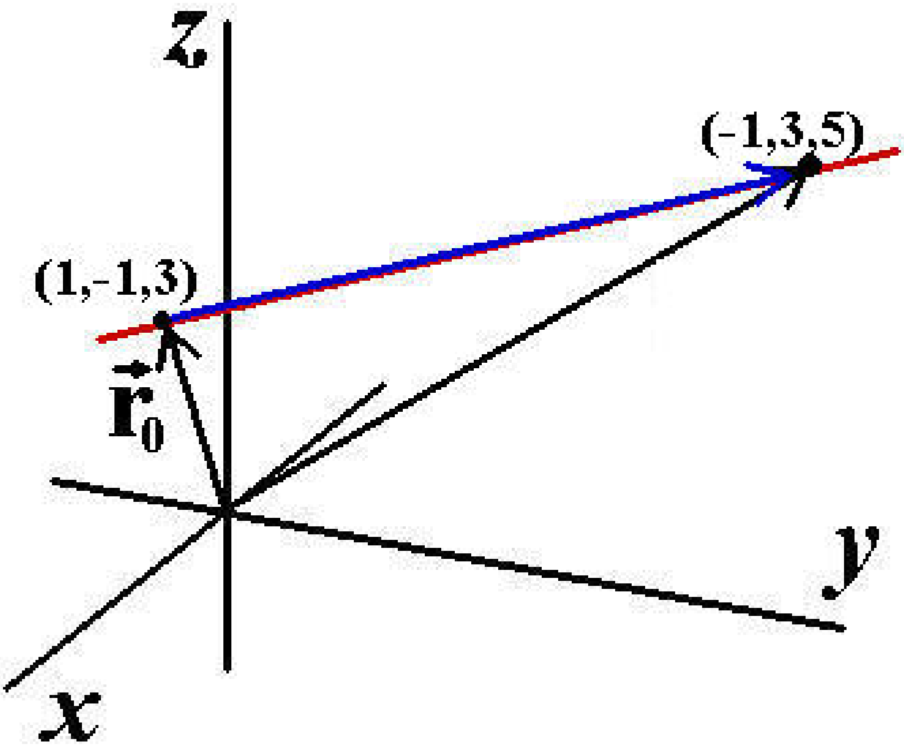


Find the equation of the line through $(1,\ -1,\ 3)$ and $(-1,\ 3,\ 5)$



Find the equation of the line through $(1,\ -1,\ 3)$ and $(-1,\ 3,\ 5)$

$$\vec{\mathbf{v}} = \langle -1,\ 3,\ 5\rangle - \langle 1,\ -1,\ 3\rangle = \langle -2,\ 4,\ 2\rangle$$



$$\begin{aligned} \vec{\mathbf{r}} &= \vec{\mathbf{r}}_0 + t\vec{\mathbf{v}} \\ \langle x,\ y,\ z\rangle &= \langle 1,\ -1,\ 3\rangle + t\langle -2,\ 4,\ 2\rangle \end{aligned}$$

$$
\begin{aligned}
\vec{\mathbf{r}} &= \vec{\mathbf{r}}_0 + t\vec{\mathbf{v}} \\
\langle x,\ y,\ z\rangle &= \langle 1,\ -1,\ 3\rangle + t\langle -2,\ 4,\ 2\rangle \\
&= \langle 1-2t,\ -1+4t,\ 3+2t\rangle
\end{aligned}
$$

$$\begin{aligned}\vec{\mathbf{r}} &= \vec{\mathbf{r}}_0 + t\vec{\mathbf{v}} \\ \langle x,\ y,\ z\rangle &= \langle 1,\ -1,\ 3\rangle + t\langle -2,\ 4,\ 2\rangle \\ &= \langle 1-2t,\ -1+4t,\ 3+2t\rangle\end{aligned}$$

$$x = 1 - 2t \qquad y = -1 + 4t \qquad z = 3 + 2t$$

$$
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\vec{\mathbf{r}} &= \vec{\mathbf{r}}_0 + t\vec{\mathbf{v}} \\
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x = 1 - 2t \qquad y = -1 + 4t \qquad z = 3 + 2t
$$

$$
\frac{x-1}{-2} = t \qquad \frac{y+1}{4} = t \qquad \frac{z-3}{2} = t
$$

$$
\begin{aligned}
\vec{\mathbf{r}} &= \vec{\mathbf{r}}_0 + t\vec{\mathbf{v}} \\
\langle x,\ y,\ z\rangle &= \langle 1,\ -1,\ 3\rangle + t\langle -2,\ 4,\ 2\rangle \\
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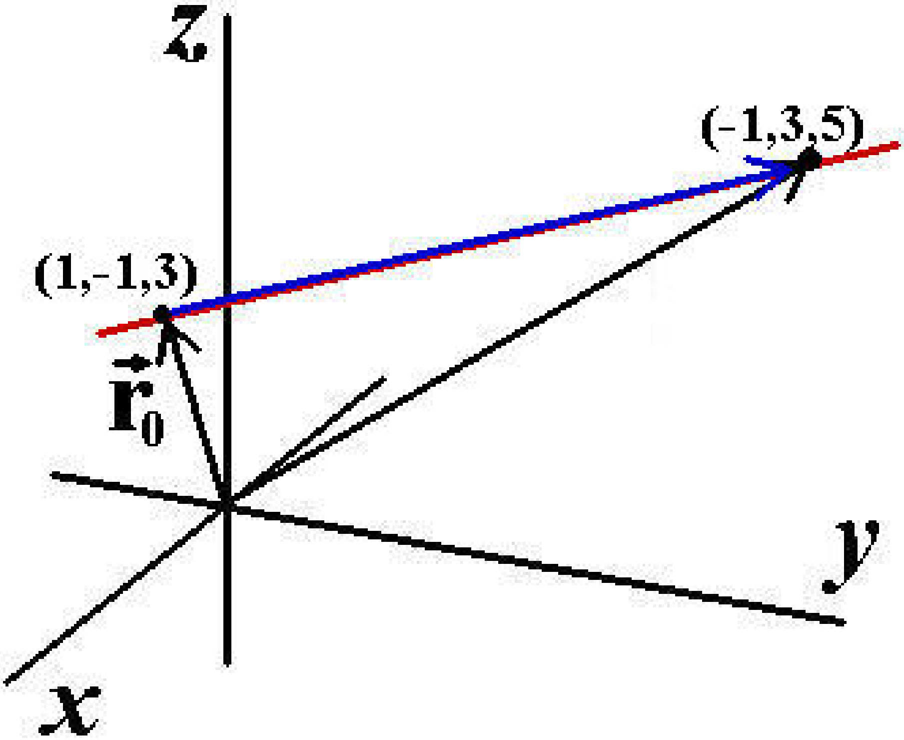
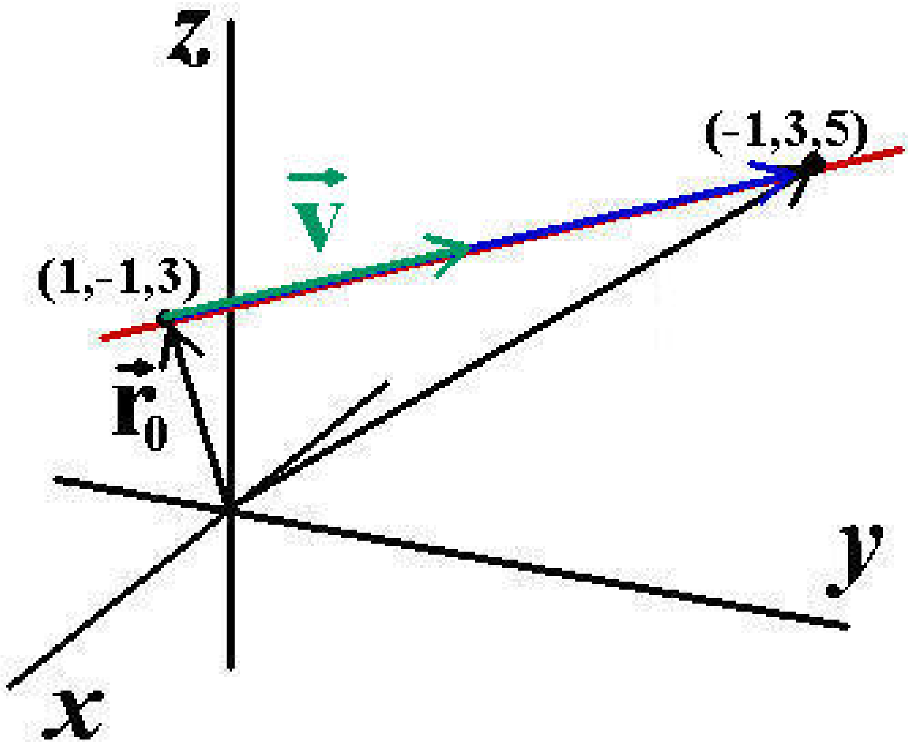
$$

$$
\frac{x-1}{-2} = \frac{y+1}{4} = \frac{z-3}{2}
$$

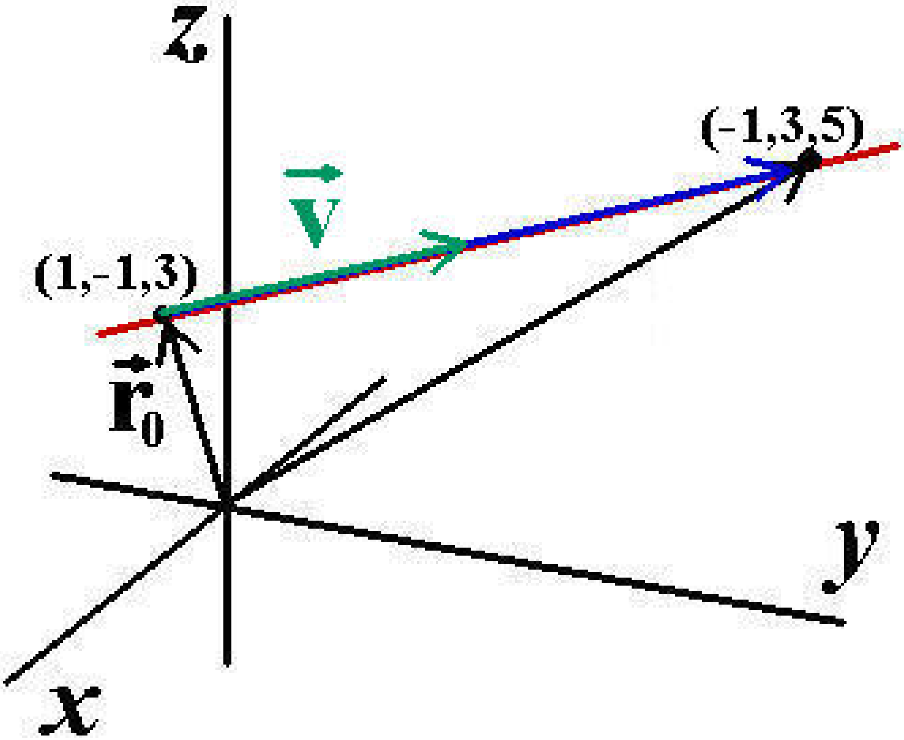
We took $\vec{\mathbf{v}} = \langle -2,\ 4,\ 2\rangle$



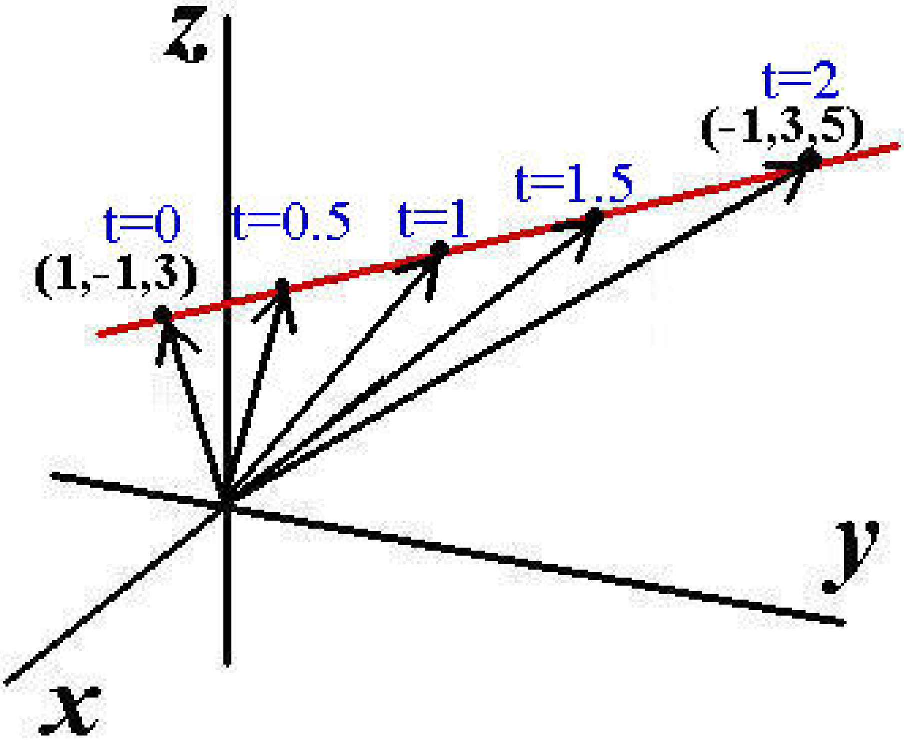
We could have taken $\vec{\mathbf{v}} = \frac{1}{2}\langle -2,\ 4,\ 2\rangle = \langle -1,\ 2,\ 1\rangle$



$$\vec{\mathbf{r}} = \vec{\mathbf{r}}_0 + t\vec{\mathbf{v}} = \langle 1,\ -1,\ 3\rangle + t\langle -1,\ 2,\ 1\rangle$$

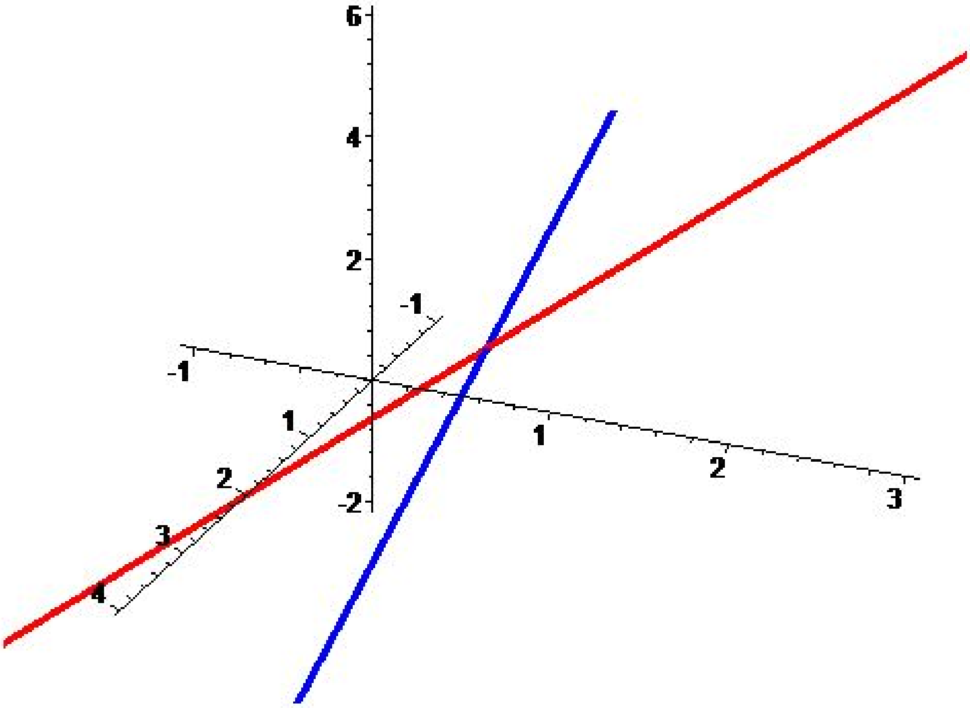


$$\vec{\mathbf{r}} = \vec{\mathbf{r}}_0 + t\vec{\mathbf{v}} = \langle 1,\ -1,\ 3\rangle + t\langle -1,\ 2,\ 1\rangle$$



Line 1 $\vec{\mathbf{r}} = \langle 2,\ 0,\ 0\rangle + t\langle -1,\ 1,\ 2\rangle$

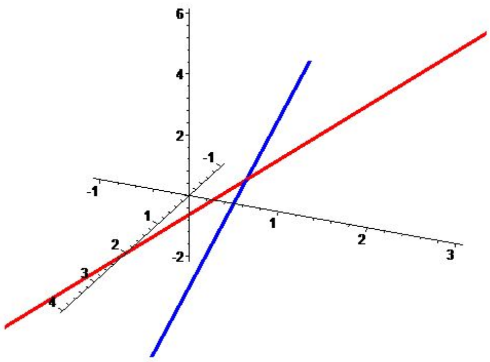
Line 2 $\vec{\mathbf{r}} = \langle 3,\ 1,\ 0\rangle + t\langle -1,\ 0,\ 1\rangle$



Line 1 $\qquad \vec{\mathbf{r}} = \langle 2,\ 0,\ 0\rangle + t\langle -1,\ 1,\ 2\rangle$

Line 2 $\qquad \vec{\mathbf{r}} = \langle 3,\ 1,\ 0\rangle + t\langle -1,\ 0,\ 1\rangle$

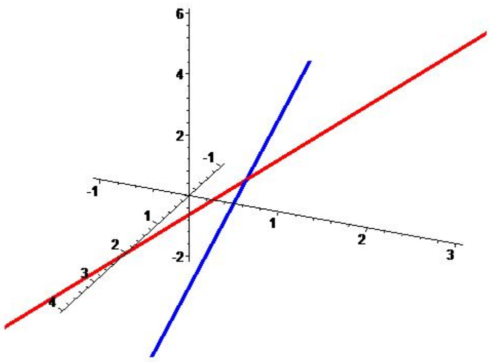
Do the lines intersect? If so, where?



Line 1 $\qquad \vec{\mathbf{r}} = \langle 2,\ 0,\ 0\rangle + t_1\langle -1,\ 1,\ 2\rangle$

Line 2 $\qquad \vec{\mathbf{r}} = \langle 3,\ 1,\ 0\rangle + t_2\langle -1,\ 0,\ 1\rangle$

Do the lines intersect? If so, where?



Line 1 $\vec{\mathbf{r}} = \langle 2,\ 0,\ 0\rangle + t_1\langle -1,\ 1,\ 2\rangle$

Line 2 $\vec{\mathbf{r}} = \langle 3,\ 1,\ 0\rangle + t_2\langle -1,\ 0,\ 1\rangle$

Do the lines intersect? If so, where?

$$\langle 2,\ 0,\ 0\rangle + t_1\langle -1,\ 1,\ 2\rangle = \langle 3,\ 1,\ 0\rangle + t_2\langle -1,\ 0,\ 1\rangle$$

$$\langle 2 - t_1,\ t_1,\ 2t_1\rangle = \langle 3 - t_2,\ 1,\ t_2\rangle$$

Line 1 $\qquad \vec{\mathbf{r}} = \langle 2,\ 0,\ 0\rangle + t_1\langle -1,\ 1,\ 2\rangle$

Line 2 $\qquad \vec{\mathbf{r}} = \langle 3,\ 1,\ 0\rangle + t_2\langle -1,\ 0,\ 1\rangle$

Do the lines intersect? If so, where?

$$\langle 2,\ 0,\ 0\rangle + t_1\langle -1,\ 1,\ 2\rangle = \langle 3,\ 1,\ 0\rangle + t_2\langle -1,\ 0,\ 1\rangle$$

$$\langle 2 - t_1,\ t_1,\ 2t_1\rangle = \langle 3 - t_2,\ 1,\ t_2\rangle$$

$$2 - t_1 = 3 - t_2 \qquad t_1 = 1 \qquad 2t_1 = t_2$$

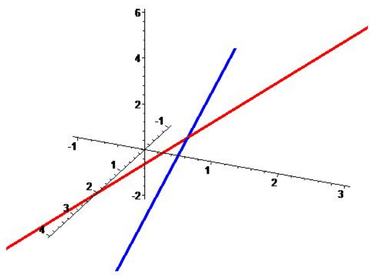
Conclusion: $t_1 = 1$ and $t_2 = 2$

Line 1 at $t_1 = 1$:

$$\vec{\mathbf{r}} = \langle 2,\ 0,\ 0\rangle + t_1\langle -1,\ 1,\ 2\rangle = \langle 2,\ 0,\ 0\rangle + 1\langle -1,\ 1,\ 2\rangle = \langle 1,\ 1,\ 2\rangle$$

Line 2 at $t_2 = 2$:

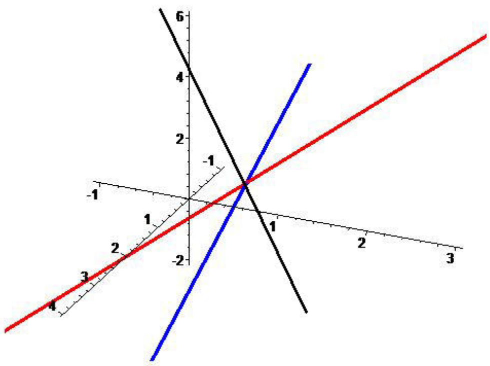
$$\vec{\mathbf{r}} = \langle 3,\ 1,\ 0\rangle + t_2\langle -1,\ 0,\ 1\rangle = \langle 3,\ 1,\ 0\rangle + 2\langle -1,\ 0,\ 1\rangle = \langle 1,\ 1,\ 2\rangle$$



Line 1 $\quad\quad\quad\quad \vec{\mathbf{r}} = \langle 2,\ 0,\ 0\rangle + t\langle -1,\ 1,\ 2\rangle$

Line 2 $\quad\quad\quad\quad \vec{\mathbf{r}} = \langle 3,\ 1,\ 0\rangle + t\langle -1,\ 0,\ 1\rangle$

Find the equation of the straight line containing $(1,\ 1,\ 2)$ that is perpendicular to both Lines 1 and 2.



Line 1 $\vec{\mathbf{r}} = \langle 2,\ 0,\ 0\rangle + t\langle -1,\ 1,\ 2\rangle$

Line 2 $\vec{\mathbf{r}} = \langle 3,\ 1,\ 0\rangle + t\langle -1,\ 0,\ 1\rangle$

Find the equation of the straight line containing $(1,\ 1,\ 2)$ that is perpendicular to both Lines 1 and 2.

$$\vec{\mathbf{r}} = \langle 1,\ 1,\ 2\rangle + t\vec{\mathbf{v}}$$

Line 1 $\quad \vec{\mathbf{r}} = \langle 2,\ 0,\ 0\rangle + t\langle -1,\ 1,\ 2\rangle$

Line 2 $\quad \vec{\mathbf{r}} = \langle 3,\ 1,\ 0\rangle + t\langle -1,\ 0,\ 1\rangle$

Find the equation of the straight line containing $(1,\ 1,\ 2)$ that is perpendicular to both Lines 1 and 2.

$$\vec{\mathbf{r}} = \langle 1,\ 1,\ 2\rangle + t\vec{\mathbf{v}}$$

$$\vec{\mathbf{v}} = \langle -1,\ 0,\ 1\rangle \times \langle -1,\ 1,\ 2\rangle$$

$$\begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ -1 & 0 & 1 \\ -1 & 1 & 2 \end{vmatrix} = -\vec{\mathbf{i}} + \vec{\mathbf{j}} - \vec{\mathbf{k}}$$

Line 1 $\qquad \vec{\mathbf{r}} = \langle 2,\ 0,\ 0\rangle + t\langle -1,\ 1,\ 2\rangle$

Line 2 $\qquad \vec{\mathbf{r}} = \langle 3,\ 1,\ 0\rangle + t\langle -1,\ 0,\ 1\rangle$

Find the equation of the straight line containing $(1,\ 1,\ 2)$ that is perpendicular to both Lines 1 and 2.

$$\vec{\mathbf{r}} = \langle 1,\ 1,\ 2\rangle + t\vec{\mathbf{v}}$$

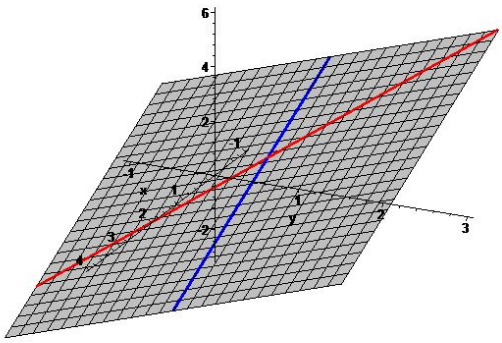
$$\vec{\mathbf{v}} = \langle -1,\ 0,\ 1\rangle \times \langle -1,\ 1,\ 2\rangle = \langle -1,\ 1,\ -1\rangle$$

$$\vec{\mathbf{r}} = \langle 1,\ 1,\ 2\rangle + t\langle -1,\ 1,\ -1\rangle$$

Line 1 $\vec{\mathbf{r}} = \langle 2,\ 0,\ 0\rangle + t\langle -1,\ 1,\ 2\rangle$

Line 2 $\vec{\mathbf{r}} = \langle 3,\ 1,\ 0\rangle + t\langle -1,\ 0,\ 1\rangle$

Find the equation of the plane containing both lines.



Line 1 $\qquad \vec{\mathbf{r}} = \langle 2,\ 0,\ 0\rangle + t\langle -1,\ 1,\ 2\rangle$

Line 2 $\qquad \vec{\mathbf{r}} = \langle 3,\ 1,\ 0\rangle + t\langle -1,\ 0,\ 1\rangle$

Find the equation of the plane containing both lines.

$$\vec{\mathbf{n}} \bullet (\vec{\mathbf{r}} - \vec{\mathbf{r}}_0) = 0$$

$$\vec{\mathbf{n}} \bullet \langle x-1,\ y-1,\ z-2\rangle = 0$$

Line 1 $\quad \vec{\mathbf{r}} = \langle 2,\ 0,\ 0\rangle + t\langle -1,\ 1,\ 2\rangle$

Line 2 $\quad \vec{\mathbf{r}} = \langle 3,\ 1,\ 0\rangle + t\langle -1,\ 0,\ 1\rangle$

Find the equation of the plane containing both lines.

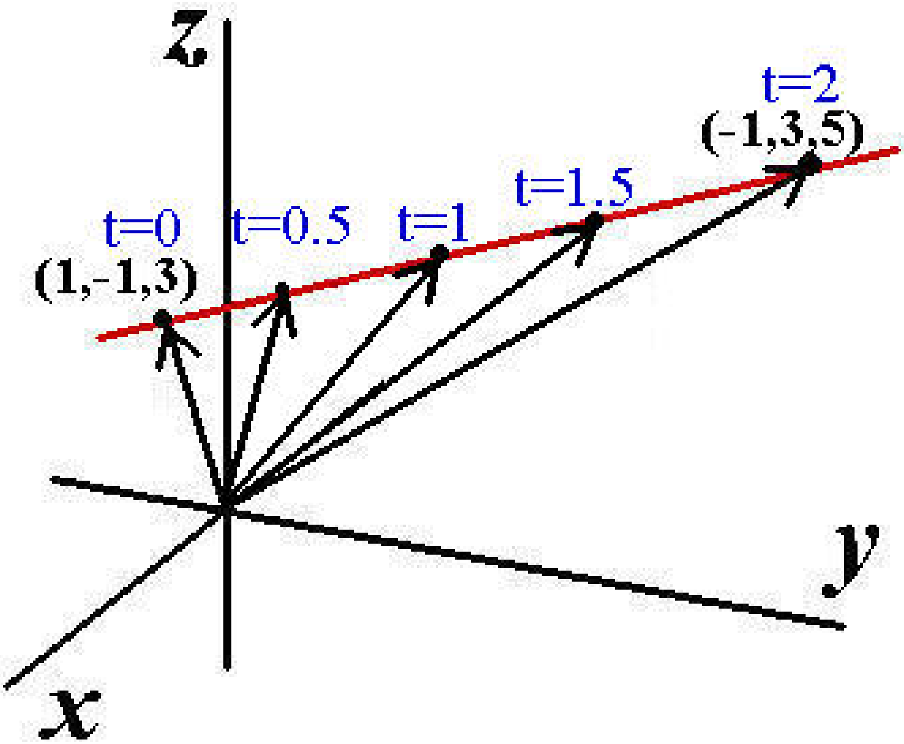
$$\vec{\mathbf{n}} \bullet (\vec{\mathbf{r}} - \vec{\mathbf{r}}_0) = 0$$

$$\vec{\mathbf{n}} \bullet \langle x-1,\ y-1,\ z-2\rangle = 0$$

$$\langle -1,\ 1,\ -1\rangle \bullet \langle x-1,\ y-1,\ z-2\rangle = 0$$

$$x - y + z = 2$$

Motion of a particle through space at different times



Motion of a particle through space at different times