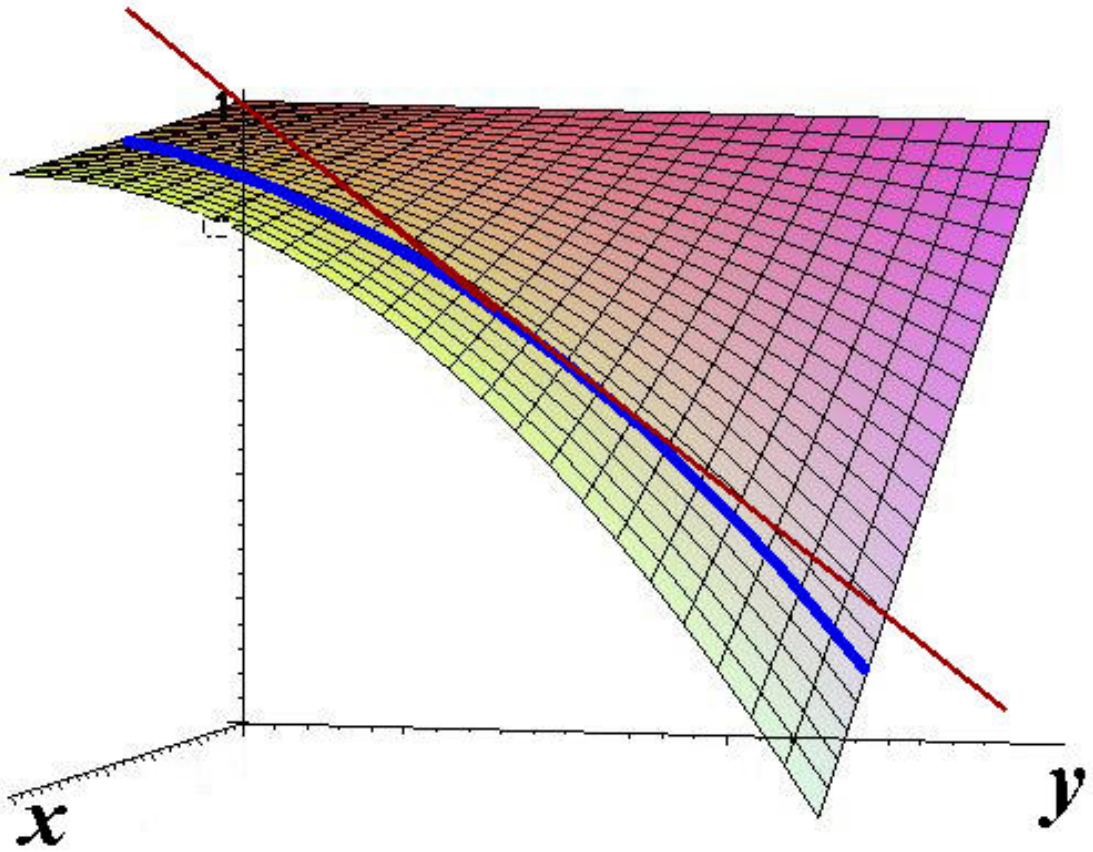
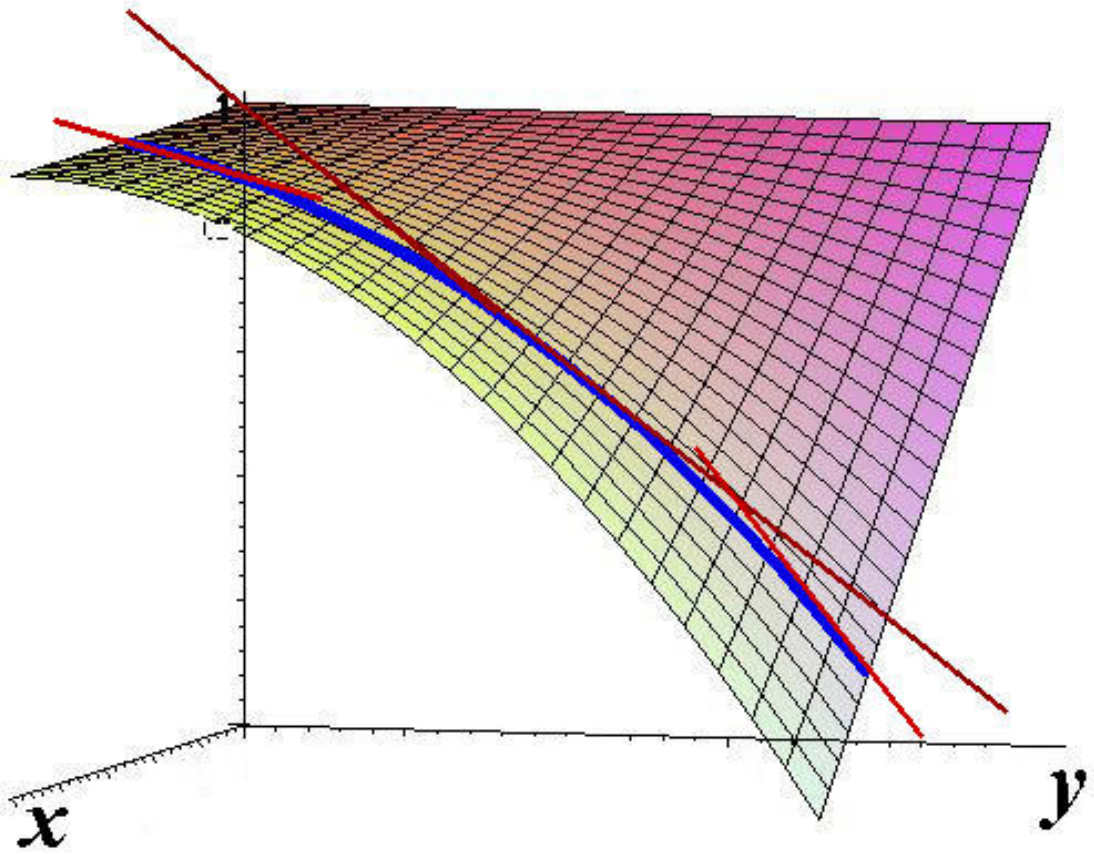
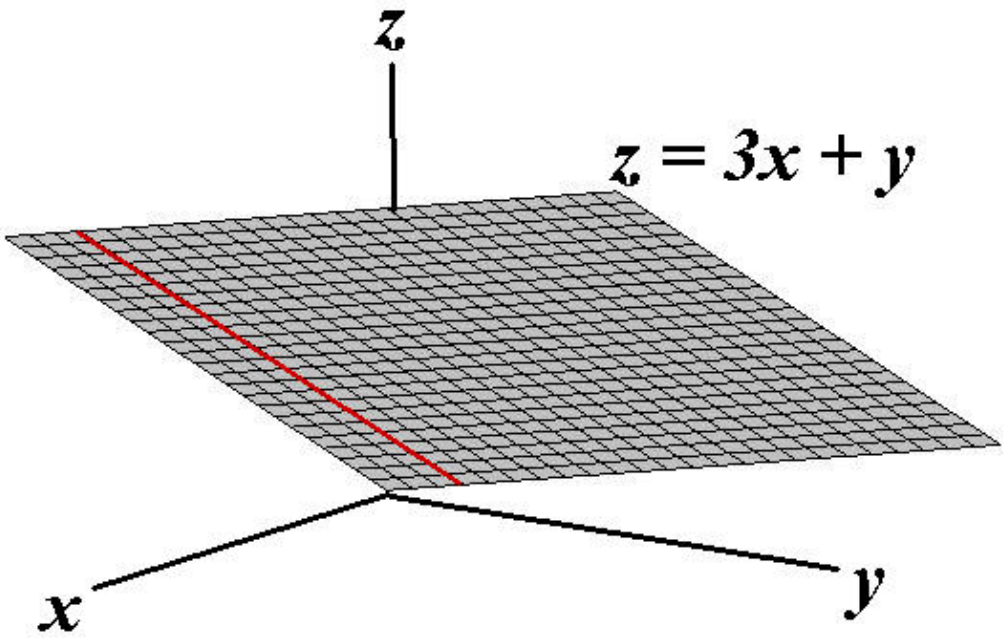
Partial Derivatives - More Examples



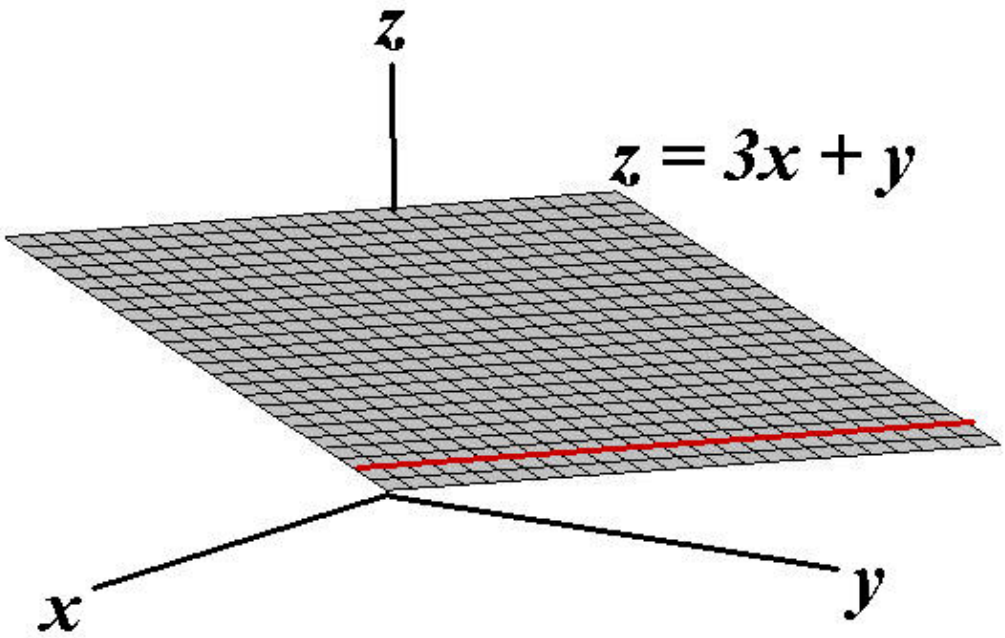
Partial Derivatives - More Examples



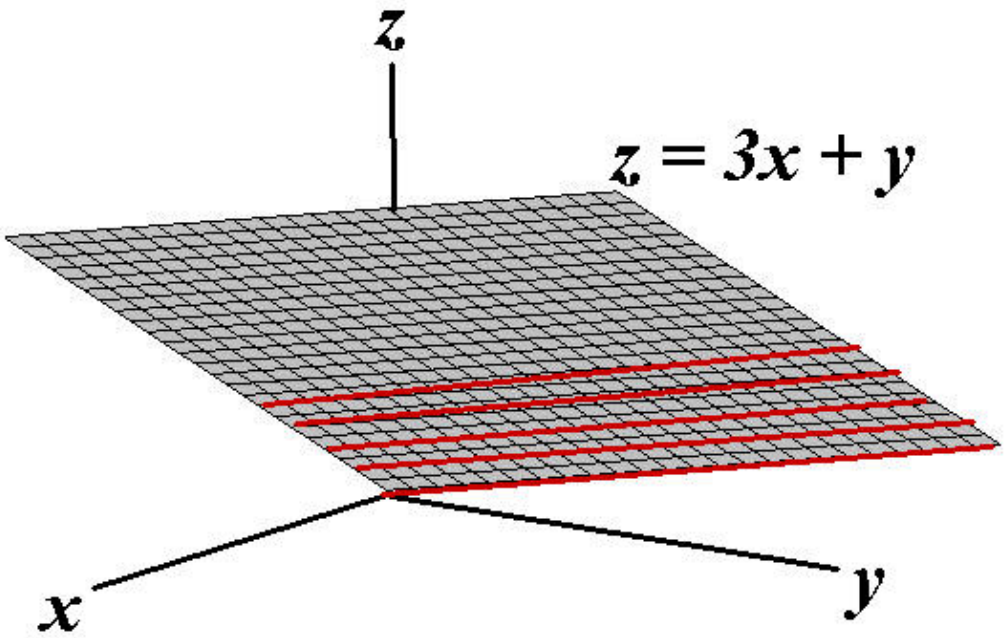
$$z = 3x + y \qquad \frac{\partial z}{\partial x} = 3$$



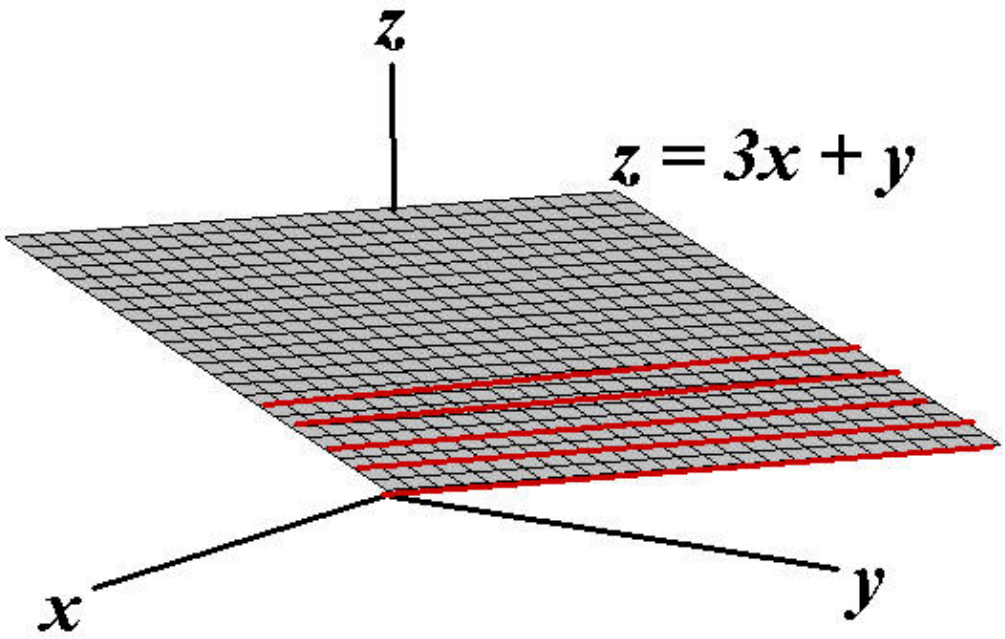
$$z = 3x + y \qquad \frac{\partial z}{\partial y} = 1$$



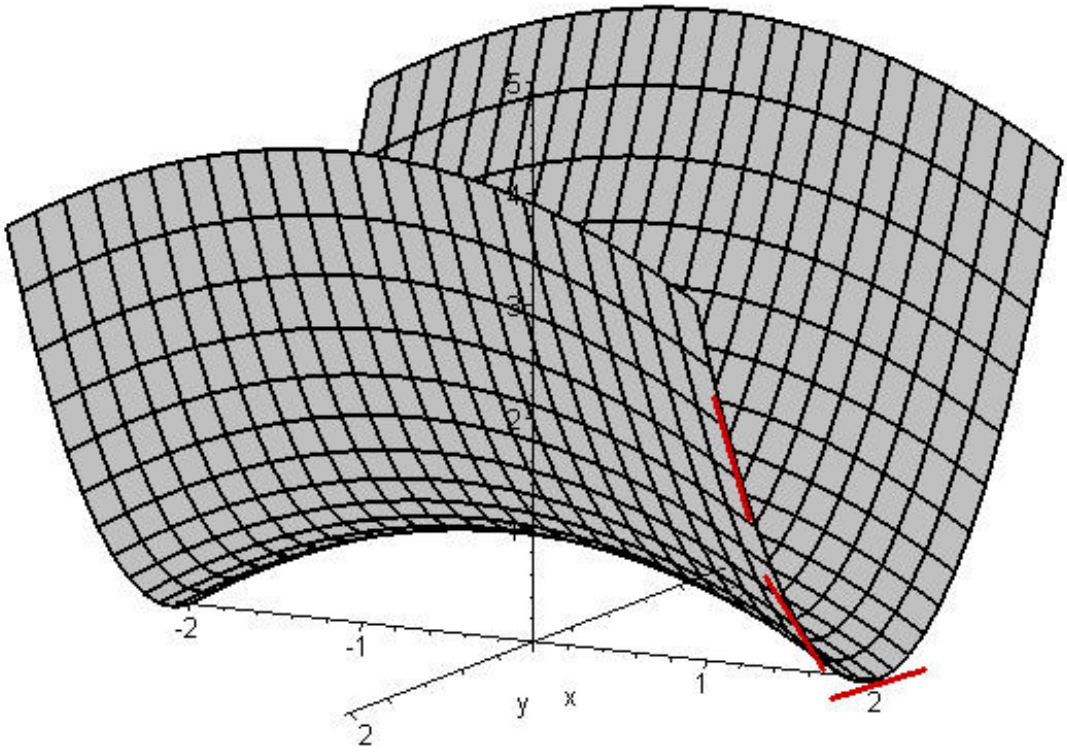
$$z = 3x + y \qquad \frac{\partial z}{\partial y} = 1$$



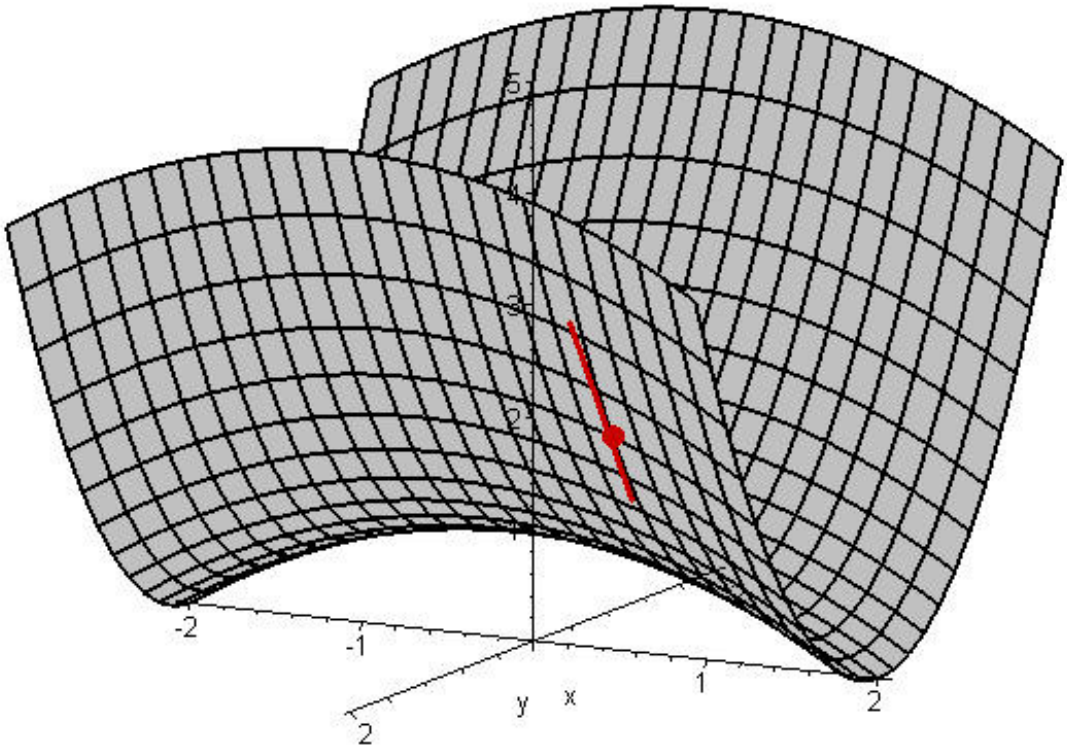
$$z = 3x + y \qquad \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = 0$$



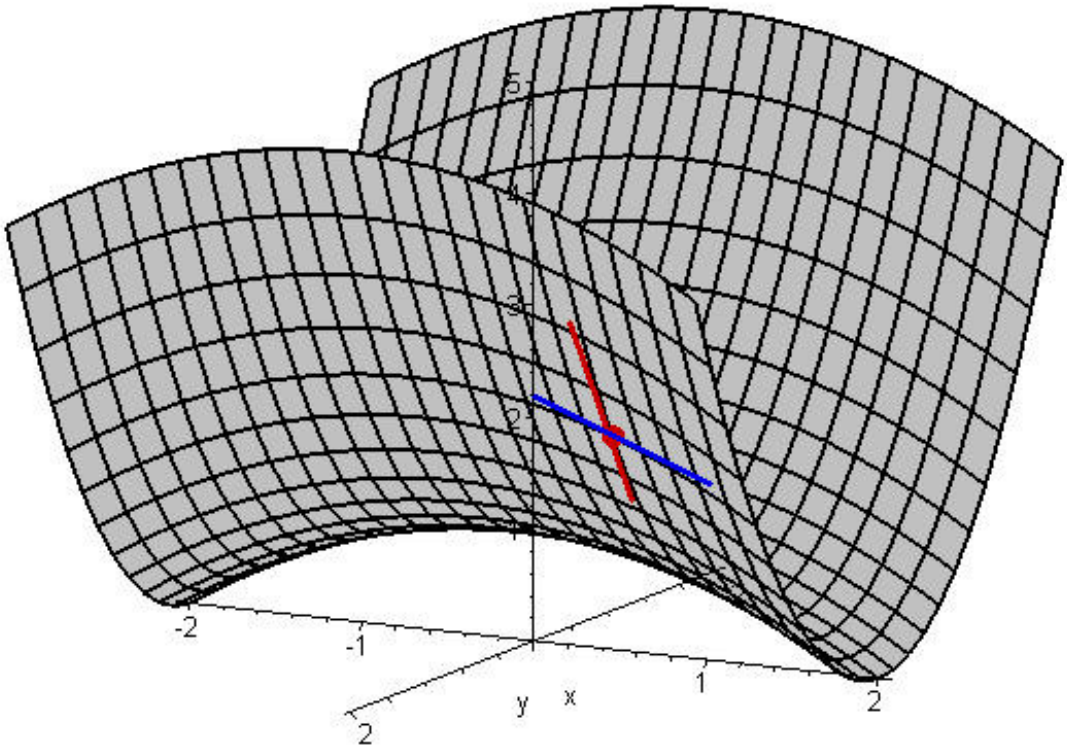
$$z = 1 + x^2 - y^2 \qquad \frac{\partial z}{\partial x} = 2x$$



$$z = 1 + x^2 - y^2 \qquad \frac{\partial z}{\partial x} = 2x \qquad \frac{\partial z}{\partial x}(1,1) = 2$$

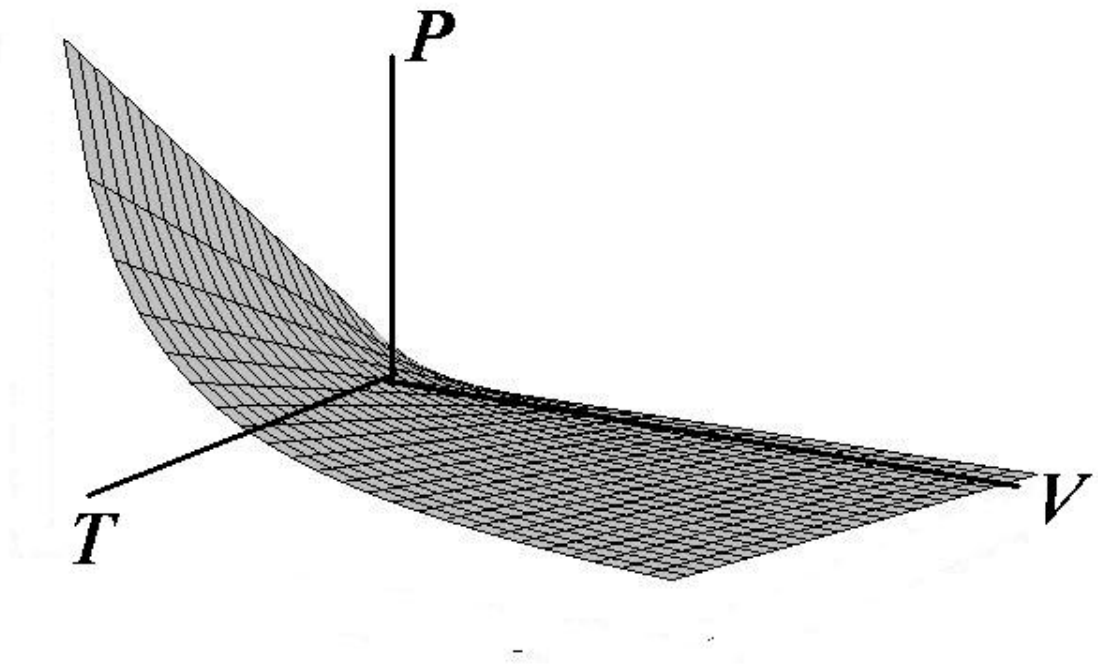


$$z = 1 + x^2 - y^2 \qquad \frac{\partial z}{\partial y} = -2y \qquad \frac{\partial z}{\partial y}(1,1) = -2$$

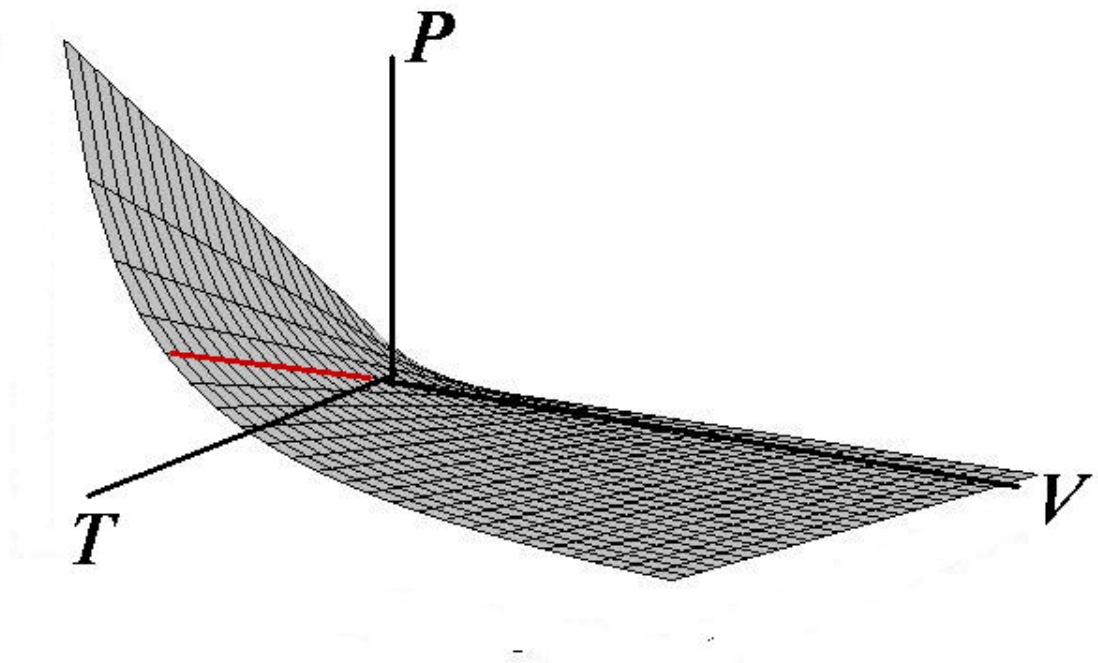


$$PV = nRT$$

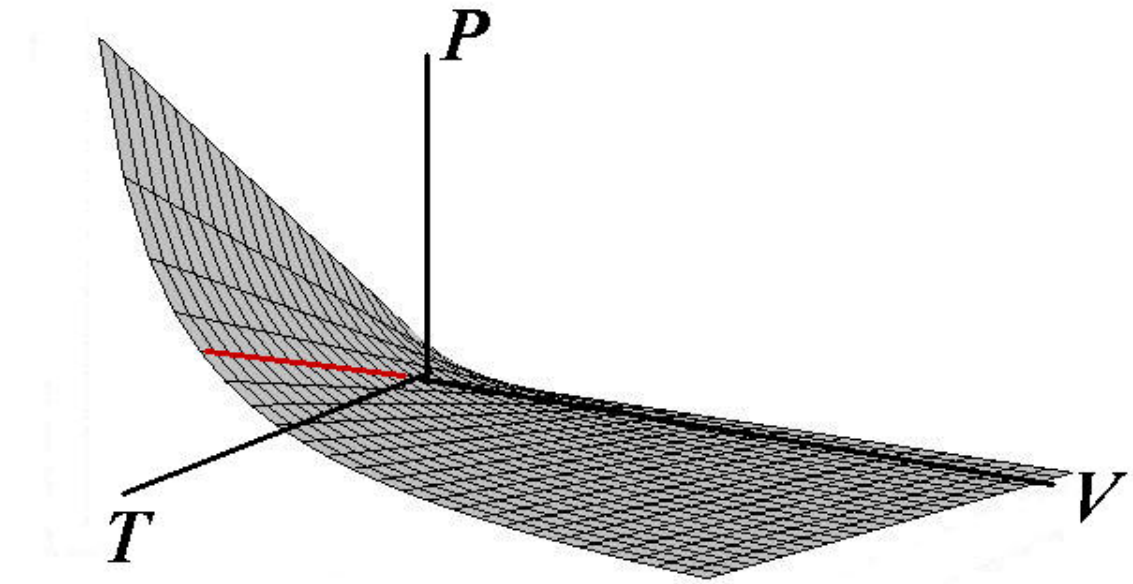
$$P = \frac{nRT}{V}$$



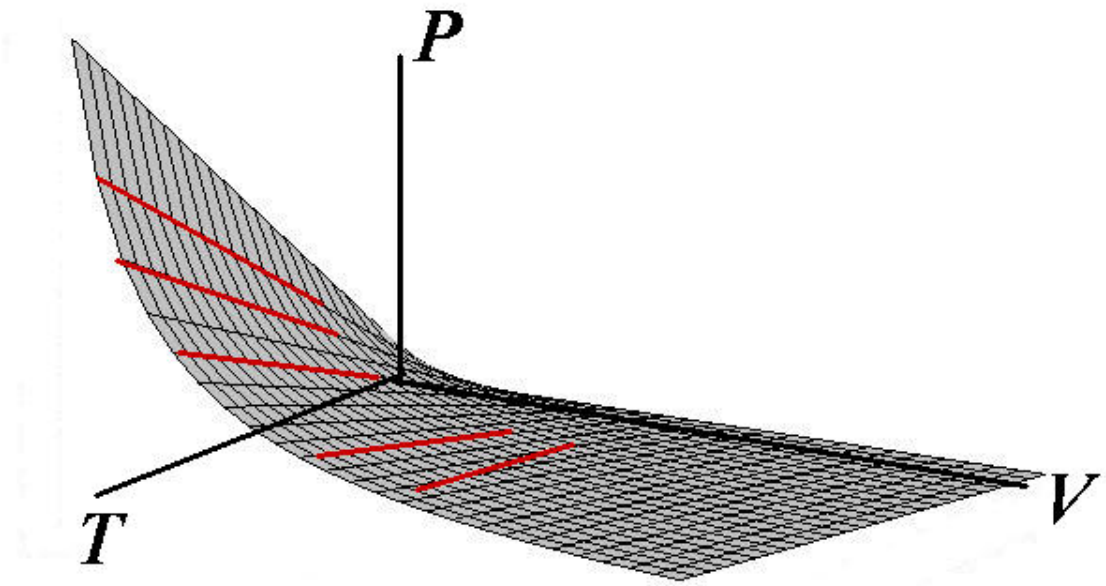
$$P = \frac{nRT}{V} \qquad \frac{\partial P}{\partial T} = \frac{nR}{V}$$



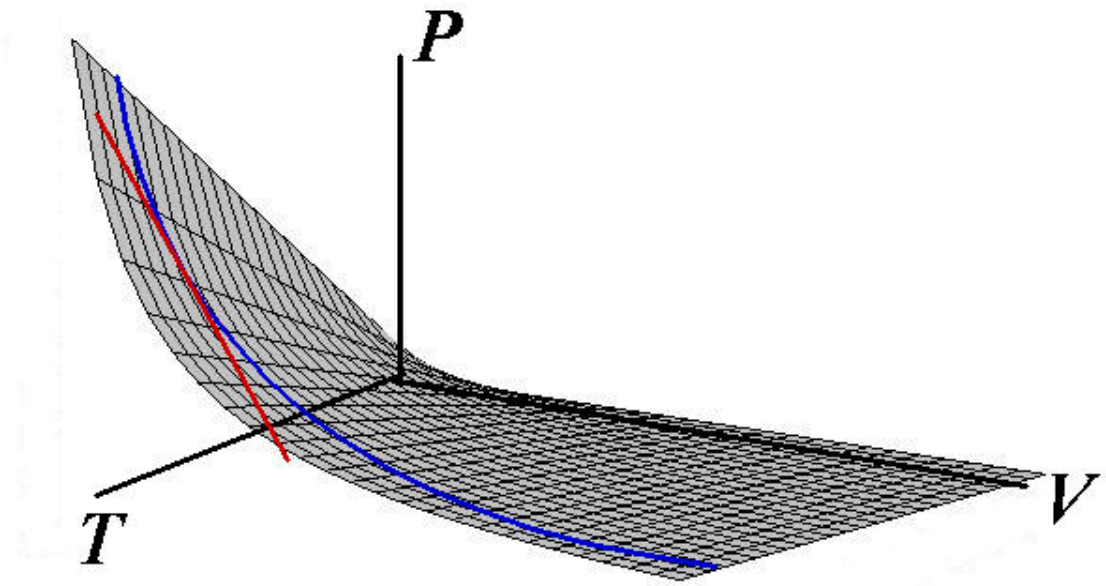
$$P = \frac{nRT}{V} \qquad \frac{\partial P}{\partial T} = \frac{nR}{V} \qquad \frac{\partial^2 P}{\partial T^2} = 0$$



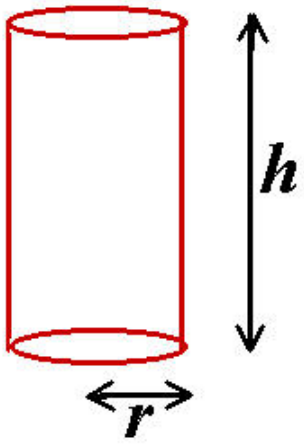
$$P = \frac{nRT}{V} \qquad \frac{\partial P}{\partial T} = \frac{nR}{V} \qquad \frac{\partial^2 P}{\partial V \partial T} = -\frac{nR}{V^2}$$



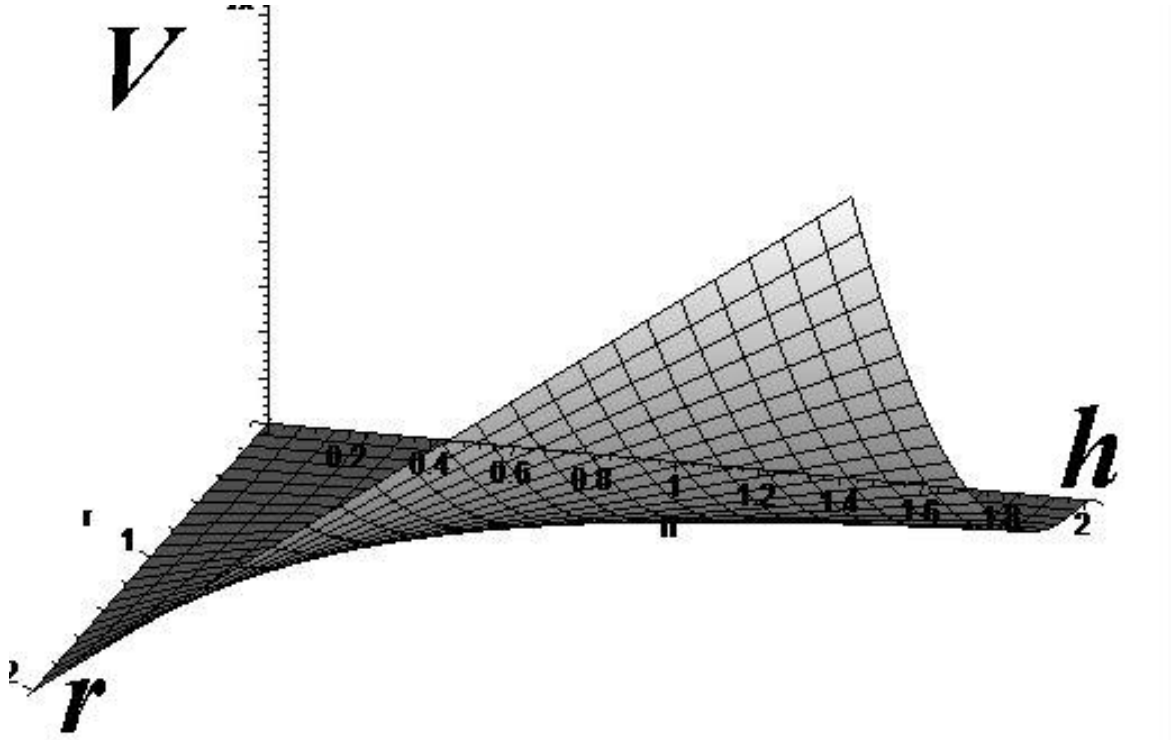
$$P = \frac{nRT}{V} \qquad \frac{\partial P}{\partial V} = -\frac{nRT}{V^2} \qquad \frac{\partial^2 P}{\partial V^2} = \frac{2nR}{V^3}$$



$$V = \pi r^2 h$$



$$V = \pi r^2 h \qquad \frac{\partial V}{\partial r} = 2\pi r h \qquad \frac{\partial V}{\partial h} = \pi r^2$$



$$z = x^{2\sqrt{y}}$$

Find the partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

$$z = x^{n}$$ constant

$$\frac{dz}{dx} = nx^{n-1}$$

$$z = x^{2\sqrt{y}}$$

$$\frac{\partial z}{\partial x} = 2\sqrt{y}\, x^{2\sqrt{y}-1}$$

$$z = x^{2\sqrt{y}}$$

$$\ln z = \ln\left(x^{2\sqrt{y}}\right)$$

$$\ln z = 2\sqrt{y}\ln x$$

$$\frac{d}{dy}\ln(f(y)) = \frac{1}{f(y)}f'(y)$$

$$z = x^{2\sqrt{y}}$$

$$\ln z = \ln\left(x^{2\sqrt{y}}\right)$$

$$\ln z = 2\sqrt{y}\ln x$$

$$\frac{\partial}{\partial y}(\ln z) = \frac{\partial}{\partial y}\left(2y^{1/2}\ln x\right)$$

$$\frac{1}{z}\frac{\partial z}{\partial y} = y^{-1/2}\ln x$$

$$z = x^{2\sqrt{y}}$$

$$\ln z = \ln\left(x^{2\sqrt{y}}\right)$$

$$\ln z = 2\sqrt{y}\ln x$$

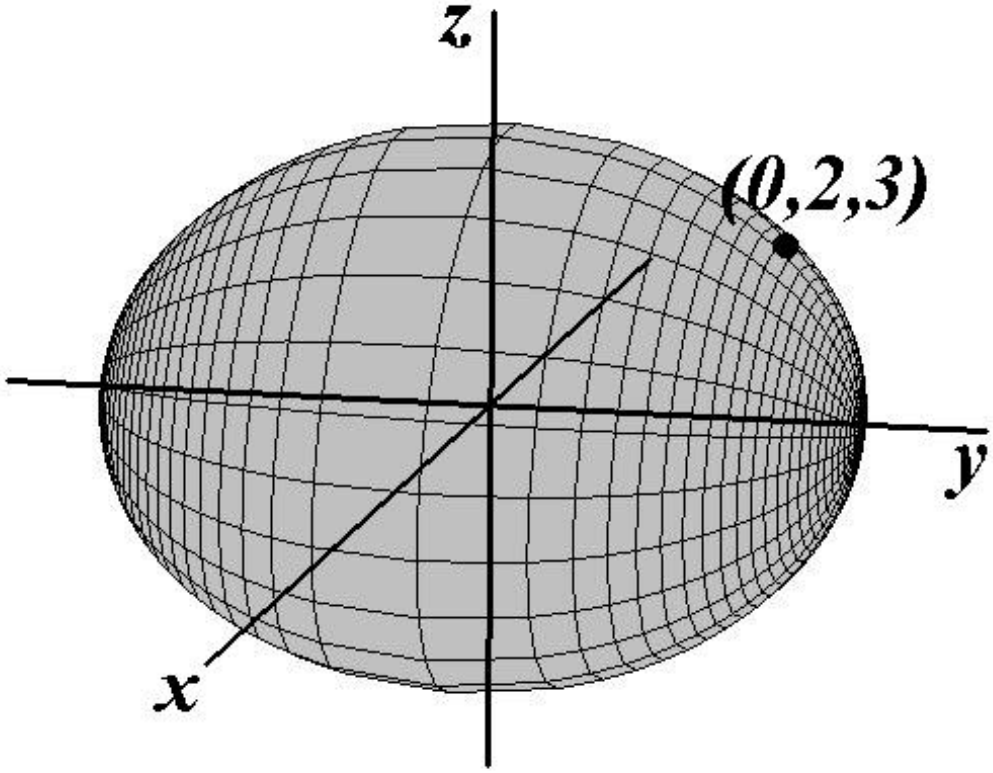
$$\frac{\partial}{\partial y}(\ln z) = \frac{\partial}{\partial y}\left(2y^{1/2}\ln x\right)$$

$$\frac{1}{z}\frac{\partial z}{\partial y} = y^{-1/2}\ln x$$

$$\frac{\partial z}{\partial y} = z\cdot y^{-1/2}\ln x = x^{2\sqrt{y}}\frac{\ln x}{\sqrt{y}}$$

$$4x^2 + 3y^2 + 4z^2 = 48$$

Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $(0,\ 2,\ 3)$



$$4x^2 + 3y^2 + 4z^2 = 48$$

$$4z^2 = 48 - 4x^2 - 3y^2$$

$$z = \frac{1}{2}\left(48 - 4x^2 - 3y^2\right)^{1/2}$$

$$z = \frac{1}{2}\left(48 - 4x^2 - 3y^2\right)^{1/2}$$

$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{1}{4}\left(48 - 4x^2 - 3y^2\right)^{-1/2}(-8x) \\ &= \frac{-2x}{\sqrt{48 - 4x^2 - 3y^2}}\end{aligned}$$

$$z = \frac{1}{2}\left(48 - 4x^2 - 3y^2\right)^{1/2}$$

$$\begin{aligned}\frac{\partial z}{\partial y} &= \frac{1}{4}\left(48 - 4x^2 - 3y^2\right)^{-1/2}(-6y) \\ &= \frac{-3y}{2\sqrt{48 - 4x^2 - 3y^2}}\end{aligned}$$

$$4x^2 + 3y^2 + 4z^2 = 48$$

$$\frac{\partial z}{\partial x} = \frac{-2x}{\sqrt{48 - 4x^2 - 3y^2}}$$

$$\frac{\partial z}{\partial y} = \frac{-3y}{2\sqrt{48 - 4x^2 - 3y^2}}$$

$$4x^2 + 3y^2 + 4z^2 = 48$$

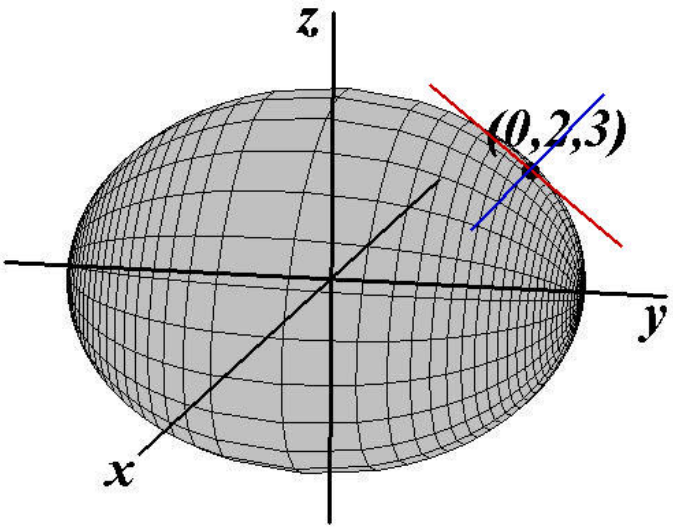
$$\frac{\partial z}{\partial x} = \frac{-2x}{\sqrt{48 - 4x^2 - 3y^2}}$$

$$\frac{\partial z}{\partial y} = \frac{-3y}{2\sqrt{48 - 4x^2 - 3y^2}}$$

$$\text{At } x = 0 \text{ and } y = 2 \quad \frac{\partial z}{\partial x} = 0 \text{ and } \frac{\partial z}{\partial y} = -\frac{1}{2}$$

$$4x^2 + 3y^2 + 4z^2 = 48$$

$$\frac{\partial z}{\partial x}(0,2) = 0 \text{ and } \frac{\partial z}{\partial y}(0,2) = -\frac{1}{2}$$



$$4x^2 + 3y^2 + 4z^2 = 48$$

$$\frac{\partial}{\partial x}\left(4x^2 + 3y^2 + 4z^2\right) = \frac{\partial}{\partial x}(48)$$

$$4x^2 + 3y^2 + 4z^2 = 48$$

$$\frac{\partial}{\partial x}\left(4x^2 + 3y^2 + 4z^2\right) = \frac{\partial}{\partial x}(48)$$

$$8x + 0 + 8z\frac{\partial z}{\partial x} = 0$$

$$\frac{\partial z}{\partial x} = -\frac{x}{z}$$

At $(0,\ 2,\ 3)$, $\frac{\partial z}{\partial x} = \frac{0}{3} = 0$

$$4x^2 + 3y^2 + 4z^2 = 48$$

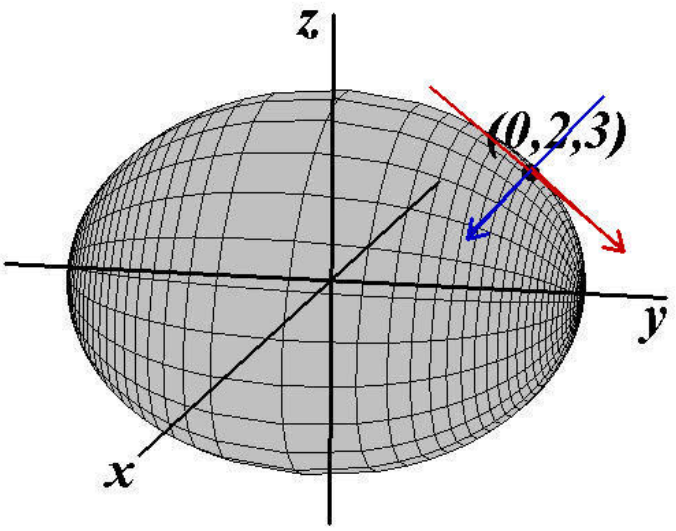
$$\frac{\partial}{\partial y}\left(4x^2 + 3y^2 + 4z^2\right) = \frac{\partial}{\partial y}(48)$$

$$0 + 6y + 8z\frac{\partial z}{\partial y} = 0$$

$$\frac{\partial z}{\partial y} = -\frac{3y}{4z}$$

At $(0,\ 2,\ 3)$, $\frac{\partial z}{\partial y} = -\frac{1}{2}$

Tangent vectors



Tangent plane