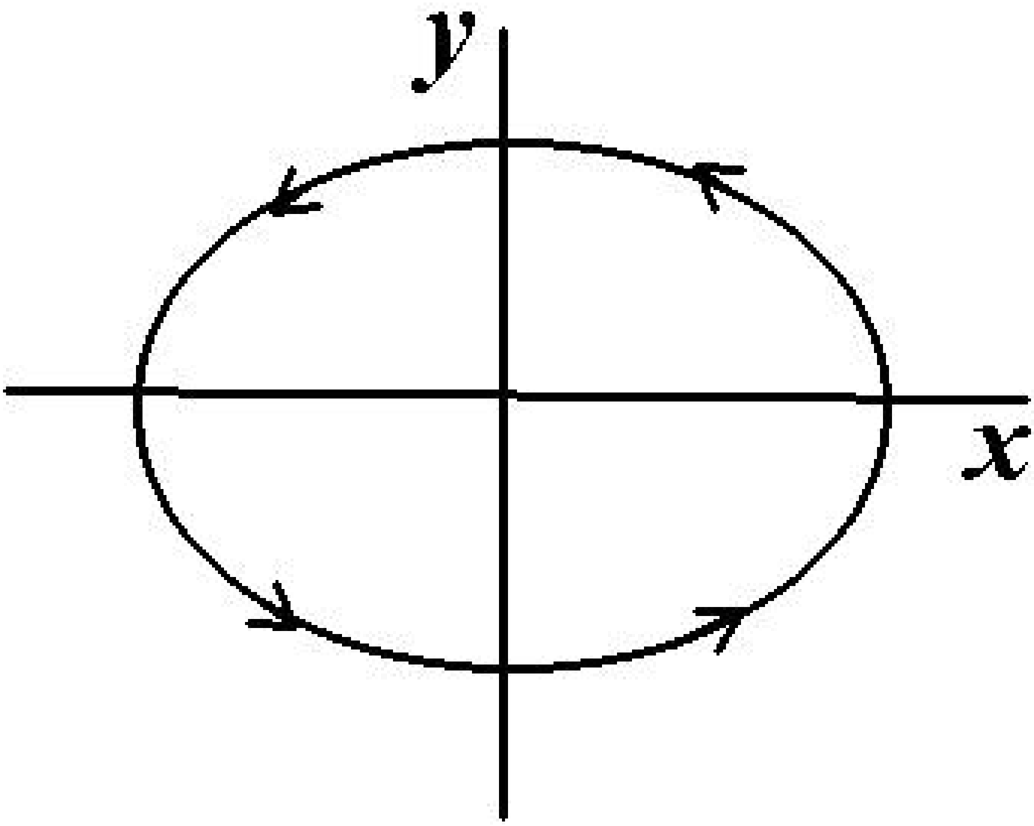
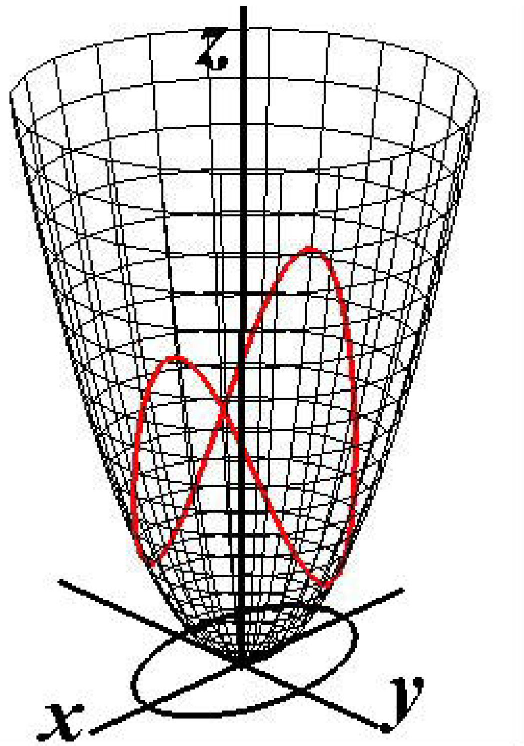
The Chain Rule for Partial Derivatives

$$x = 2\cos t \qquad y = \sin t$$



$$z = x^2 + y^2$$



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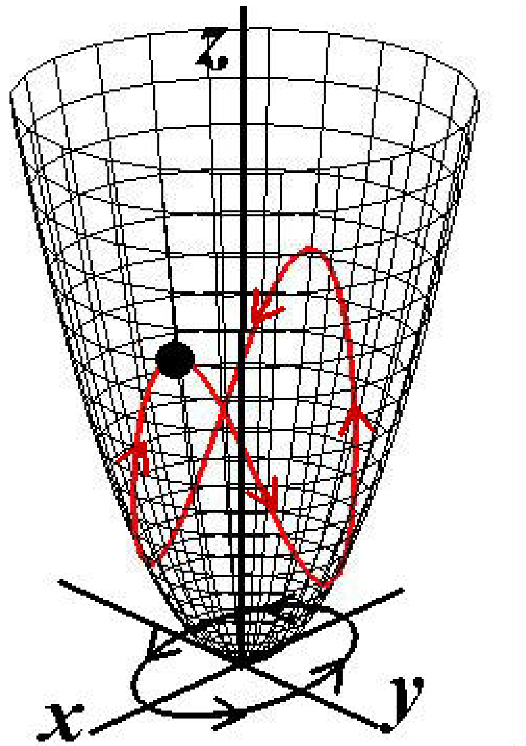

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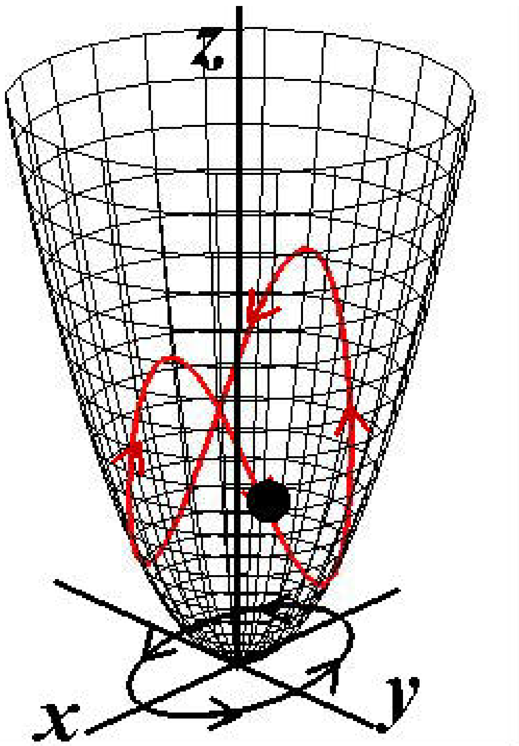

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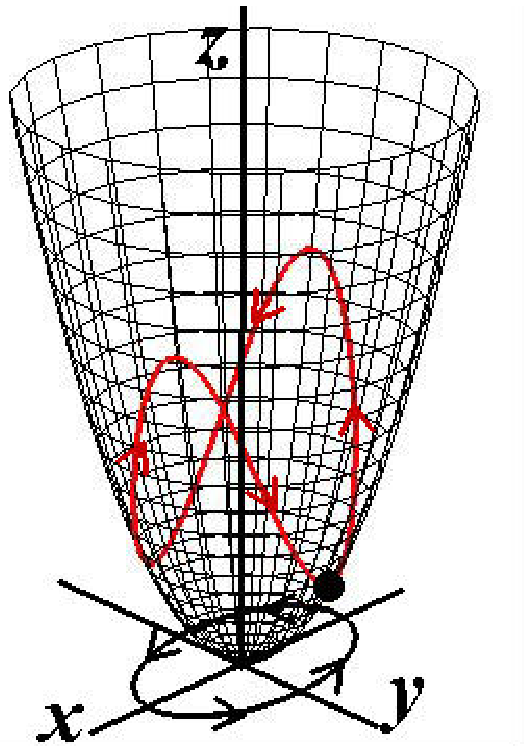

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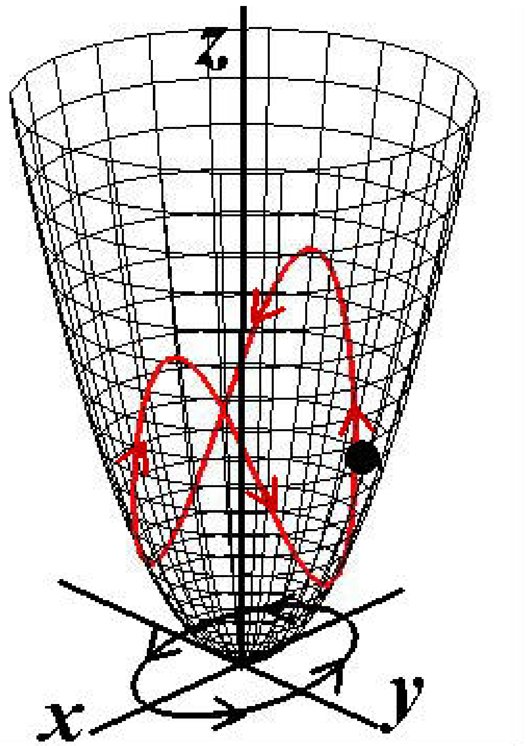

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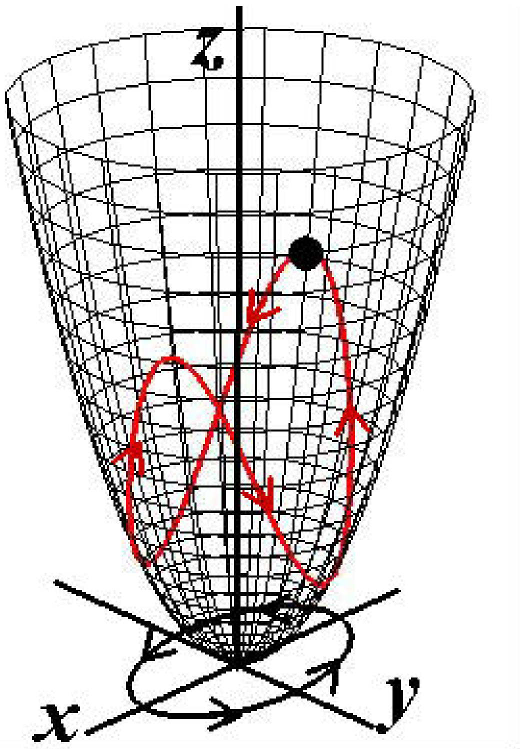
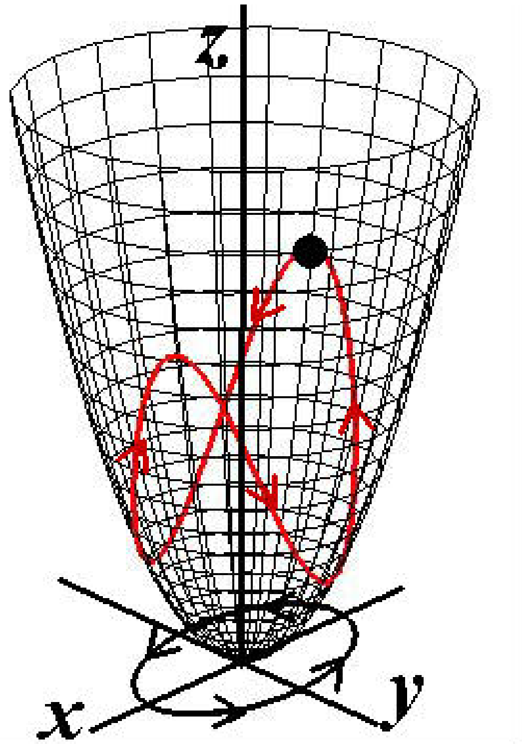


$$z = x^2 + y^2 \qquad \text{Find } \frac{dz}{dt}$$



$$z = x^2 + y^2$$

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$$\frac{dz}{dt} = -8\sin t\cos t + 2\sin t\cos t = -6\sin t\cos t$$

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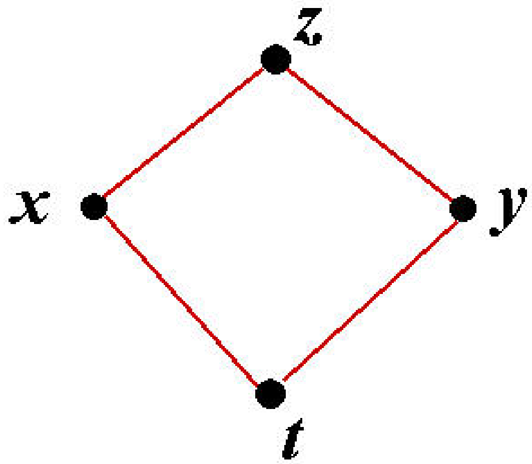
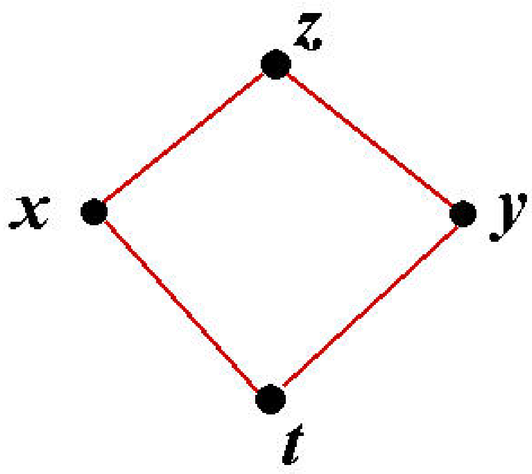
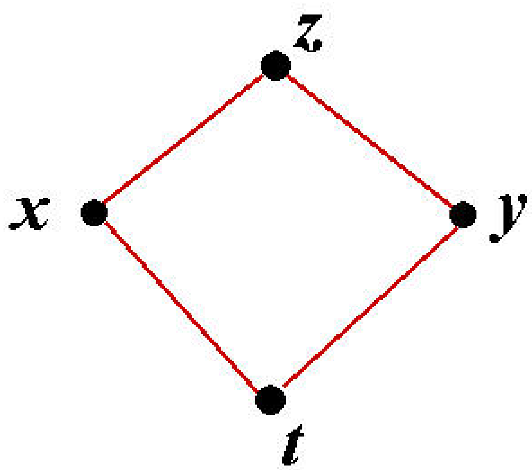


$$z = f(x, y)$$

$$x = x(t) \qquad y = y(t)$$

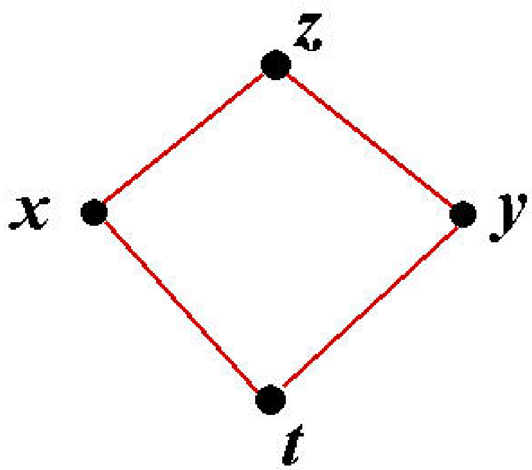


$$dz = \frac{\partial z}{\partial x}\, dx + \frac{\partial z}{\partial y}\, dy$$



$$dz = \frac{\partial z}{\partial x}\,dx + \frac{\partial z}{\partial y}\,dy$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\,\frac{dx}{dt} + \frac{\partial z}{\partial y}\,\frac{dy}{dt}$$



$$z = x^2 + y^2$$

$$x = 2\cos t \qquad y = \sin t$$

$$\begin{aligned}
\frac{dz}{dt} &= \frac{\partial z}{\partial x}\,\frac{dx}{dt} + \frac{\partial z}{\partial y}\,\frac{dy}{dt} \\
&= (2x)(-2\sin t) + (2y)(\cos t)
\end{aligned}$$

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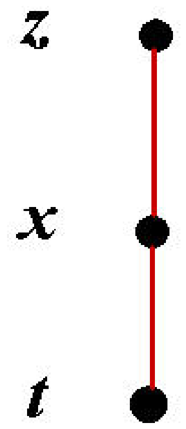
\frac{dz}{dt} &= \frac{\partial z}{\partial x}\,\frac{dx}{dt} + \frac{\partial z}{\partial y}\,\frac{dy}{dt} \\
&= (2x)(-2\sin t) + (2y)(\cos t) \\
&= (4\cos t)(-2\sin t) + (2\sin t)(\cos t) \\
&= -6\sin t\cos t
\end{aligned}$$

Chain Rule from Calculus I

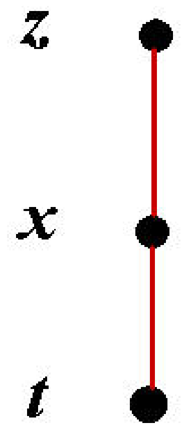
Suppose $z = f(x)$ and $x = g(t)$



Chain Rule from Calculus I

Suppose $z = f(x)$ and $x = g(t)$

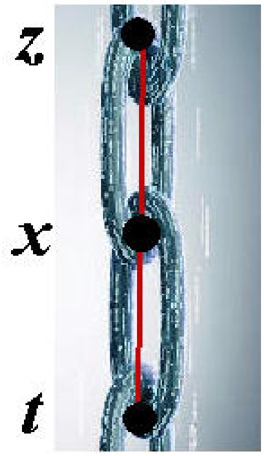
$$\frac{dz}{dt} = \frac{dz}{dx}\,\frac{dx}{dt}$$



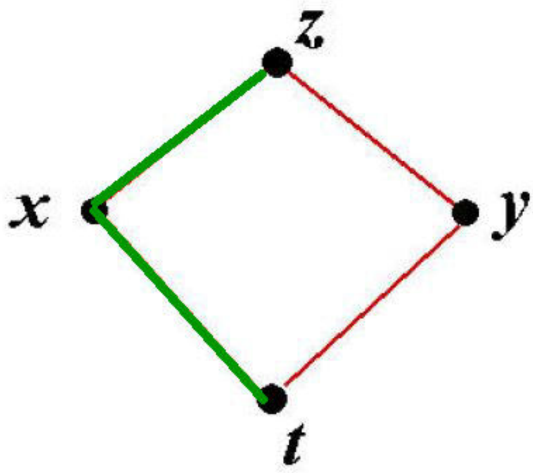
Chain Rule from Calculus I

Suppose $z = f(x)$ and $x = g(t)$

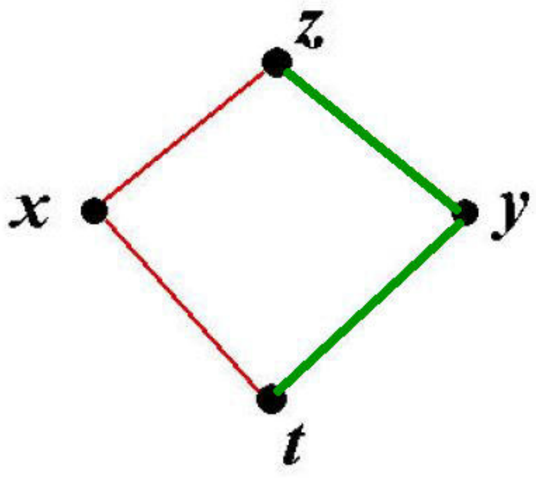
$$\frac{dz}{dt} = \frac{dz}{dx}\,\frac{dx}{dt}$$



$$\frac{dz}{dt} = \boxed{\frac{\partial z}{\partial x}\frac{dx}{dt}} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

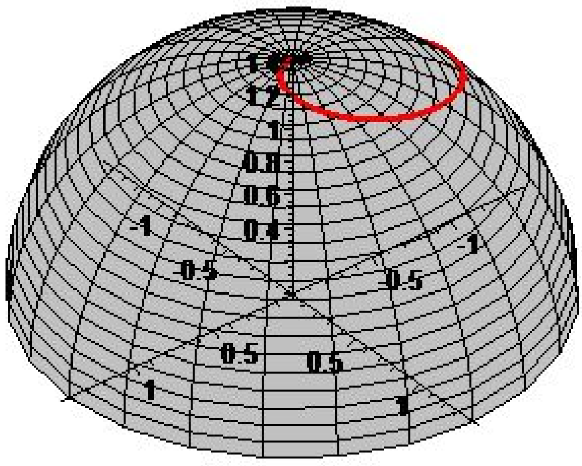


$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \boxed{\frac{\partial z}{\partial y}\frac{dy}{dt}}$$



$$x = t\cos \pi t \qquad y = t\sin \pi t \qquad z = \sqrt{2 - x^2 - y^2}$$

Use the Chain Rule for Partial Derivatives to find $\frac{dz}{dt}$ at $t = 1$.



$$x = t\cos\pi t \qquad y = t\sin\pi t \qquad z = \sqrt{2 - x^2 - y^2}$$

$$\frac{dx}{dt} = \cos\pi t - \pi t\sin\pi t$$

$$\frac{dy}{dt} = \sin\pi t + \pi t\cos\pi t$$

At $t = 1$,

$$\frac{dx}{dt} = -1 \qquad \frac{dy}{dt} = -\pi$$

$$x = t\cos \pi t \qquad y = t\sin \pi t \qquad z = \sqrt{2 - x^2 - y^2}$$

$$\frac{\partial z}{\partial x} = \frac{-x}{\sqrt{2 - x^2 - y^2}} \qquad \frac{\partial z}{\partial y} = \frac{-y}{\sqrt{2 - x^2 - y^2}}$$

At $t = 1$, $x = -1$ and $y = 0$ so:

$$\frac{\partial z}{\partial x} = 1 \qquad \frac{\partial z}{\partial y} = 0$$

$$x = t\cos\pi t \qquad y = t\sin\pi t \qquad z = \sqrt{2 - x^2 - y^2}$$

$$\begin{aligned}
\frac{dz}{dt} &= \frac{\partial z}{\partial x}\,\frac{dx}{dt} + \frac{\partial z}{\partial y}\,\frac{dy}{dt} \\
&= (1)(-1) + (0)(-\pi) \\
&= -1
\end{aligned}$$

$$x = t\cos\pi t \qquad y = t\sin\pi t \qquad z = \sqrt{2 - x^2 - y^2}$$

$$z = \sqrt{2 - (t\cos\pi t)^2 - (t\sin\pi t)^2} = \sqrt{2 - t^2}$$

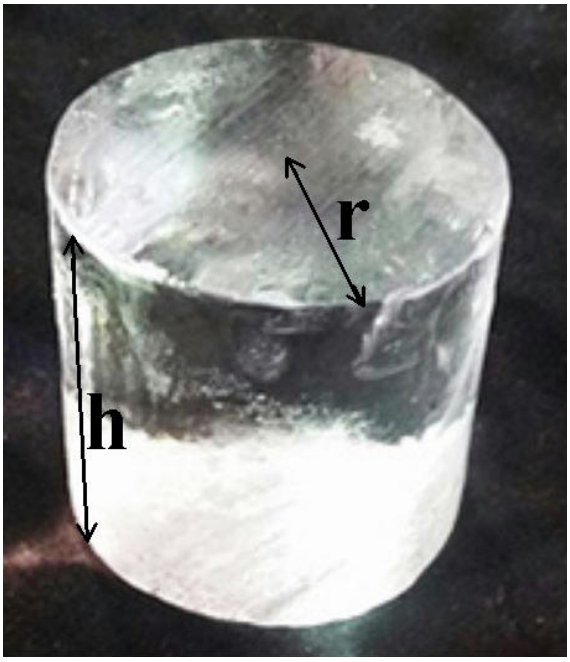
$$\frac{dz}{dt} = \frac{-t}{\sqrt{2 - t^2}}$$

At $t = 1$,

$$\frac{dz}{dt} = \frac{-1}{\sqrt{2 - 1}} = -1$$

Cylindrical Block of Ice

$$V = \pi r^2 h$$



$$V = \pi r^2 h$$

Suppose this block of ice is melting. At the point in time when the radius is 6 cm and the height is 24 cm, the radius is shrinking at 3 cm/hour and the height is shrinking at 1 cm/hour.

How fast is the volume changing at this point in time?

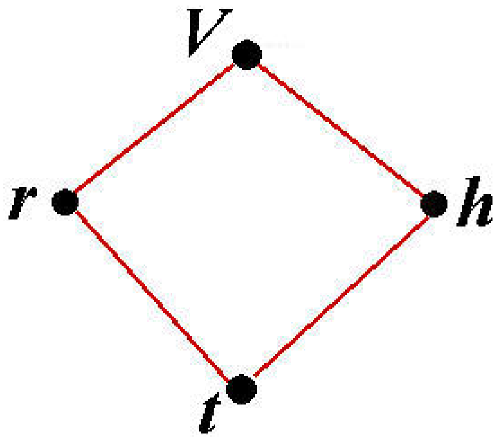
$$V = \pi r^2 h$$

Suppose this block of ice is melting. At the point in time when the radius is 6 cm and the height is 24 cm, the radius is shrinking at 3 cm/hour and the height is shrinking at 1 cm/hour.

At $r = 6$ and $h = 24$ then $\frac{dr}{dt} = -3$ and $\frac{dh}{dt} = -1$. Find $\frac{dV}{dt}$

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = \frac{\partial V}{\partial r}\frac{dr}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt}$$



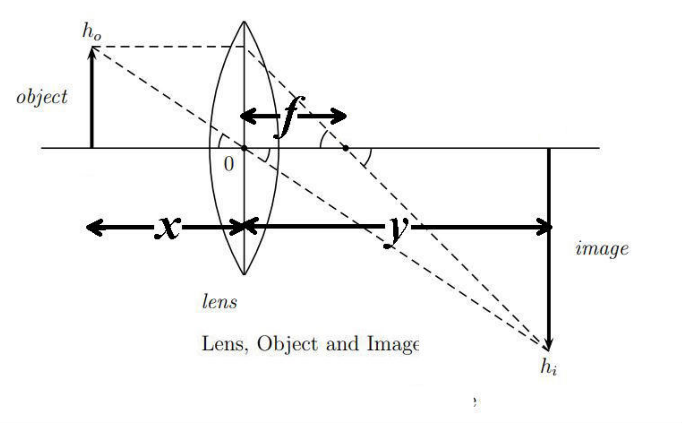
$$V = \pi r^2 h$$

$$\begin{aligned}
\frac{dV}{dt} &= \frac{\partial V}{\partial r}\frac{dr}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt} \\
&= 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}
\end{aligned}$$

$$V = \pi r^2 h$$

$$\begin{aligned}
\frac{dV}{dt} &= \frac{\partial V}{\partial r}\frac{dr}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt} \\
&= 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt} \\
&= 2\pi(6)(24)(-3) + \pi(6)^2(-1) \\
&= -900\pi \\
&\approx 2827 \text{ cm}^3/\text{hour}
\end{aligned}$$

Let x be the distance from an object to a lens
Let y be the distance from the lens to the image
Let f be the focal length



$$\frac{1}{f} = \frac{1}{x} + \frac{1}{y}$$

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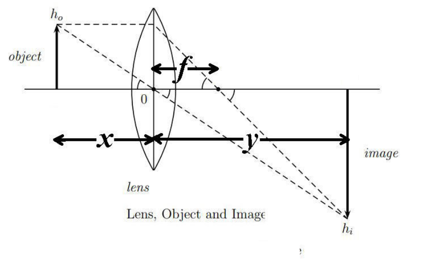
At the point in time when $x = 4$ cm and $y = 4$ cm, the positions of the object and its image are changing at the following rates:

$$\frac{dx}{dt} = \frac{1}{8} \text{ cm/min} \qquad \frac{dy}{dt} = \frac{3}{8} \text{ cm/min}$$

Find $\frac{df}{dt}$



$$\frac{1}{f} = \frac{1}{x} + \frac{1}{y} = \frac{y + x}{xy}$$

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$$f = \frac{xy}{y+x}$$

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

$$\frac{1}{f} = \frac{1}{x} + \frac{1}{y} = \frac{y+x}{xy}$$

$$f = \frac{xy}{y+x}$$

$$\begin{aligned}
\frac{df}{dt} &= \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} \\
&= \left(\frac{y}{x+y}\right)^2 \frac{dx}{dt} + \left(\frac{x}{x+y}\right)^2 \frac{dy}{dt}
\end{aligned}$$

When x = y = 4 then $\frac{dx}{dt} = \frac{1}{8}$ and $\frac{dy}{dt} = \frac{3}{8}$

$$\begin{aligned}
\frac{df}{dt} &= \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} \\
&= \left(\frac{y}{x+y}\right)^2 \frac{dx}{dt} + \left(\frac{x}{x+y}\right)^2 \frac{dy}{dt} \\
&= \left(\frac{4}{4+4}\right)^2 \cdot \frac{1}{8} + \left(\frac{4}{4+4}\right)^2 \cdot \frac{3}{8} \\
&= \frac{1}{8} \text{ cm/min}
\end{aligned}$$