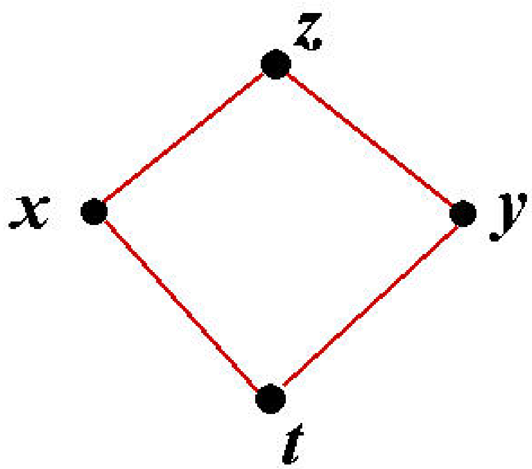
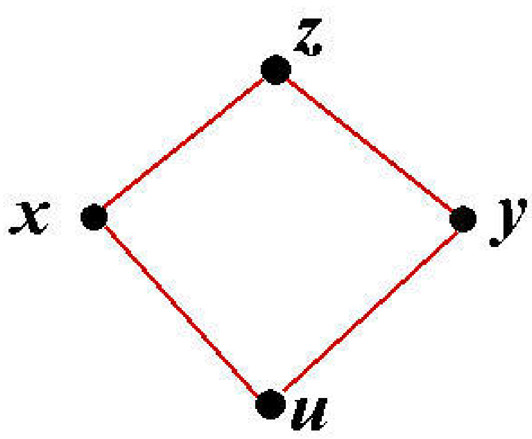
Chain Rule for Partial Derivatives - Generalization

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

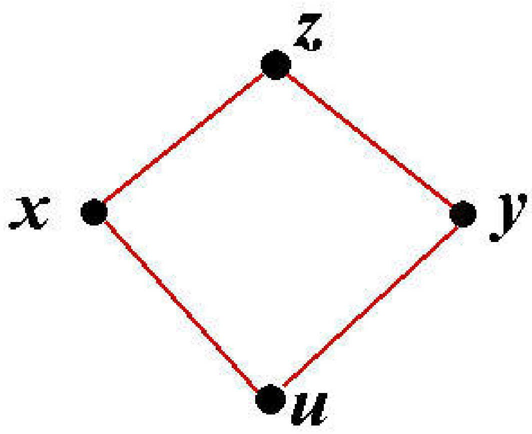


$$z = xy^2 \qquad x = 4u + 3 \qquad y = 2u^3 - 1$$



$$z = xy^2 \qquad x = 4u + 3 \qquad y = 2u^3 - 1$$

$$\frac{dz}{du} = \frac{\partial z}{\partial x}\frac{dx}{du} + \frac{\partial z}{\partial y}\frac{dy}{du}$$



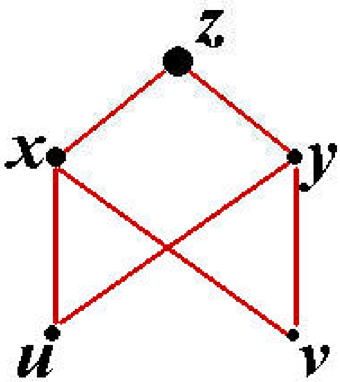
$$z = xy^2 \qquad x = 4u + 3 \qquad y = 2u^3 - 1$$

$$\begin{aligned} \frac{dz}{du} &= \frac{\partial z}{\partial x}\frac{dx}{du} + \frac{\partial z}{\partial y}\frac{dy}{du} \\ &= \left(y^2\right)(4) + (2xy)(6u^2) \end{aligned}$$

$$z = xy^2 \qquad x = 4u + 3 \qquad y = 2u^3 - 1$$

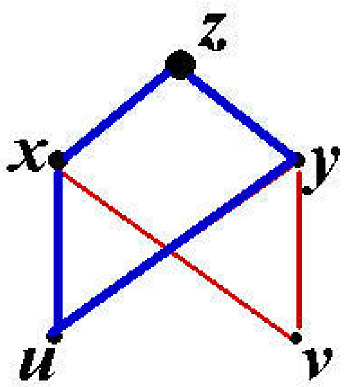
$$\begin{aligned} \frac{dz}{du} &= \frac{\partial z}{\partial x}\frac{dx}{du} + \frac{\partial z}{\partial y}\frac{dy}{du} \\ &= \left(y^2\right)(4) + (2xy)(6u^2) \\ &= 4\left(2u^3 - 1\right)^2 + 12u^2(4u+3)\left(2u^3 - 1\right) \end{aligned}$$

$$z = xy^2 \qquad x = 4u + 7v \qquad y = 2u^3 - v$$



$$z = xy^2 \qquad x = 4u + 7v \qquad y = 2u^3 - v$$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$$



$$z = xy^2 \qquad x = 4u + 7v \qquad y = 2u^3 - v$$

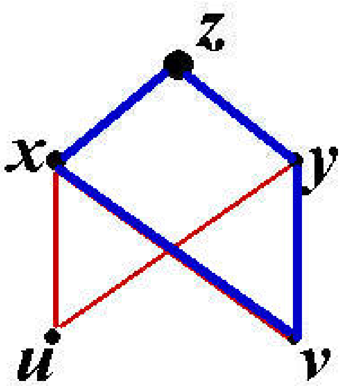
$$\begin{aligned} \frac{\partial z}{\partial u} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} \\ &= \left(y^2\right)(4) + (2xy)\left(6u^2\right) \end{aligned}$$

$$z = xy^2 \qquad x = 4u + 7v \qquad y = 2u^3 - v$$

$$\begin{aligned}
\frac{\partial z}{\partial u} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} \\
&= \left(y^2\right)(4) + (2xy)\left(6u^2\right) \\
&= 4\left(2u^3 - v\right)^2 + 12(4u + 7v)\left(2u^3 - v\right)u^2
\end{aligned}$$

$$z = xy^2 \qquad x = 4u + 7v \qquad y = 2u^3 - v$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$$



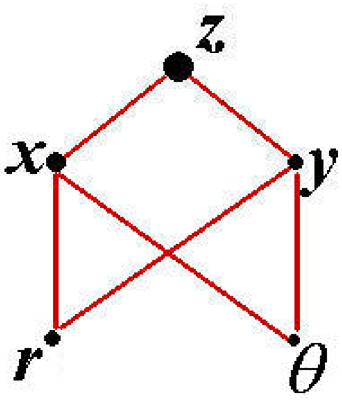
$$z = xy^2 \qquad x = 4u + 7v \qquad y = 2u^3 - v$$

$$\begin{aligned} \frac{\partial z}{\partial v} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v} \\ &= \left(y^2\right)(7) + (2xy)(-1) \end{aligned}$$

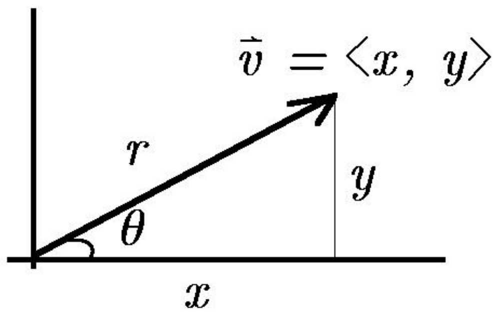
$$z = xy^2 \qquad x = 4u + 7v \qquad y = 2u^3 - v$$

$$\begin{aligned}
\frac{\partial z}{\partial v} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v} \\
&= \left(y^2\right)(7) + (2xy)(-1) \\
&= 7\left(2u^3 - v\right)^2 - 2(4u + 7)\left(2u^3 - v\right)
\end{aligned}$$

$$z = 1 - \sqrt{x^2 + y^2} \qquad x = r\cos\theta \qquad y = r\sin\theta$$

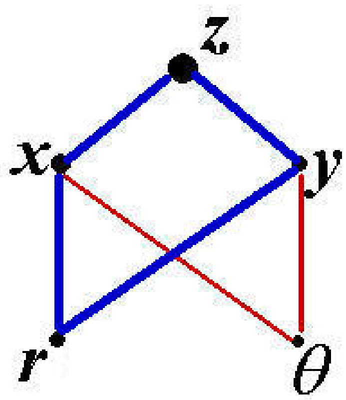


$$x = r\cos\theta \qquad y = r\sin\theta$$



$$z = 1 - \sqrt{x^2 + y^2} \qquad x = r\cos\theta \qquad y = r\sin\theta$$

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r}$$



$$z = 1 - \sqrt{x^2 + y^2} \qquad x = r\cos\theta \qquad y = r\sin\theta$$

$$\begin{aligned}
\frac{\partial z}{\partial r} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r} \\
&= \left(\frac{-x}{\sqrt{x^2 + y^2}}\right)\cos\theta + \left(\frac{-y}{\sqrt{x^2 + y^2}}\right)\sin\theta
\end{aligned}$$

$$z = 1 - \sqrt{x^2 + y^2} \qquad x = r\cos\theta \qquad y = r\sin\theta$$

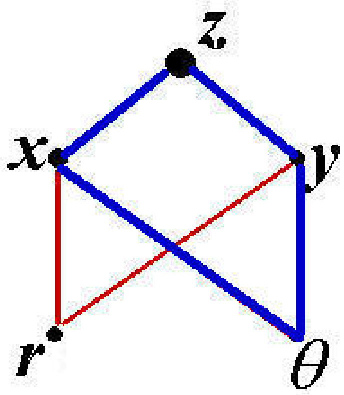
$$\begin{aligned}
\frac{\partial z}{\partial r} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r} \\
&= \left(\frac{-x}{\sqrt{x^2+y^2}}\right)\cos\theta + \left(\frac{-y}{\sqrt{x^2+y^2}}\right)\sin\theta \\
&= \left(\frac{-r\cos\theta}{r}\right)\cos\theta + \left(\frac{-r\sin\theta}{r}\right)\sin\theta
\end{aligned}$$

$$z = 1 - \sqrt{x^2 + y^2} \qquad x = r\cos\theta \qquad y = r\sin\theta$$

$$\begin{aligned}
\frac{\partial z}{\partial r} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r} \\
&= \left(\frac{-x}{\sqrt{x^2+y^2}}\right)\cos\theta + \left(\frac{-y}{\sqrt{x^2+y^2}}\right)\sin\theta \\
&= \left(\frac{-r\cos\theta}{r}\right)\cos\theta + \left(\frac{-r\sin\theta}{r}\right)\sin\theta \\
&= -1
\end{aligned}$$

$$z = 1 - \sqrt{x^2 + y^2} \qquad x = r\cos\theta \qquad y = r\sin\theta$$

$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta}$$



$$z = 1 - \sqrt{x^2 + y^2} \qquad x = r\cos\theta \qquad y = r\sin\theta$$

$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta}$$

$$= \frac{-x}{\sqrt{x^2 + y^2}} \cdot (-r\sin\theta) + \frac{-y}{\sqrt{x^2 + y^2}} \cdot (r\cos\theta)$$

$$z = 1 - \sqrt{x^2 + y^2} \qquad x = r\cos\theta \qquad y = r\sin\theta$$

$$\begin{aligned}
\frac{\partial z}{\partial \theta} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta} \\
&= \frac{-x}{\sqrt{x^2+y^2}} \cdot (-r\sin\theta) + \frac{-y}{\sqrt{x^2+y^2}} \cdot (r\cos\theta) \\
&= \left(\frac{-r\cos\theta}{r}\right)(-r\sin\theta) + \left(\frac{-r\sin\theta}{r}\right)(r\cos\theta)
\end{aligned}$$

$$z = 1 - \sqrt{x^2 + y^2} \qquad x = r\cos\theta \qquad y = r\sin\theta$$

$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta}$$

$$= \frac{-x}{\sqrt{x^2 + y^2}} \cdot (-r\sin\theta) + \frac{-y}{\sqrt{x^2 + y^2}} \cdot (r\cos\theta)$$

$$= \left(\frac{-r\cos\theta}{r}\right)(-r\sin\theta) + \left(\frac{-r\sin\theta}{r}\right)(r\cos\theta)$$

$$= 0$$

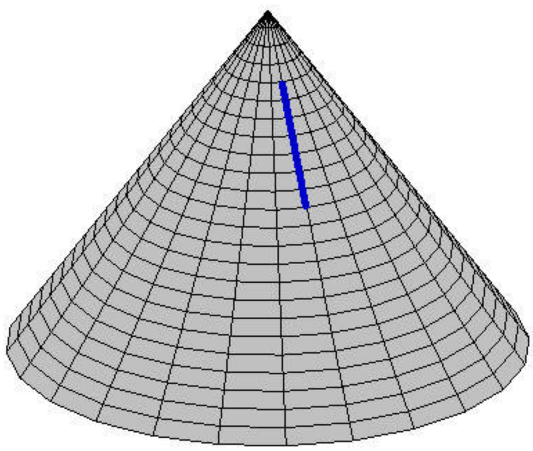
$$z = 1 - \sqrt{x^2 + y^2} \qquad x = r\cos\theta \qquad y = r\sin\theta$$

$$z = 1 - \sqrt{r^2\cos^2\theta + r^2\sin^2\theta} = 1 - \sqrt{r^2} = 1 - r$$

$$\frac{\partial z}{\partial r} = -1 \qquad \frac{\partial z}{\partial \theta} = 0$$

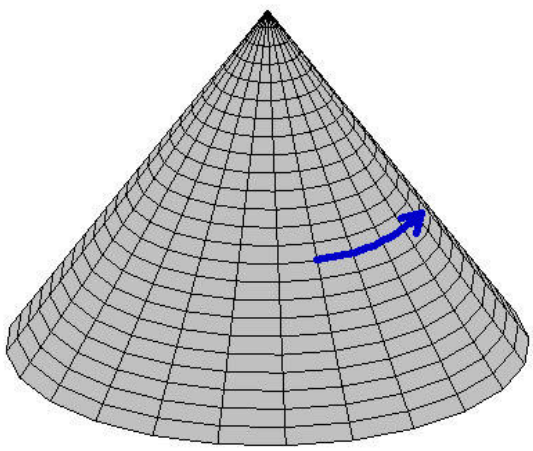
$$z = 1 - r$$

$$\frac{\partial z}{\partial r} = -1$$



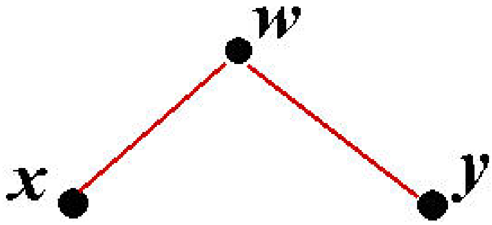
$$z = 1 - r$$

$$\frac{\partial z}{\partial \theta} = 0$$



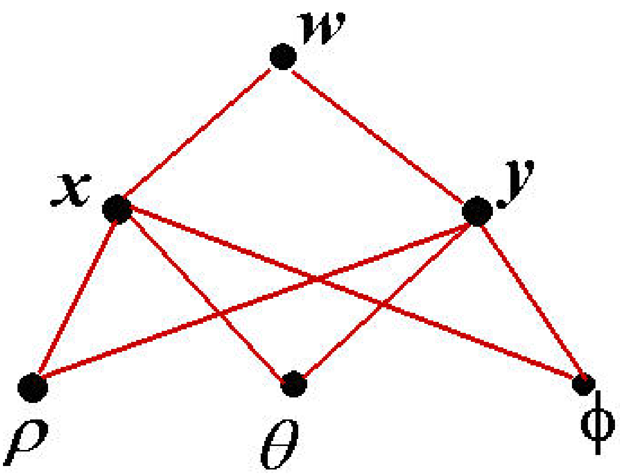
$$w = f(x,\ y)$$

$$\frac{\partial w}{\partial x} = 2x + \frac{y}{x^2} \qquad \frac{\partial w}{\partial y} = 2y - \frac{1}{x}$$



$$w = f(x,\ y)$$

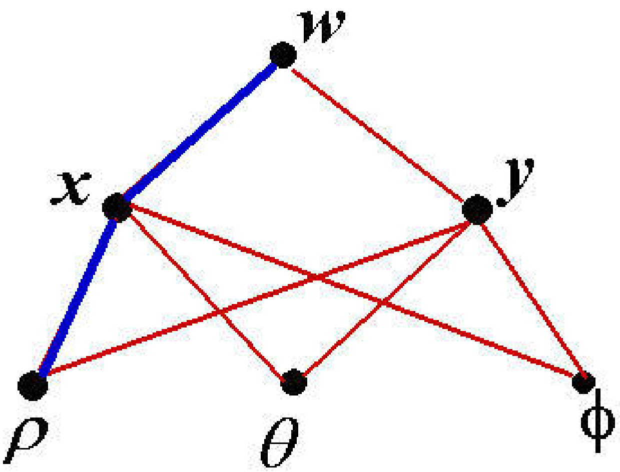
$$x = \rho \cos\theta \sin\phi \qquad y = \rho \sin\theta \sin\phi$$



$$\frac{\partial w}{\partial x} = 2x + \frac{y}{x^2} \qquad \frac{\partial w}{\partial y} = 2y - \frac{1}{x}$$

$$x = \rho \cos\theta \sin\phi \qquad y = \rho \sin\theta \sin\phi$$

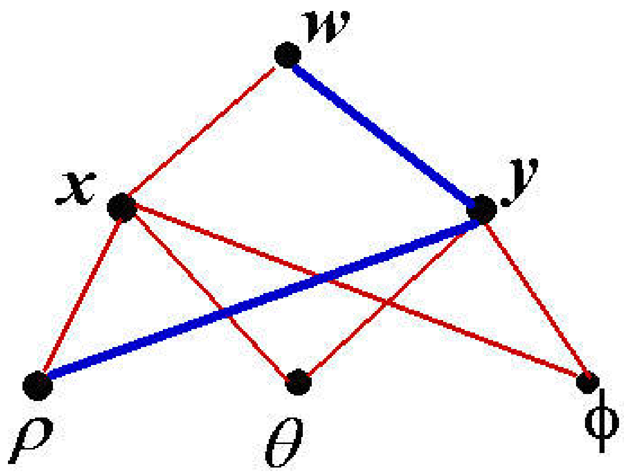
$$\frac{\partial w}{\partial \rho} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial \rho} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \rho}$$



$$\frac{\partial w}{\partial x} = 2x + \frac{y}{x^2} \qquad \frac{\partial w}{\partial y} = 2y - \frac{1}{x}$$

$$x = \rho \cos\theta \sin\phi \qquad y = \rho \sin\theta \sin\phi$$

$$\frac{\partial w}{\partial \rho} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial \rho} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \rho}$$



$$\frac{\partial w}{\partial x} = 2x + \frac{y}{x^2} \qquad \frac{\partial w}{\partial y} = 2y - \frac{1}{x}$$

$$x = \rho \cos\theta \sin\phi \qquad y = \rho \sin\theta \sin\phi$$

$$\begin{aligned}
\frac{\partial w}{\partial \rho} &= \frac{\partial w}{\partial x}\frac{\partial x}{\partial \rho} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \rho} \\
&= \left(2x + \frac{y}{x^2}\right)\cos\theta \sin\phi + \left(2y - \frac{1}{x}\right)\sin\theta \sin\phi
\end{aligned}$$

$$\frac{\partial w}{\partial x} = 2x + \frac{y}{x^2} \qquad \frac{\partial w}{\partial y} = 2y - \frac{1}{x}$$

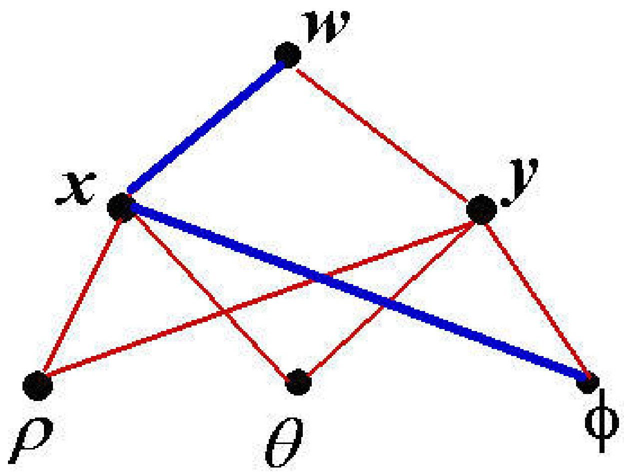
$$x = \rho \cos\theta \sin\phi \qquad y = \rho \sin\theta \sin\phi$$

$$\begin{aligned}
\frac{\partial w}{\partial \rho} &= \frac{\partial w}{\partial x}\frac{\partial x}{\partial \rho} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \rho} \\
&= \left(2x + \frac{y}{x^2}\right)\cos\theta\sin\phi + \left(2y - \frac{1}{x}\right)\sin\theta\sin\phi \\
&= 2\rho\sin^2\phi
\end{aligned}$$

$$\frac{\partial w}{\partial x} = 2x + \frac{y}{x^2} \qquad \frac{\partial w}{\partial y} = 2y - \frac{1}{x}$$

$$x = \rho \cos\theta \sin\phi \qquad y = \rho \sin\theta \sin\phi$$

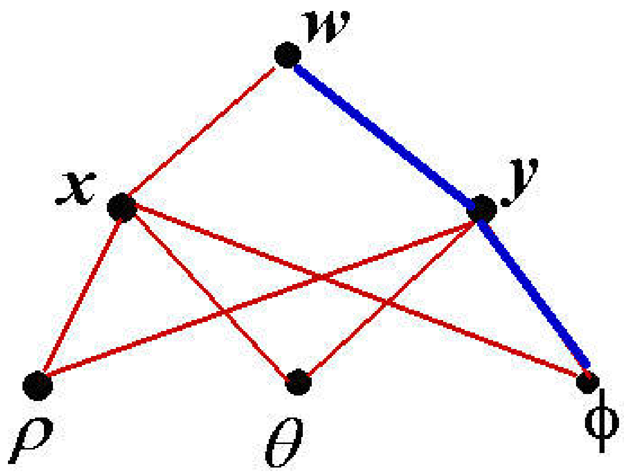
$$\frac{\partial w}{\partial \phi} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial \phi} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \phi}$$



$$\frac{\partial w}{\partial x} = 2x + \frac{y}{x^2} \qquad \frac{\partial w}{\partial y} = 2y - \frac{1}{x}$$

$$x = \rho \cos\theta \sin\phi \qquad y = \rho \sin\theta \sin\phi$$

$$\frac{\partial w}{\partial \phi} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial \phi} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \phi}$$



$$\frac{\partial w}{\partial x} = 2x + \frac{y}{x^2} \qquad \frac{\partial w}{\partial y} = 2y - \frac{1}{x}$$

$$x = \rho \cos\theta \sin\phi \qquad y = \rho \sin\theta \sin\phi$$

$$\frac{\partial w}{\partial \phi} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial \phi} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \phi}$$

$$= \left(2x + \frac{y}{x^2}\right)\rho\cos\theta\cos\phi + \left(2y - \frac{1}{x}\right)\rho\sin\theta\cos\phi$$

$$\frac{\partial w}{\partial x} = 2x + \frac{y}{x^2} \qquad \frac{\partial w}{\partial y} = 2y - \frac{1}{x}$$

$$x = \rho \cos\theta \sin\phi \qquad y = \rho \sin\theta \sin\phi$$

$$\frac{\partial w}{\partial \phi} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial \phi} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \phi}$$

$$= \left(2x + \frac{y}{x^2}\right)\rho\cos\theta\cos\phi + \left(2y - \frac{1}{x}\right)\rho\sin\theta\cos\phi$$

$$= 2\rho^2 \sin\phi\cos\phi$$

$$x = \rho \cos\theta \sin\phi \qquad y = \rho \sin\theta \sin\phi \qquad z = \rho \cos\phi$$