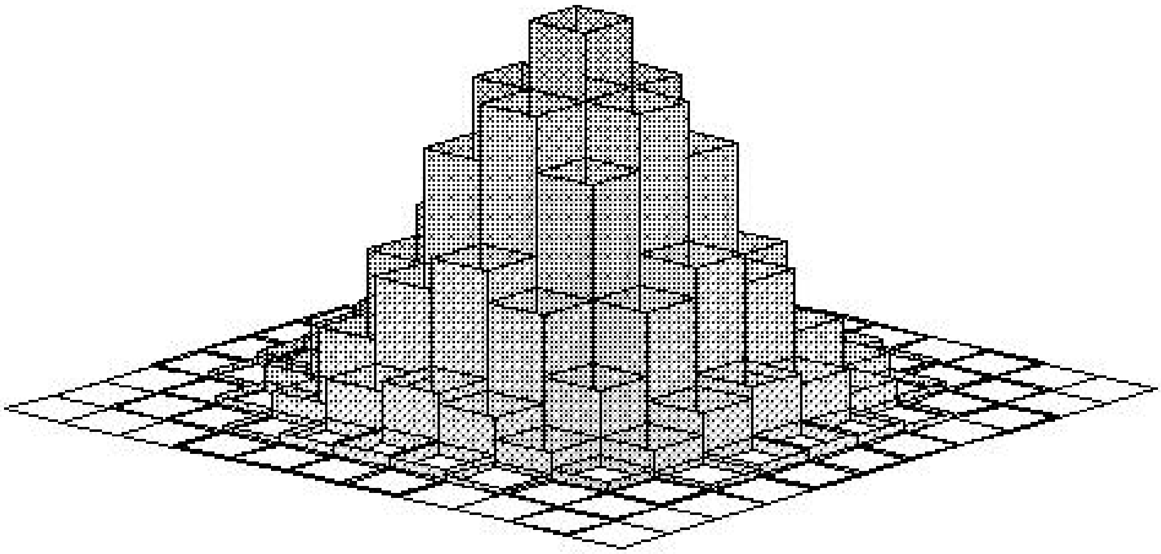
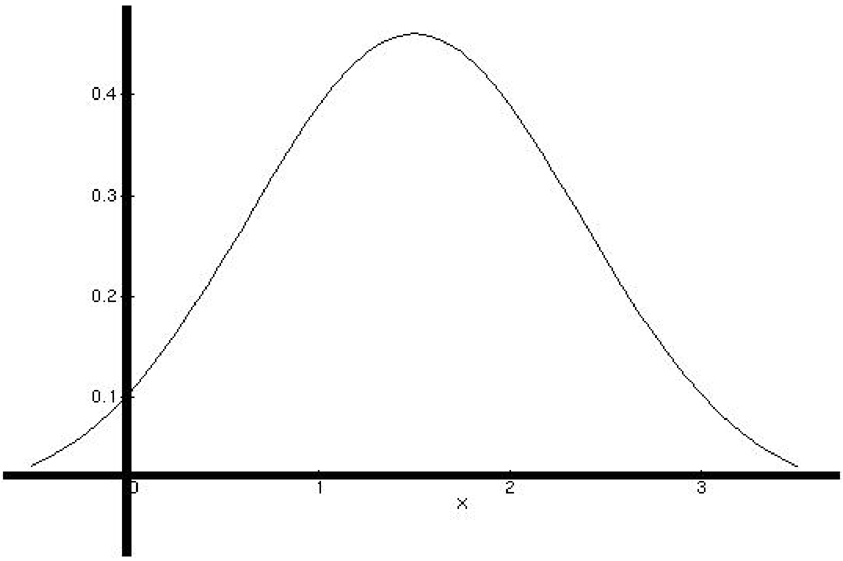
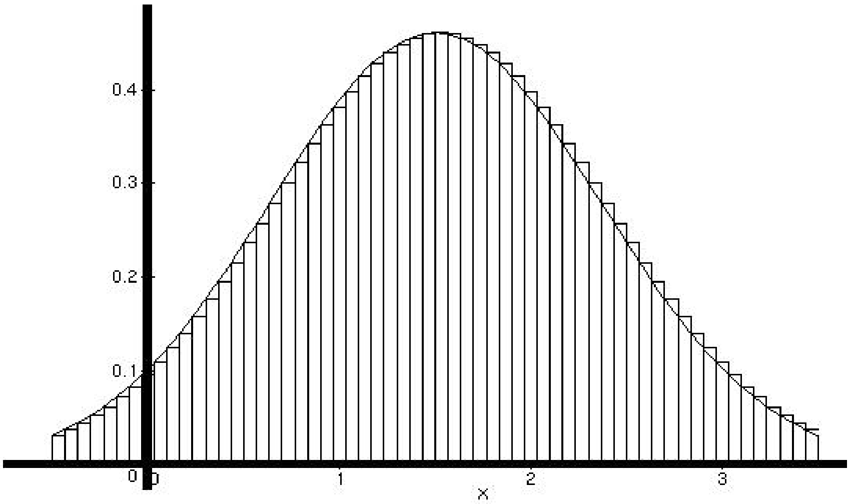
Introduction to Double Integrals



$$\text{Area under the curve} = \int_a^b f(x)\, dx$$

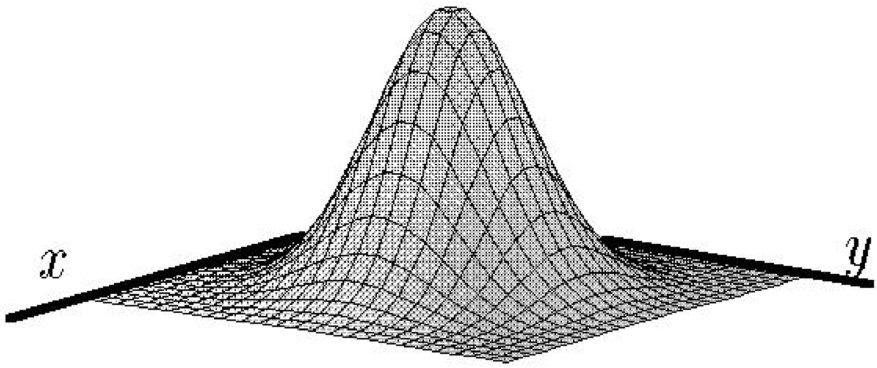


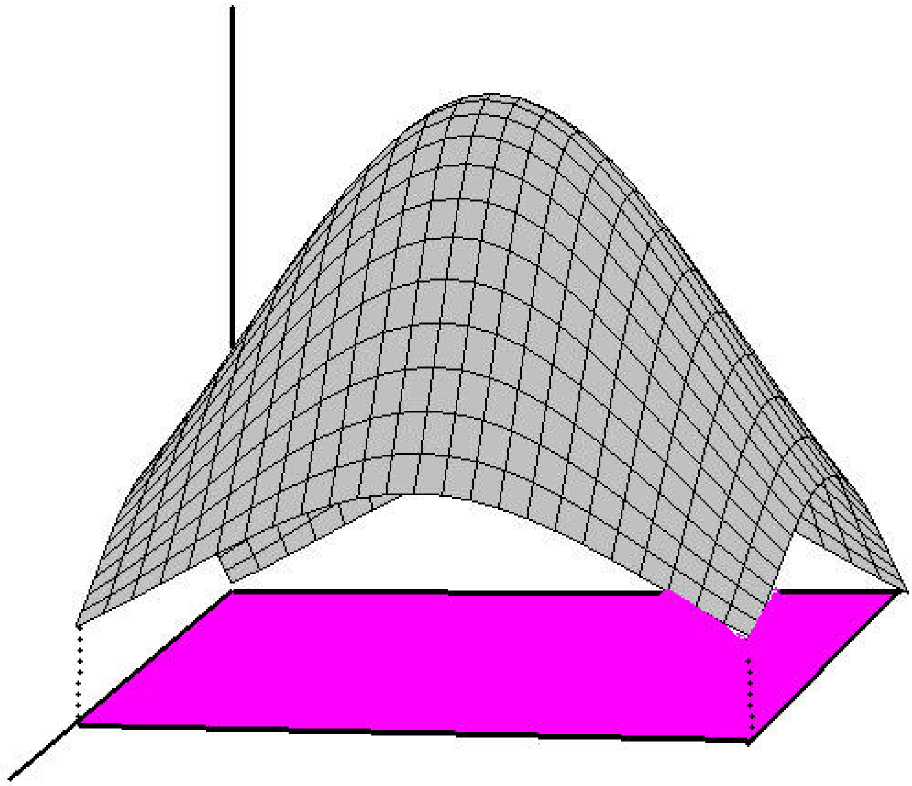
$$\int_a^b f(x)\ dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

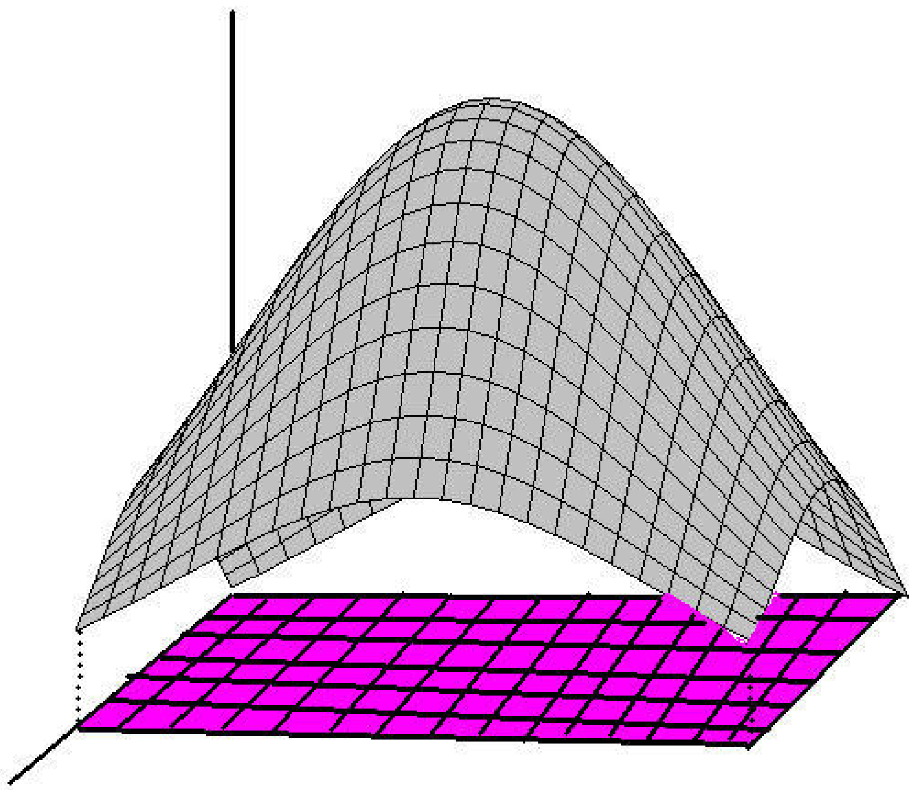


Find the volume under a section of a surface

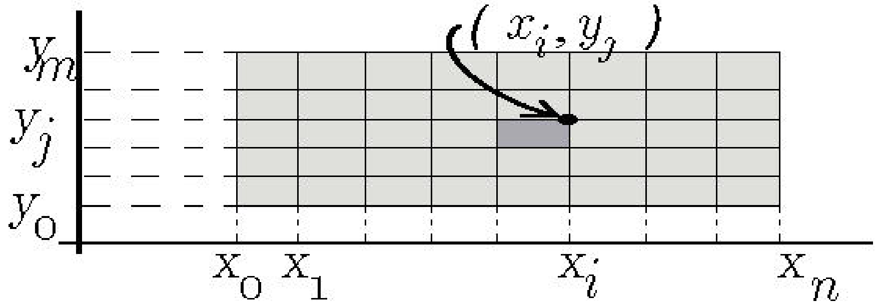
$$z = f(x, y)$$



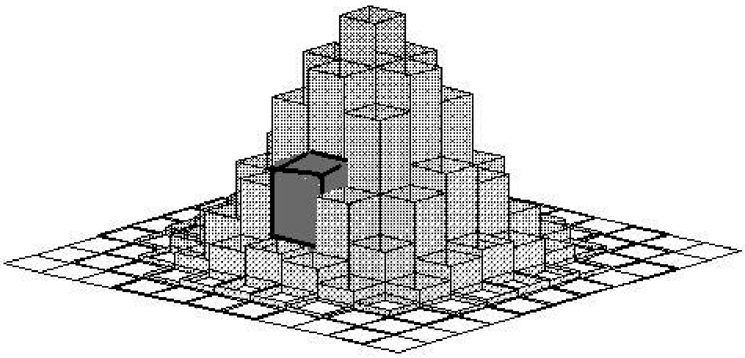




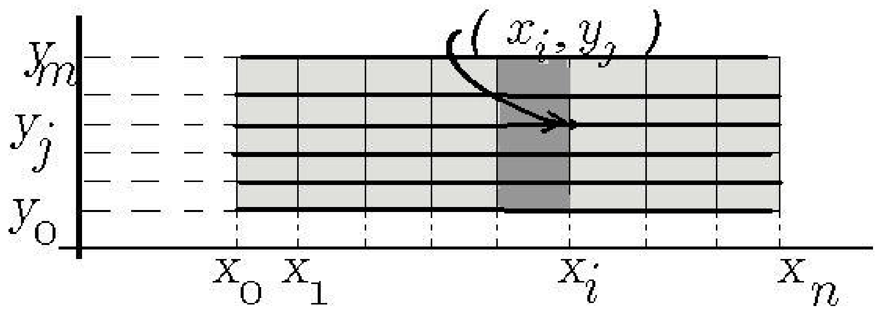
Take the section directly underneath the section of the surface that we are interested in. Subdivide it into a grid.



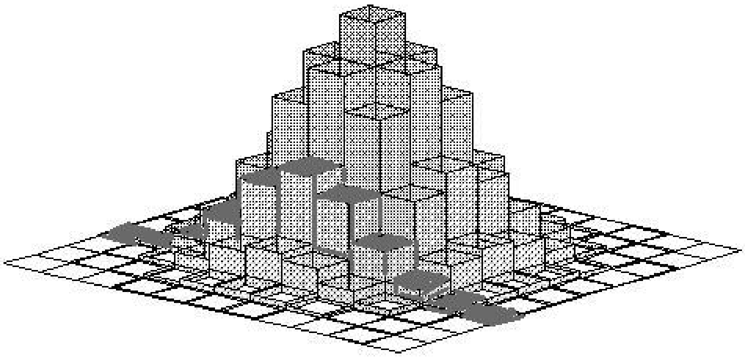
$$\text{Volume of One Section} = f(x_i,\ y_j)\,\Delta x\,\Delta y$$



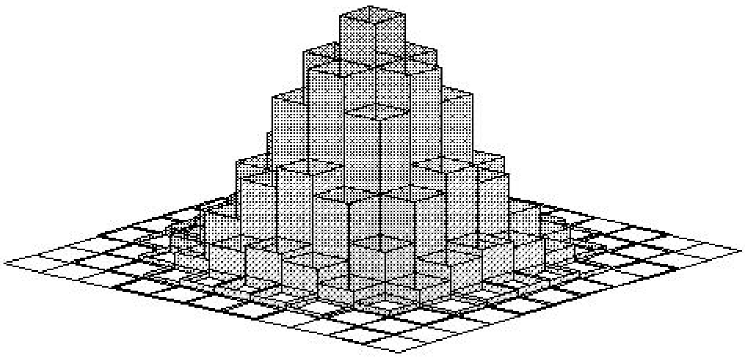
Repeat the procedure along an entire column in the grid



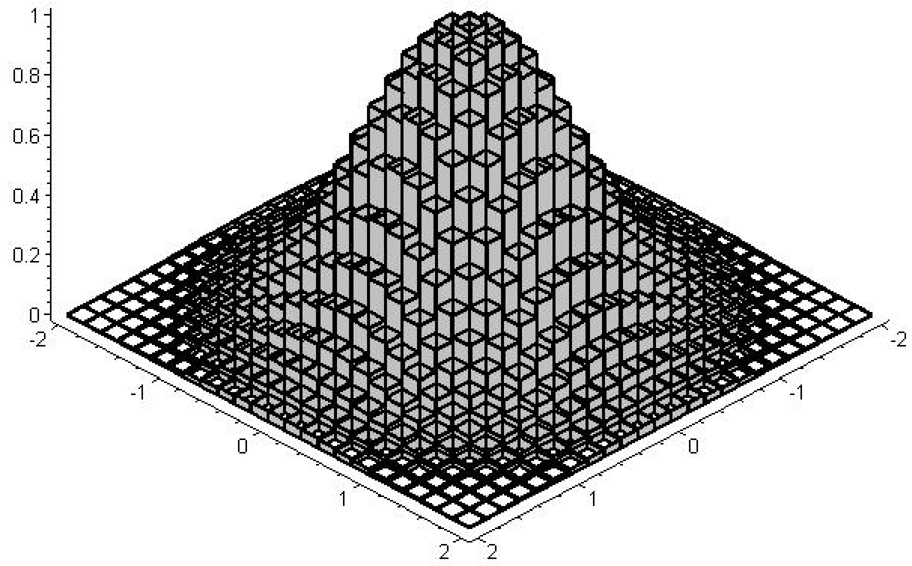
$$\text{Volume Over Grid Column} = \sum_{j=1}^{m} f(x_i,\ y_j)\,\Delta x\,\Delta y$$



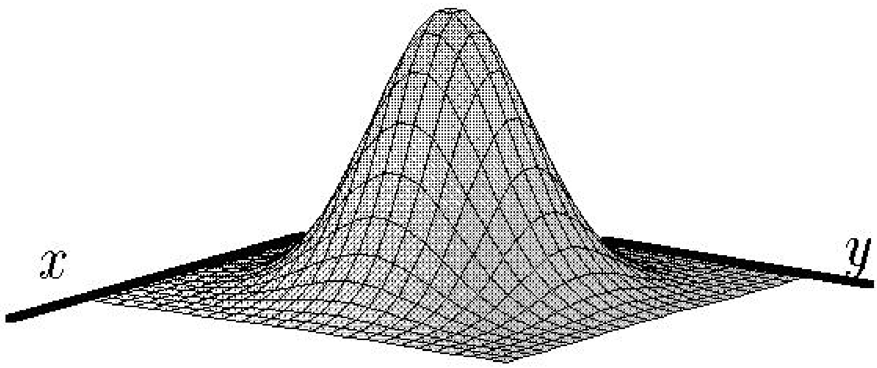
$$\text{Total Volume} \approx \sum_{i=1}^{n} \sum_{j=1}^{m} f(x_i,\ y_j)\,\Delta x\,\Delta y$$



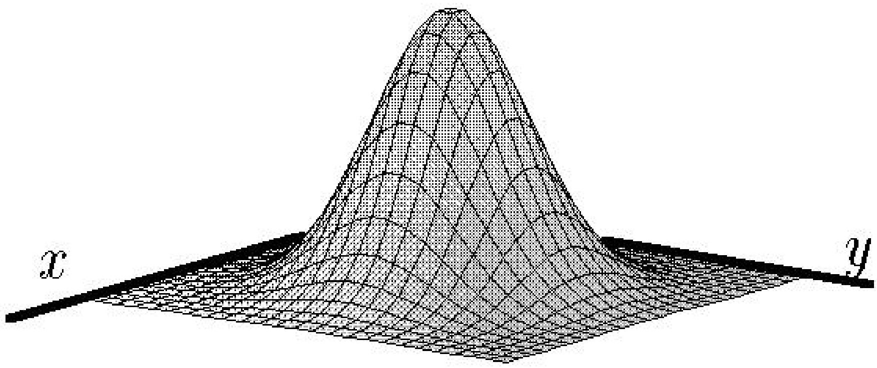
$$\text{Total Volume} = \lim_{n,m\to\infty} \sum_{i=1}^{n} \sum_{j=1}^{m} f(x_i,\ y_j)\, \Delta x\, \Delta y$$



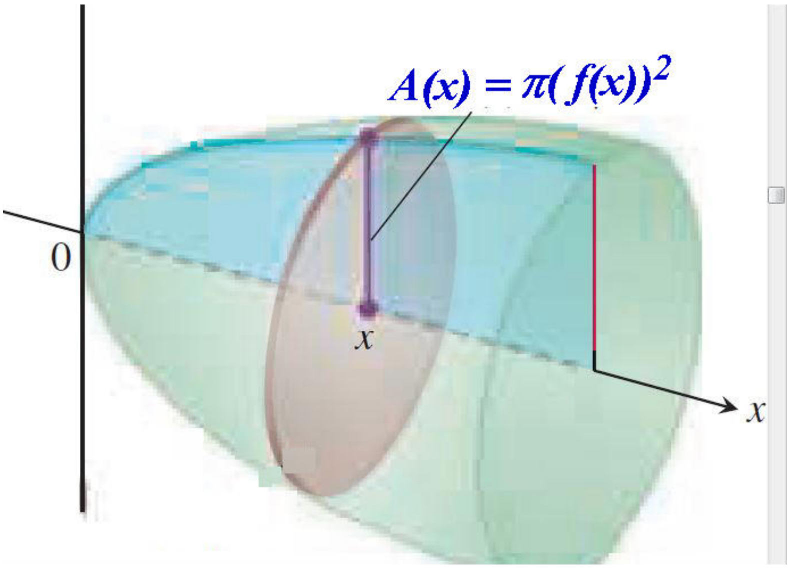
$$\iint_R f(x,y)\,dx\,dy = \lim_{n,m\to\infty} \sum_{i=1}^{n} \sum_{j=1}^{m} f(x_i,\ y_j)\,\Delta x\,\Delta y$$



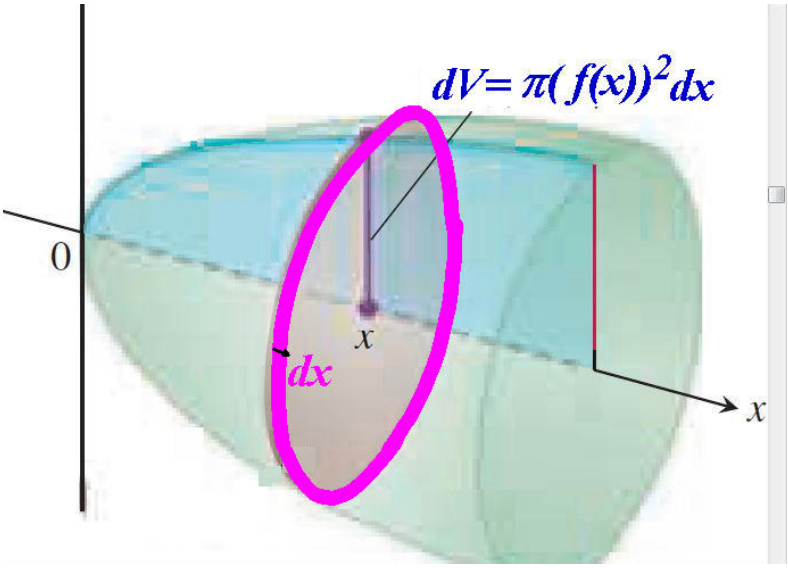
$$\iint_R f(x,y)\,dA = \lim_{n,m\to\infty} \sum_{i=1}^{n} \sum_{j=1}^{m} f(x_i,\ y_j)\,\Delta A$$



The cross-section of a solid of revolution is a circle

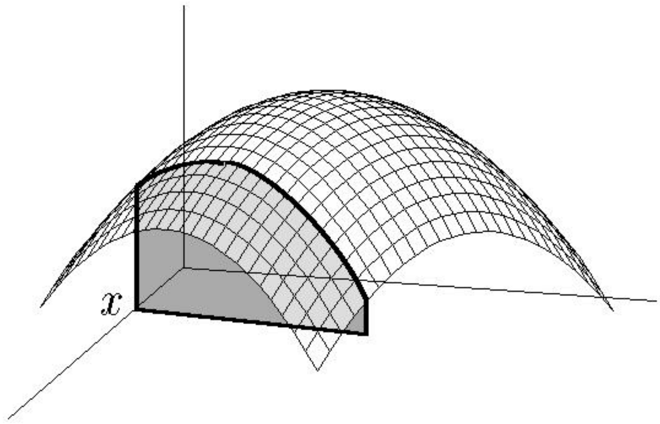


$$V = \int_a^b \pi(f(x))^2 \, dx$$



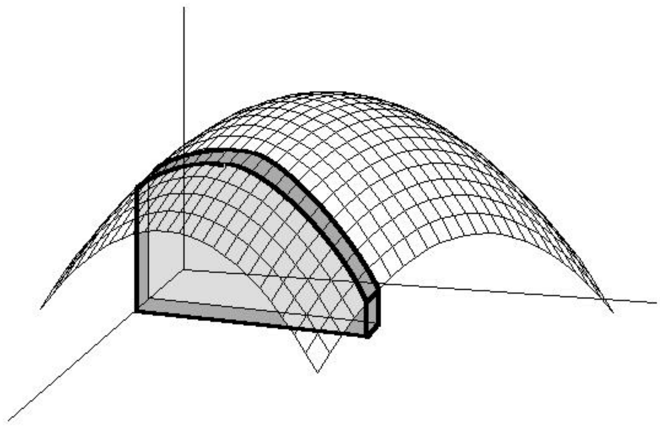
Volume - Method of Cross-sections

$$A(x) = \int_c^d f(x,y)\,dy$$



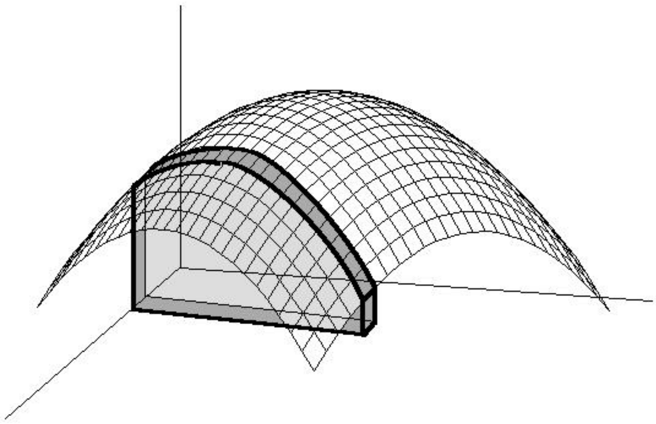
Volume - Method of Cross-sections

$$dV = A(x)\,dx$$



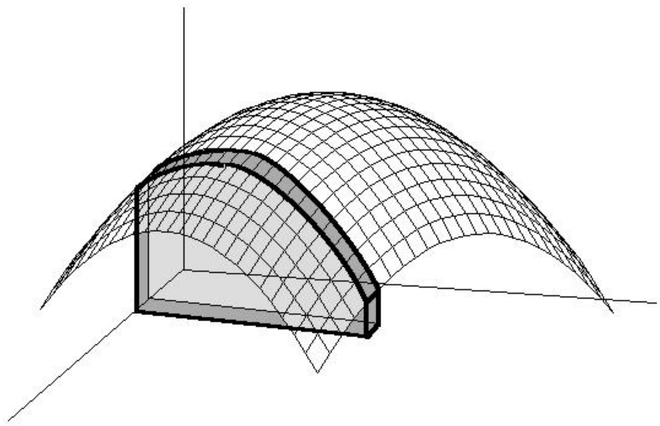
$$dV = A(x)\,dx$$

$$V = \int_a^b A(x)\,dx$$

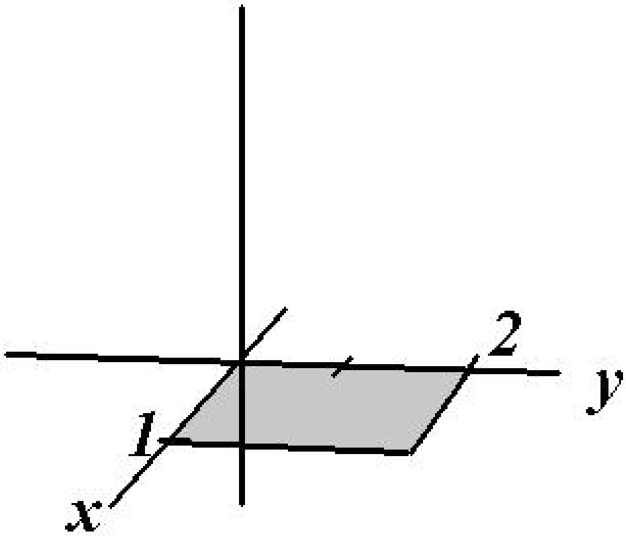


$$dV = A(x)\,dx$$

$$V = \int_a^b A(x)\,dx = \int_a^b \left(\int_c^d f(x,y)\,dy \right) dx$$

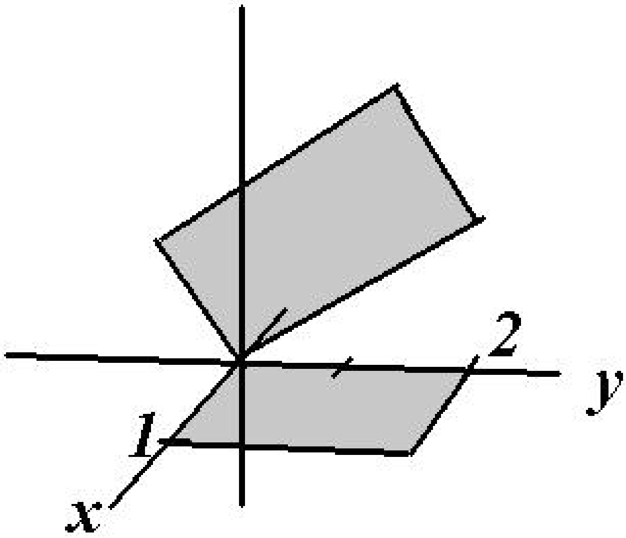


Let R be the rectangular region in the xy plane for $0 \leq x \leq 1$ and $0 \leq y \leq 2$



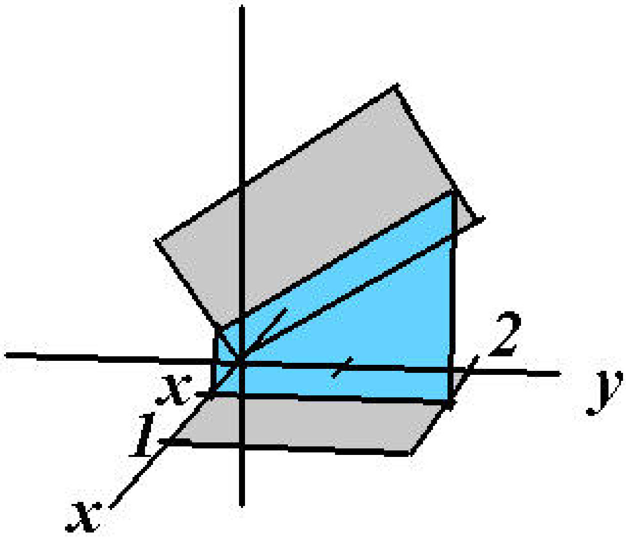
$$z = 2x + y$$

Find the volume under a section of the plane



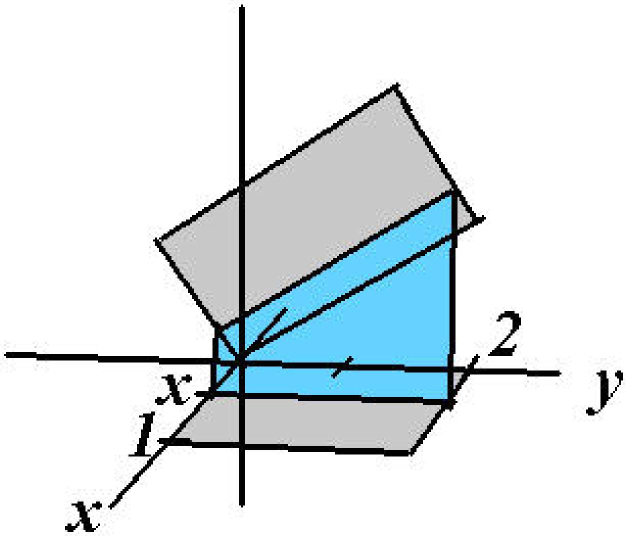
$$z = 2x + y$$

Start with a cross-section at a particular x



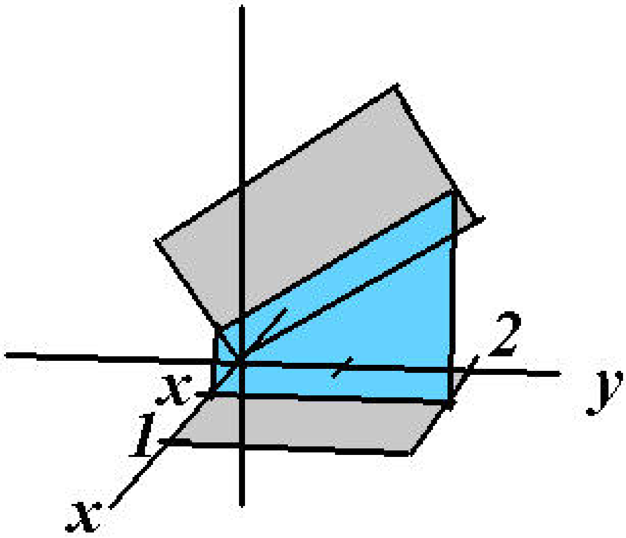
$$z = 2x + y$$

$$A(x) = \int_0^2 (2x + y)\, dy$$



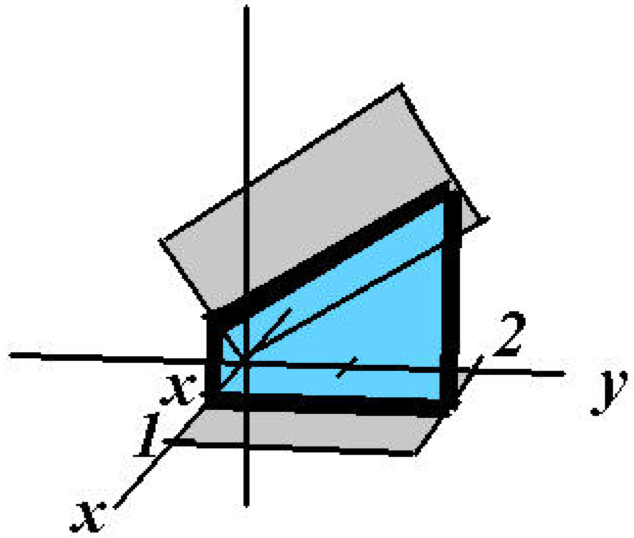
$$z = 2x + y$$

$$A(x) = \int_0^2 (2x + y)\, dy = \left[2xy + \frac{1}{2}y^2\right]_{y=0}^{2} = 4x + 2$$



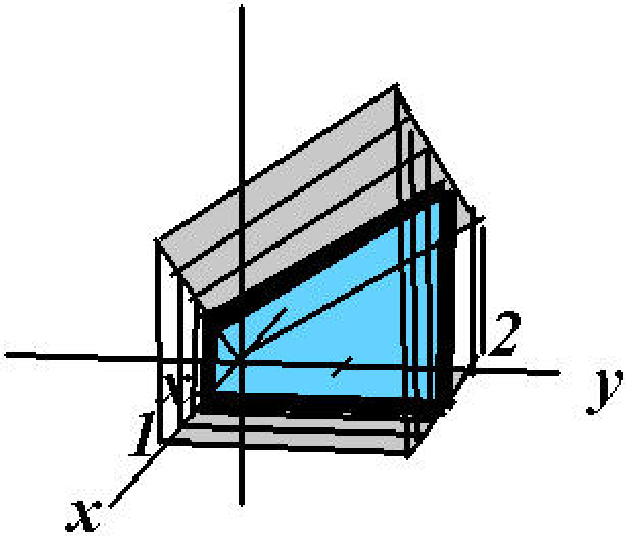
$$z = 2x + y$$

$$dV = A(x)\,dx = (4x + 2)\,dx$$



$$z = 2x + y$$

$$V = \int_0^1 A(x)\,dx = \int_0^1 (4x+2)\,dx = 4$$

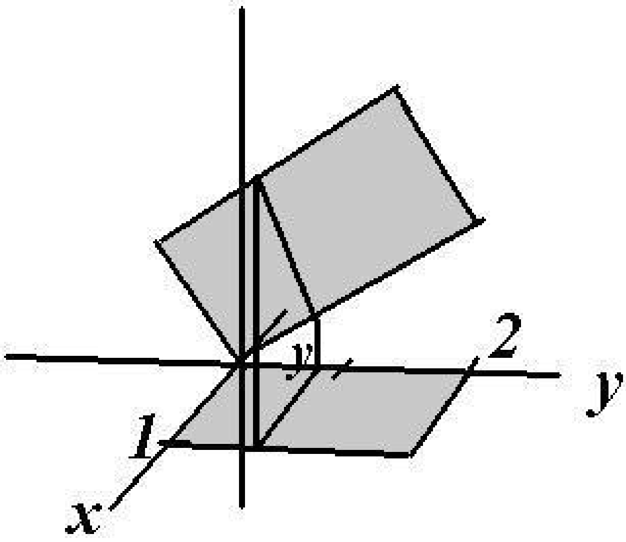


$$V = \int_0^1 A(x)\,dx = \int_0^1 (2x + 4)\,dx$$

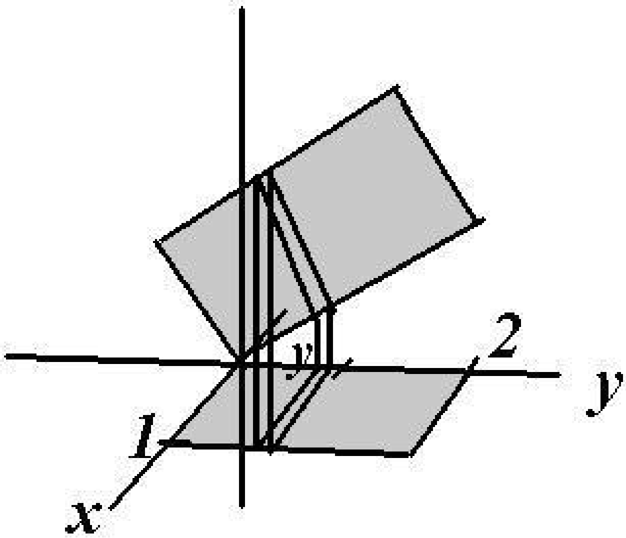
$$V = \int_0^1 A(x)\,dx = \int_0^1 \left(\int_0^2 (2x + y)\,dy \right)\,dx$$

$$V = \int_0^1 A(x)\,dx = \int_0^1 \int_0^2 (2x + y)\,dy\,dx$$

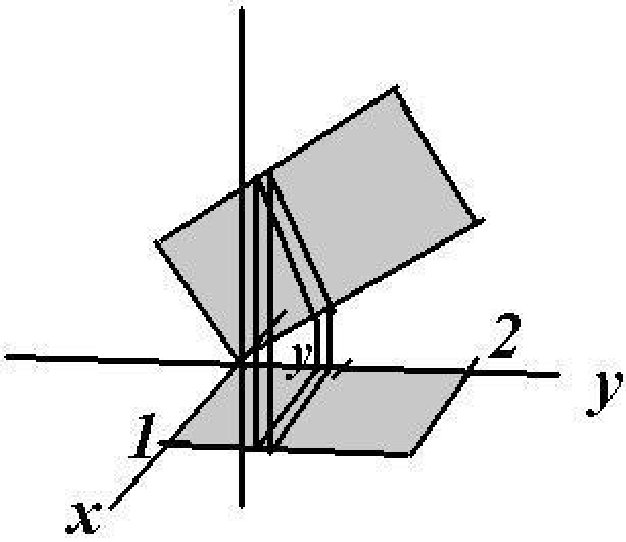
$$A(y) = \int_0^1 (2x + y)\,dx$$



$$A(y)\,dy = \left(\int_0^1 (2x + y)\,dx \right)\,dy$$



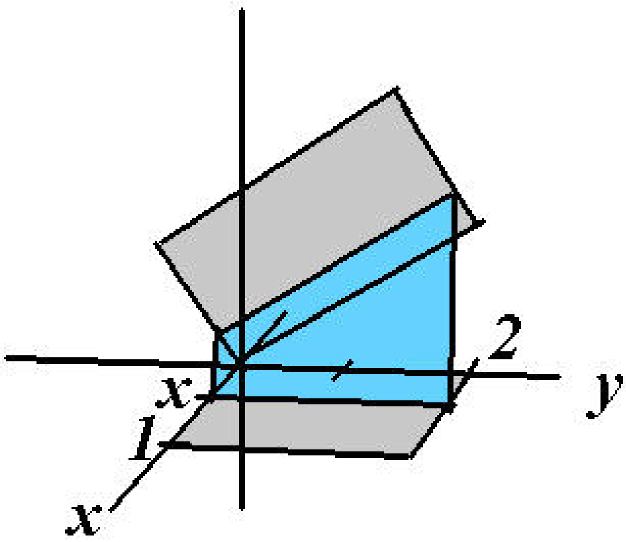
$$V = \int_0^2 \int_0^1 (2x + y)\, dx\, dy$$



$$
\begin{aligned}
V &= \int_0^2 \int_0^1 (2x + y)\, dx\, dy \\
&= \int_0^2 \left[x^2 + xy\right]_{x=0}^{1}\, dy \\
&= \int_0^2 (1 + y)\, dy
\end{aligned}
$$

$$
\begin{aligned}
V &= \int_0^2 \int_0^1 (2x + y)\, dx\, dy \\
&= \int_0^2 \left[x^2 + xy\right]_{x=0}^{1} dy \\
&= \int_0^2 (1 + y)\, dy \\
&= \left[y + \frac{1}{2}y^2\right]_0^2 = 4
\end{aligned}
$$

$$\int_0^1 \int_0^2 (2x+y)\, dy\, dx = \int_0^2 \int_0^1 (2x+y)\, dx\, dy$$



$$\int_0^1 \int_0^2 (2x + y)\, dy\, dx = \int_0^2 \int_0^1 (2x + y)\, dx\, dy$$