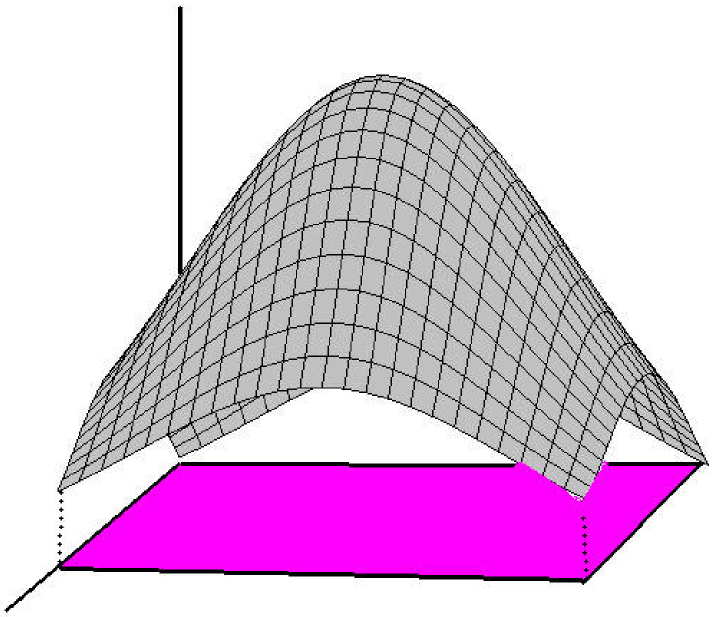
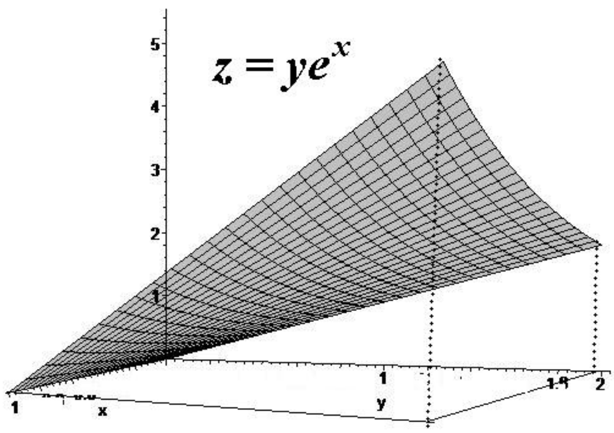
Double Integrals

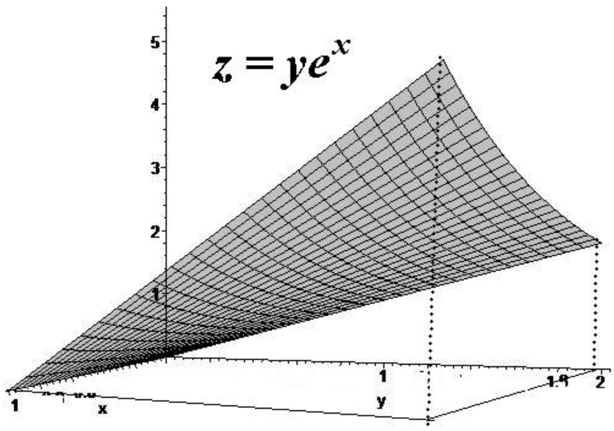
Rectangular and Nonrectangular Regions



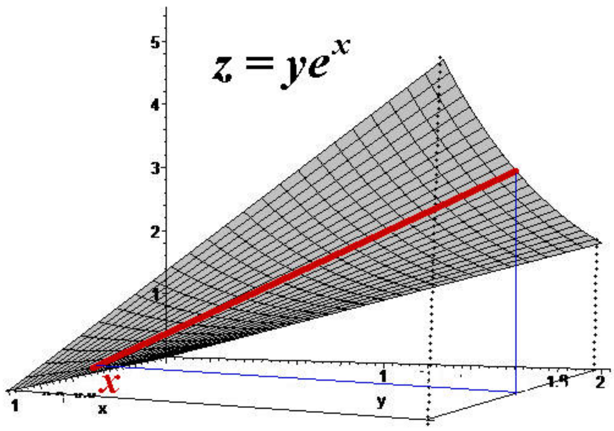
Find the volume under $z = ye^x$ over the rectangle R



$$\iint_R ye^x \, dA = \int_0^1 \int_0^2 ye^x \, dy \, dx$$



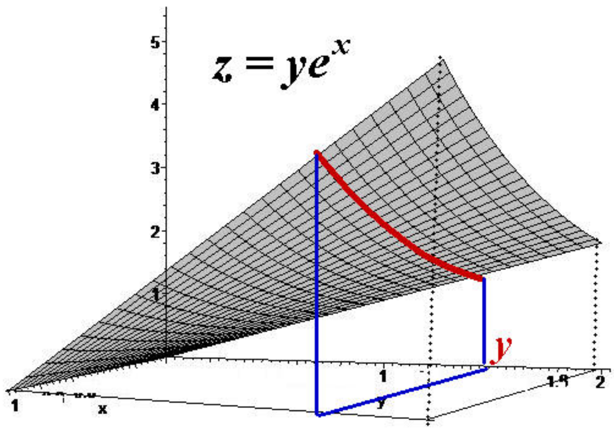
$$\iint_R ye^x \, dA = \int_0^1 \left(\int_0^2 ye^x \, dy \right) \, dx$$



$$\begin{aligned}\iint_R ye^x \, dA &= \int_0^1 \int_0^2 ye^x \, dy \, dx \\ &= \int_0^1 \left[\frac{1}{2}y^2 e^x\right]_{y=0}^{2} dx\end{aligned}$$

$$
\begin{aligned}
\iint_R ye^x \, dA &= \int_0^1 \int_0^2 ye^x \, dy \, dx \\
&= \int_0^1 \left[\frac{1}{2} y^2 e^x \right]_{y=0}^2 dx \\
&= \int_0^1 2e^x \, dx \\
&= 2(e-1)
\end{aligned}
$$

$$\iint_R ye^x \, dA = \int_0^2 \left(\int_0^1 ye^x \, dx \right) \, dy$$



$$\iint_R ye^x \, dA = \int_0^2 \int_0^1 ye^x \, dx \, dy$$
$$= \int_0^2 \Big[ye^x \Big]_{x=0}^{1} dy$$

$$
\begin{aligned}
\iint_R ye^x \, dA &= \int_0^2 \int_0^1 ye^x \, dx \, dy \\
&= \int_0^2 \Big[ye^x \Big]_{x=0}^{1} \, dy \\
&= \int_0^2 y(e-1) \, dy \\
&= 2(e-1)
\end{aligned}
$$

$$\int_a^b c\, f(y)\, dy = c \int_a^b f(y)\, dy$$

$$\int_0^2 \left(\int_0^1 y e^x \, dx \right) \, dy = \int_0^2 y \left(\int_0^1 e^x \, dx \right) \, dy$$

$$\int_0^2 \left(\int_0^1 y e^x \, dx \right) \, dy = \int_0^2 y \left(\int_0^1 e^x \, dx \right) \, dy$$

$$\text{Let } c = \int_0^1 e^x \, dx$$

$$\int_0^2 y \left(\int_0^1 e^x \, dx \right) \, dy = \int_0^2 y \cdot c \, dy = c \int_0^2 y \, dy$$

$$\int_0^2 \left(\int_0^1 y e^x \, dx \right) \, dy = \int_0^2 y \left(\int_0^1 e^x \, dx \right) \, dy$$

$$\text{Let } c = \int_0^1 e^x \, dx$$

$$\int_0^2 y \left(\int_0^1 e^x \, dx \right) \, dy = \int_0^2 y \cdot c \, dy = c \int_0^2 y \, dy$$

This is the same as:

$$\left(\int_0^1 e^x \, dx \right) \left(\int_0^2 y \, dy \right)$$

$$\int_0^1 \int_0^2 y e^x \, dy \, dx = \int_0^1 e^x \, dx \cdot \int_0^2 y \, dy$$

$$\int_0^1 \int_0^2 ye^x \, dy \, dx = \int_0^1 e^x \, dx \cdot \int_0^2 y \, dy$$

More generally:

$$\int_a^b \int_c^d f(x)g(y) \, dy \, dx = \int_a^b f(x) \, dx \cdot \int_c^d g(y) \, dy$$

$$\int_0^1 \int_0^2 ye^x \, dy \, dx = \int_0^1 e^x \, dx \cdot \int_0^2 y \, dy$$

More generally:

$$\int_a^b \int_c^d f(x)g(y) \, dy \, dx = \int_a^b f(x) \, dx \cdot \int_c^d g(y) \, dy$$

Wouldn't apply to:

$$\int_0^1 \int_0^2 \left(3x^2 + 6y^2\right) \, dy \, dx$$

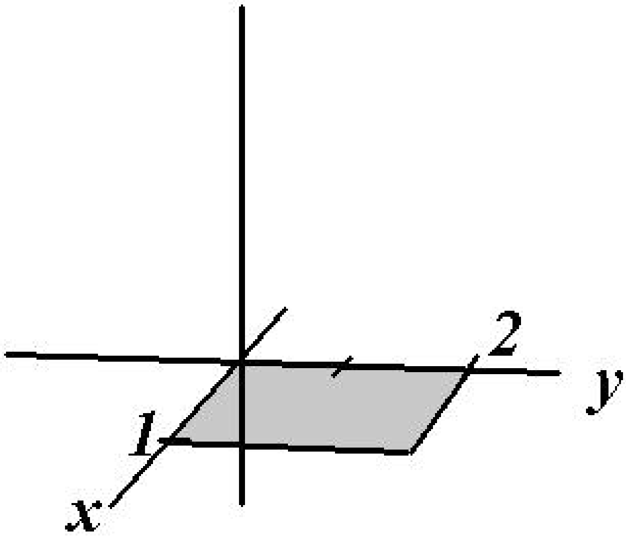
$$\int_a^b (f(x) + g(x))\, dx = \int_a^b f(x)\, dx + \int_a^b g(x)\, dx$$

$$\int_0^1 \int_0^2 \left(3x^2 + 6y^2\right)\, dy\, dx$$

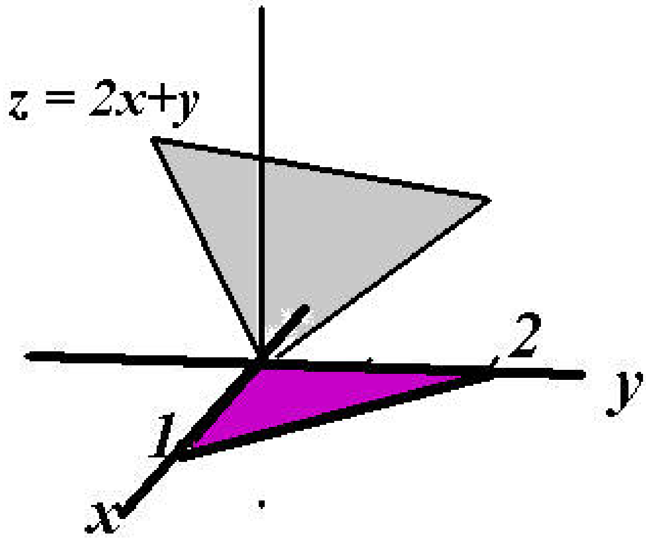
This can be split up into two integrals:

$$\int_0^1 \int_0^2 3x^2\, dy\, dx + \int_0^1 \int_0^2 6y^2\, dy\, dx$$

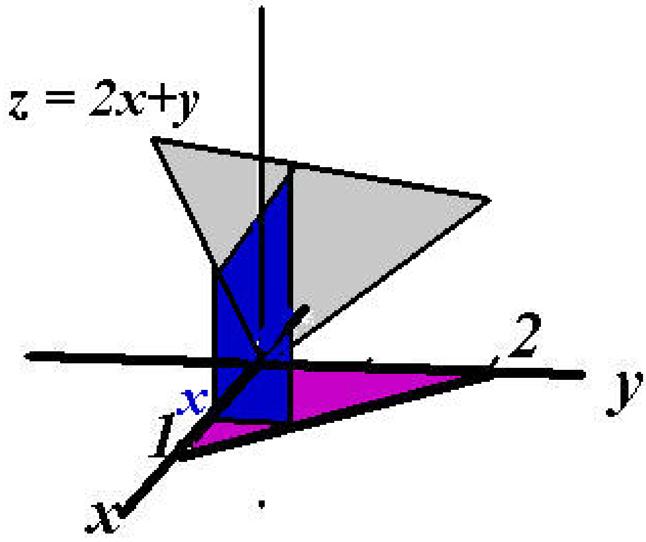
$$\iint_R (2x + y)\ dA = 4$$



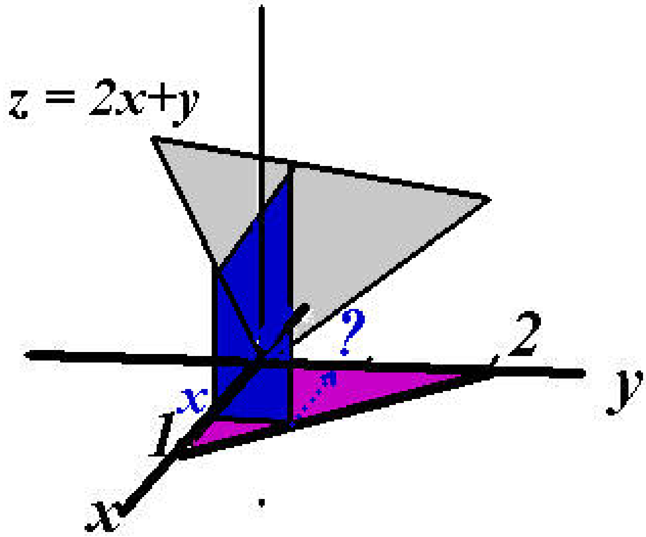
$$\iint_T (2x + y)\; dA = \textbf{?}$$



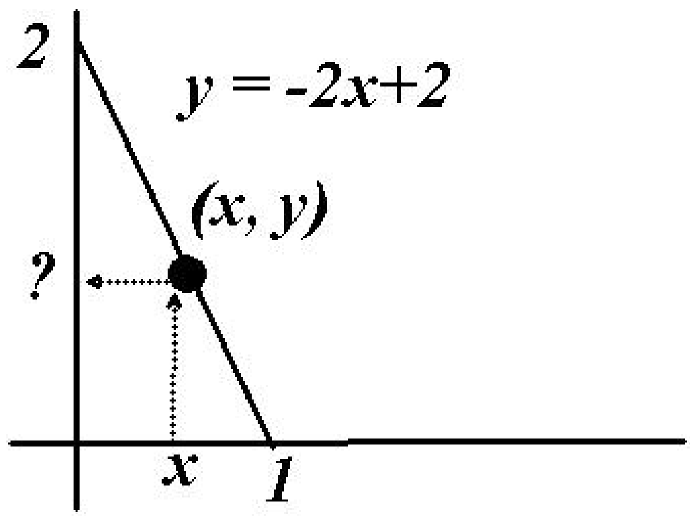
$$\iint_T (2x+y)\ dA = \int\int (2x+y)\,dy\,dx$$



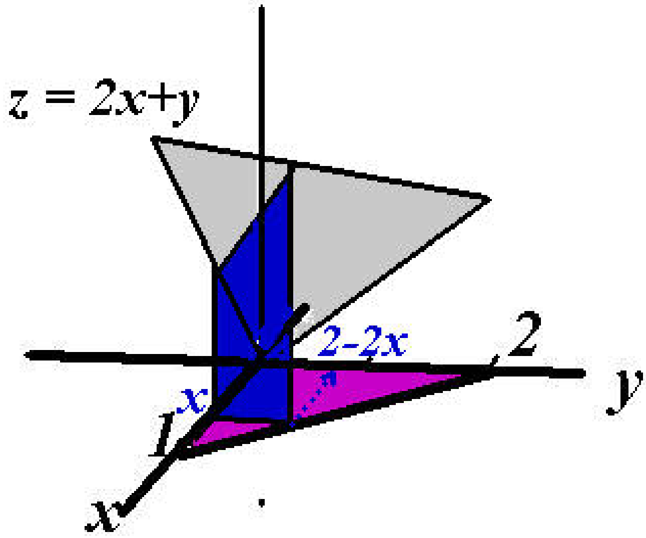
$$\int\!\!\int_T (2x+y)\;dA = \int\int_0^{?} (2x+y)\,dy\,dx$$



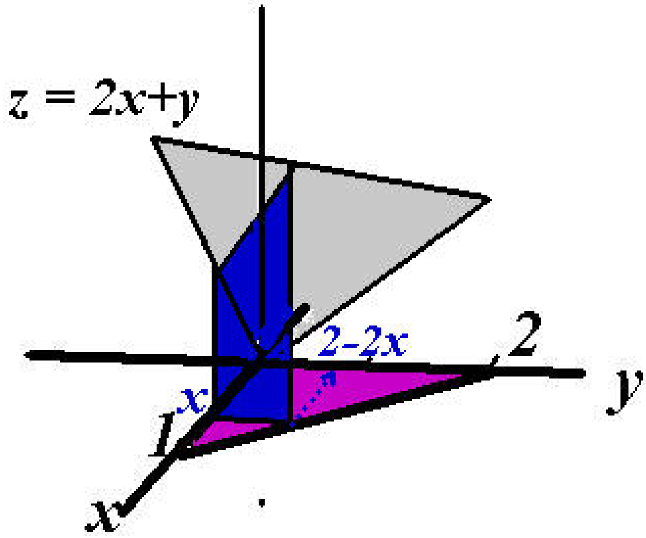
The equation of the line between (1, 0) and (0, 2) is $y = -2x + 2$



$$\iint_T (2x+y)\ dA = \int\int_0^{2-2x} (2x+y)\,dy\,dx$$



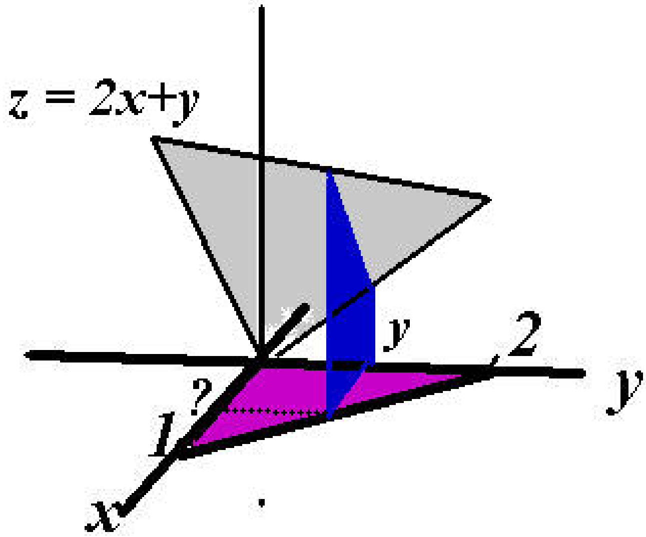
$$\iint_T (2x+y)\ dA = \int_0^1 \int_0^{2-2x} (2x+y)\, dy\, dx$$



$$\begin{aligned}\iint_T (2x+y)\ dA &= \int_0^1 \int_0^{2-2x} (2x+y)\,dy\,dx \\ &= \int_0^1 \left[2xy + \frac{1}{2}y^2\right]_{y=0}^{2-2x}\ dx\end{aligned}$$

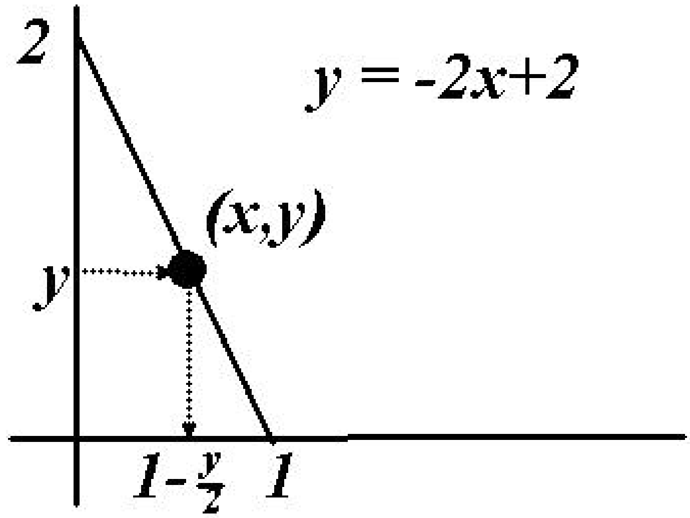
$$
\begin{aligned}
\iint_T (2x+y)\ dA &= \int_0^1 \int_0^{2-2x} (2x+y)\, dy\, dx \\
&= \int_0^1 \left[2xy + \frac{1}{2}y^2 \right]_{y=0}^{2-2x} dx \\
&= \int_0^1 \left(2 - 2x^2\right)\ dx \\
&= \frac{4}{3}
\end{aligned}
$$

$$\iint_T (2x + y)\; dA = \int \int_0^{?} (2x + y)\, dx\, dy$$

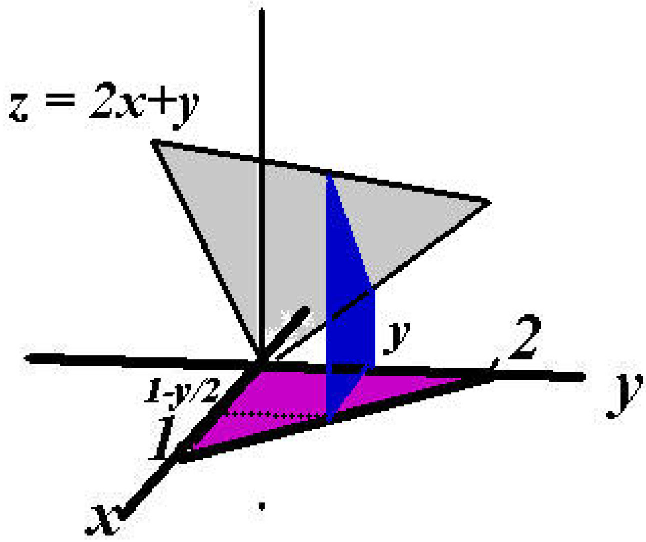


The equation of the line between (1, 0) and (0, 2) is $y = -2x + 2$

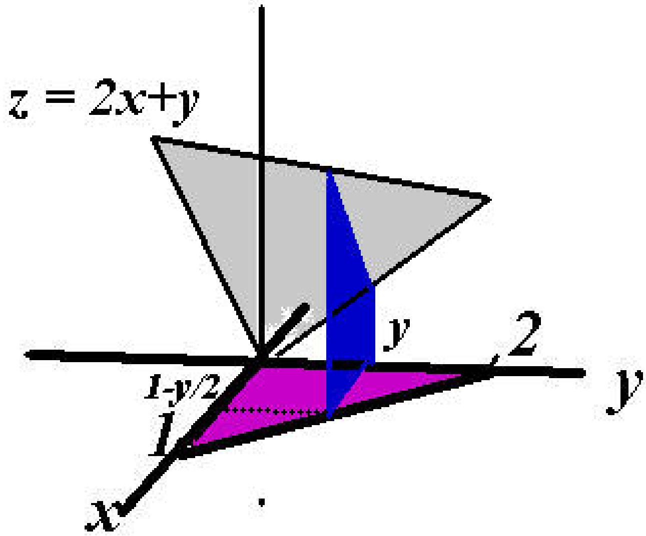
This is equivalent to $x = 1 - \frac{y}{2}$



$$\iint_T (2x+y)\ dA = \int\int_0^{1-\frac{y}{2}} (2x+y)\, dx\, dy$$

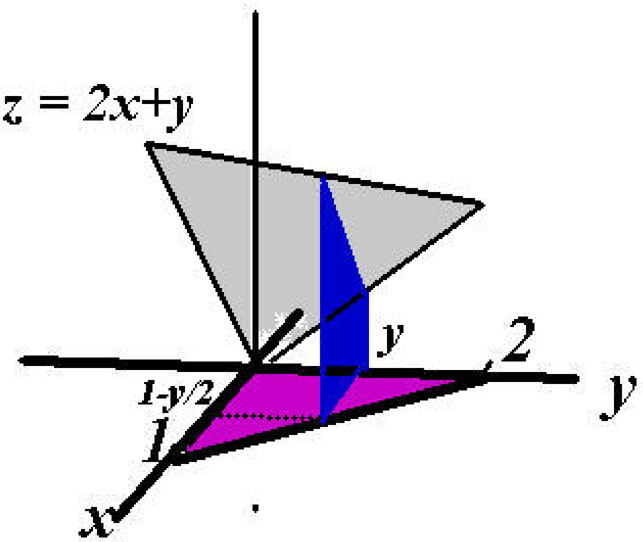


$$\iint_T (2x+y)\ dA = \int_0^2 \int_0^{1-\frac{y}{2}} (2x+y)\, dx\, dy$$



$$\iint_T (2x+y)\ dA = \int_0^2 \int_0^{1-\frac{y}{2}} (2x+y)\, dx\, dy$$

$$= \int_0^2 \left[x^2 + yx\right]_{x=0}^{1-\frac{y}{2}} dy$$



$$\begin{aligned}
\iint_T (2x+y)\ dA &= \int_0^2 \int_0^{1-\frac{y}{2}} (2x+y)\,dx\,dy \\
&= \int_0^2 \left[x^2 + yx\right]_{x=0}^{1-\frac{y}{2}} dy \\
&= \int_0^2 \left(1 - \frac{1}{4}y^2\right)\ dy \\
&= \frac{4}{3}
\end{aligned}$$

$$\int_0^1 \int_0^2 (2x + y)\, dy\, dx = \int_0^2 \int_0^1 (2x + y)\, dx\, dy$$

$$\int_0^1 \int_0^2 (2x + y)\, dy\, dx = \int_0^2 \int_0^1 (2x + y)\, dx\, dy$$

$$\int_0^1 \int_0^{2-2x} (2x + y)\, dy\, dx = \int_0^2 \int_0^{1-\frac{y}{2}} (2x + y)\, dx\, dy$$