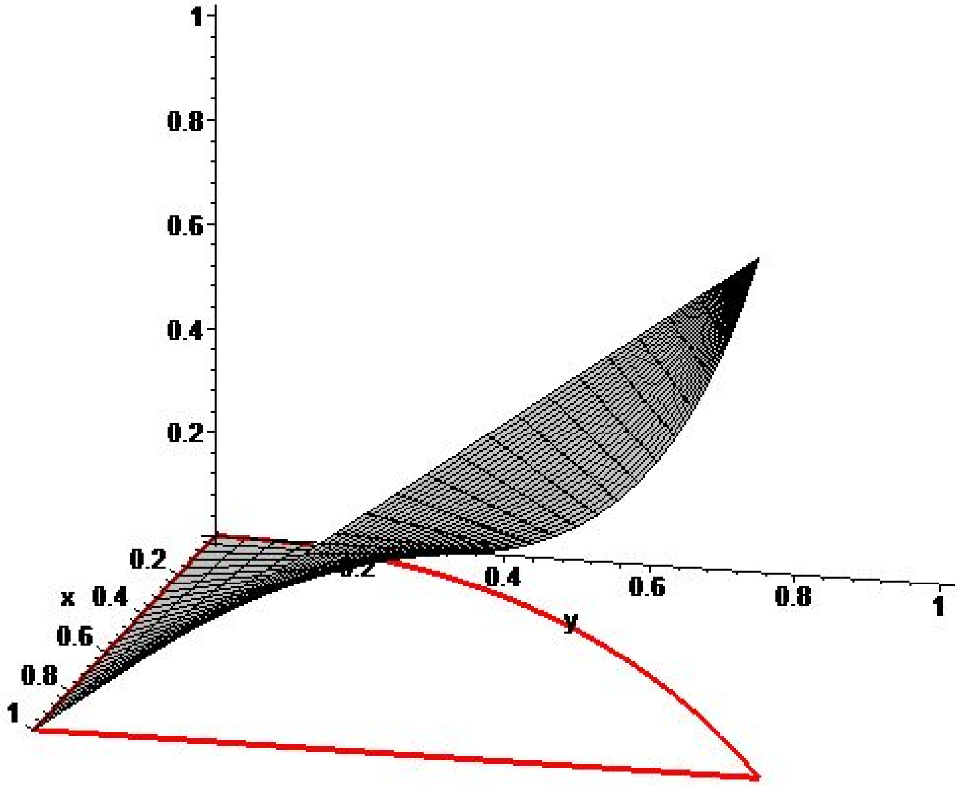
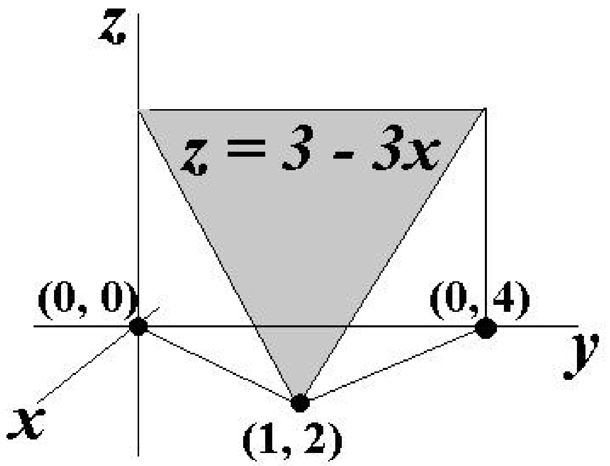
Double Integrals Over Nonrectangular Regions



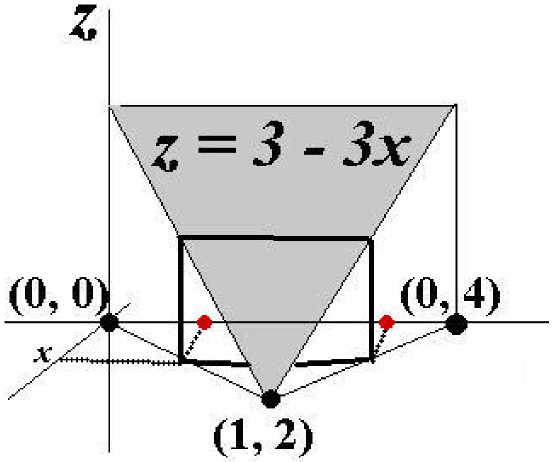
Let T be the triangle in the xy plane with vertices $(0,\ 0)$, $(1,\ 2)$ and $(0,\ 4)$.

$$\int\!\!\int_T (3-3x)\,dA = \int \quad \int \quad (3-3x)\,dy\,dx$$

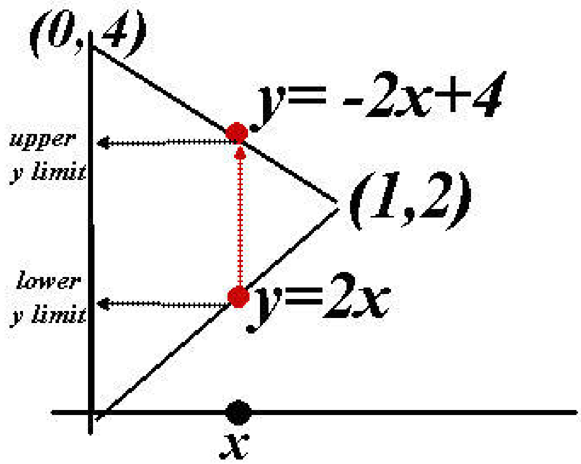


Let T be the triangle in the xy plane with vertices $(0,\ 0)$, $(1,\ 2)$ and $(0,\ 4)$.

$$\iint_T (3-3x)\,dA = \int \quad \int_?^? \quad (3-3x)\,dy\,dx$$

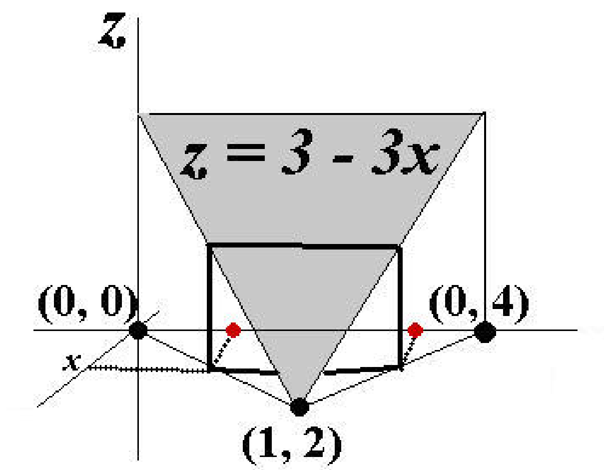


If x is held fixed, y varies from $2x$ to $-2x + 4$



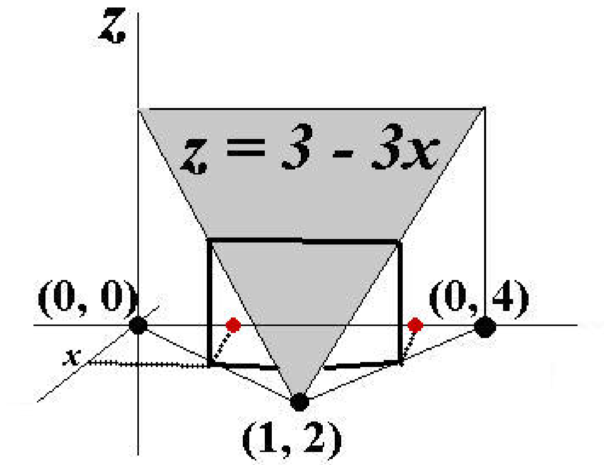
Let T be the triangle in the xy plane with vertices $(0,\ 0)$, $(1,\ 2)$ and $(0,\ 4)$.

$$\iint_T (3-3x)\,dA = \int\int_{2x}^{4-2x} (3-3x)\,dy\,dx$$



Let T be the triangle in the xy plane with vertices $(0,\ 0)$, $(1,\ 2)$ and $(0,\ 4)$.

$$\iint_T (3-3x)\,dA = \int_0^1 \int_{2x}^{4-2x} (3-3x)\,dy\,dx$$

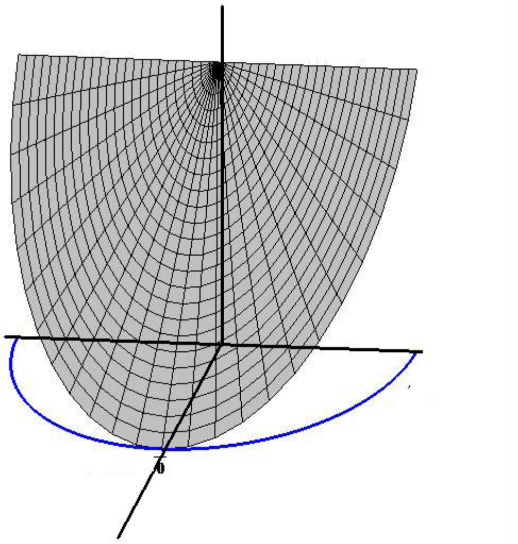


$$\iint_T (3-3x)\,dA = \int_0^1 \int_{2x}^{4-2x} (3-3x)\,dy\,dx$$
$$= \int_0^1 \Big[(3-3x)y\Big]_{y=2x}^{4-2x}\,dx$$

$$
\begin{aligned}
\iint_T (3-3x)\,dA &= \int_0^1 \int_{2x}^{4-2x} (3-3x)\,dy\,dx \\
&= \int_0^1 \Big[(3-3x)y\Big]_{y=2x}^{4-2x}\,dx \\
&= \int_0^1 (3-3x)(4-4x)\,dx \\
&= \int_0^1 12(1-x)^2\,dx \\
&= 4
\end{aligned}
$$

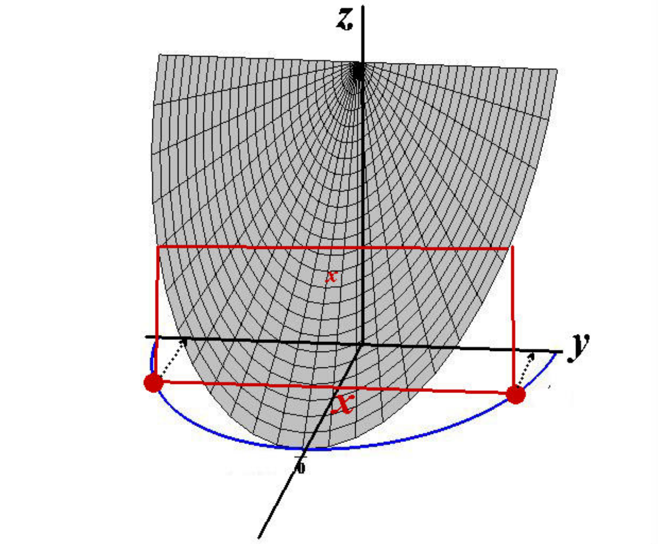
Let R be the semicircular region in the xy plane bounded by the y-axis and $x = \sqrt{1 - y^2}$

$$\int\int_R (3 - 3x)\, dA = \int \quad \int \quad (3 - 3x)\, dy\, dx$$



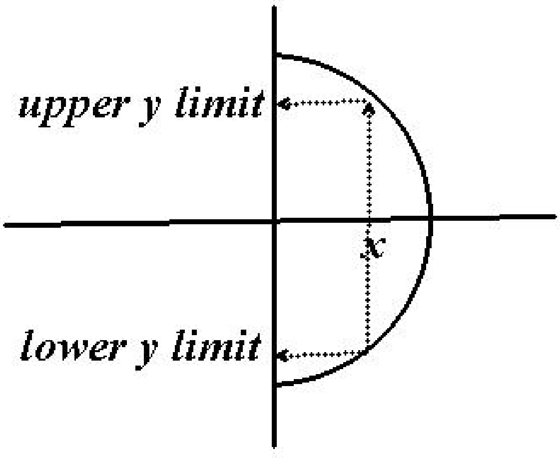
Let R be the semicircular region in the xy plane bounded by the y-axis and $x = \sqrt{1-y^2}$

$$\iint_R (3-3x)\,dA = \int \quad \int \quad (3-3x)\,dy\,dx$$

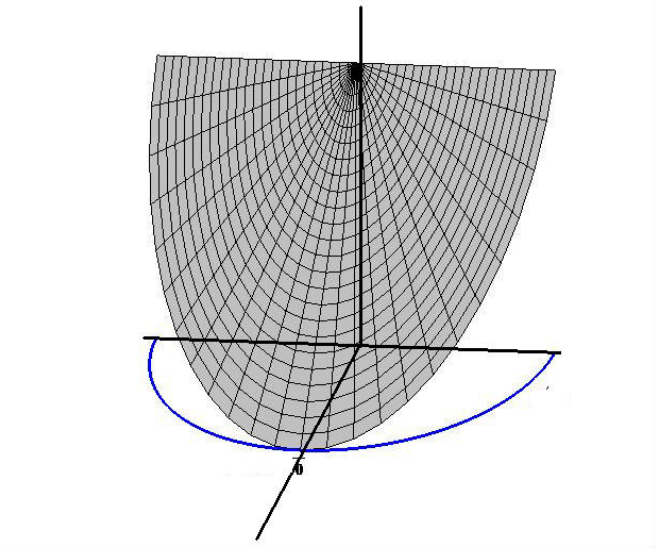


$$x^2 + y^2 = 1 \quad \text{so} \quad y = \pm\sqrt{1 - x^2}$$

$$\int\!\!\int_R (3 - 3x)\, dA = \int \quad \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (3 - 3x)\, dy\, dx$$

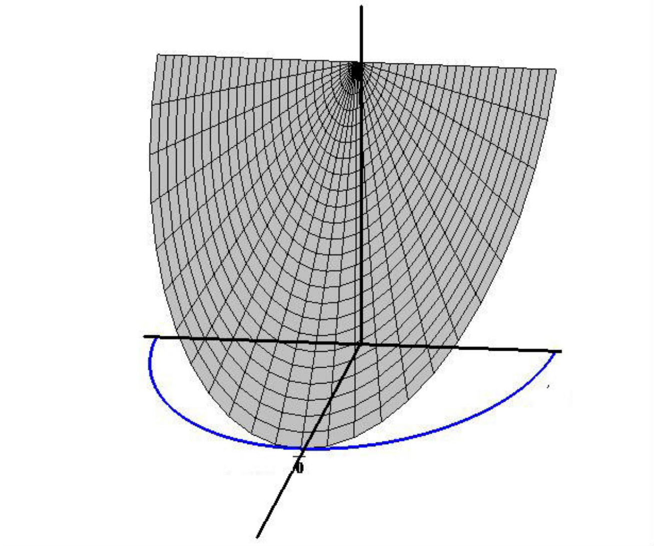


$$\iint_R (3-3x)\,dA = \int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (3-3x)\,dy\,dx$$



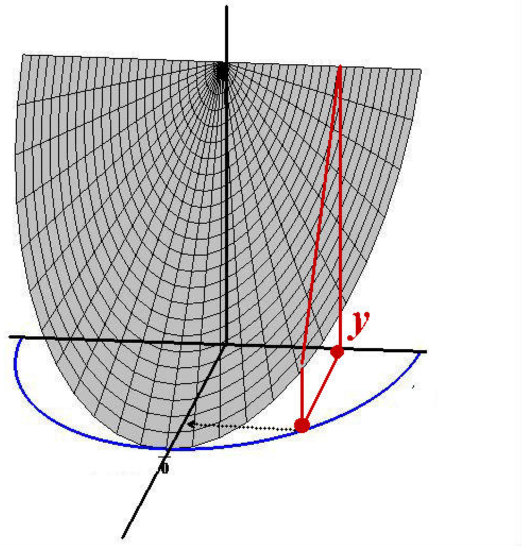
$$\iint_R (3-3x)\,dA = \int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (3-3x)\,dy\,dx$$

$$= \frac{3\pi - 4}{2}$$



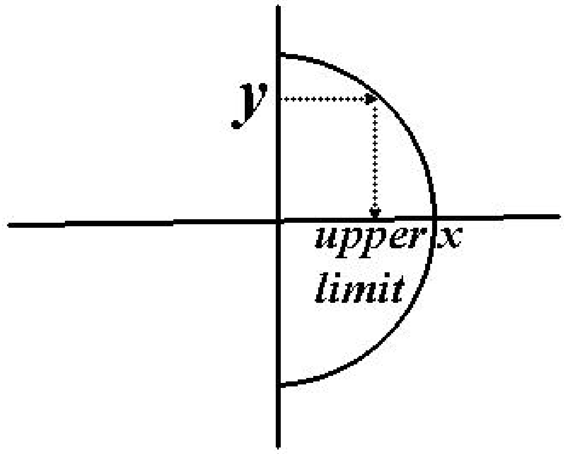
Let R be the semicircular region in the xy plane bounded by the y-axis and $x = \sqrt{1 - y^2}$

$$\iint_R (3 - 3x)\, dA = \int \quad \int \quad (3 - 3x)\, dx\, dy$$

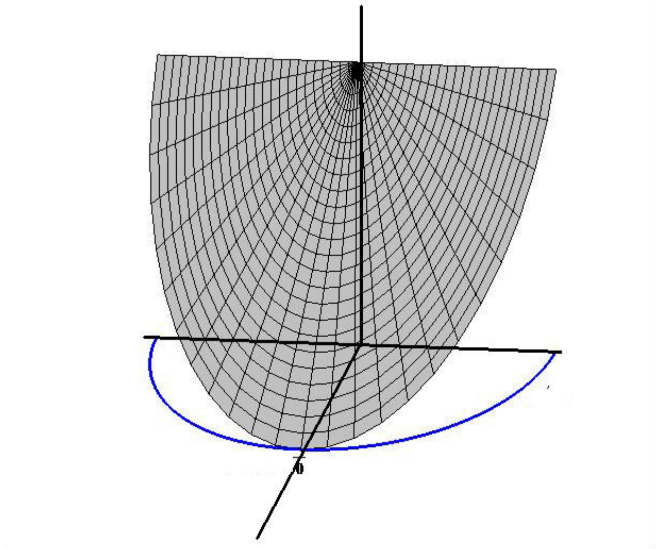


$$x^2 + y^2 = 1 \quad \text{so} \quad x = \sqrt{1 - y^2}$$

$$\iint_R (3 - 3x)\, dA = \int \quad \int_0^{\sqrt{1-y^2}} (3 - 3x)\, dx\, dy$$

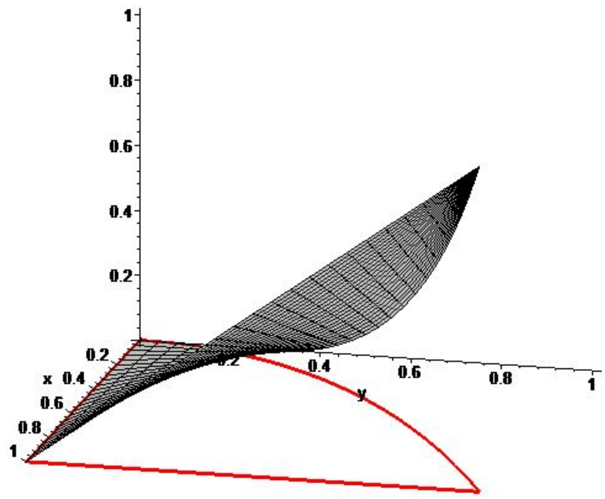


$$\iint_R (3-3x)\,dA = \int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}} (3-3x)\,dx\,dy = \frac{3\pi-4}{2}$$



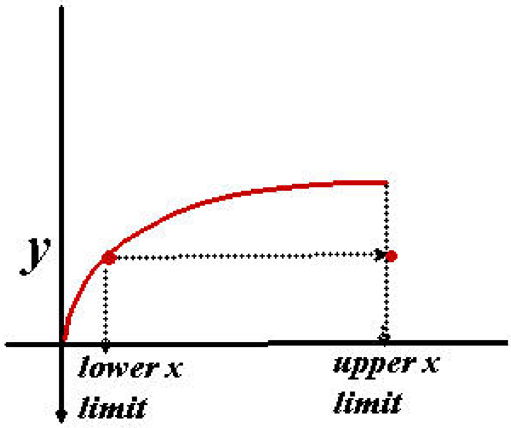
Let Ω be the region in the xy plane bounded by the curve $x = y^2$ and the line $y = 0$ and $x = 1$

$$\iint_\Omega xy \, dA = \int \quad \int \quad xy \, dx \, dy$$



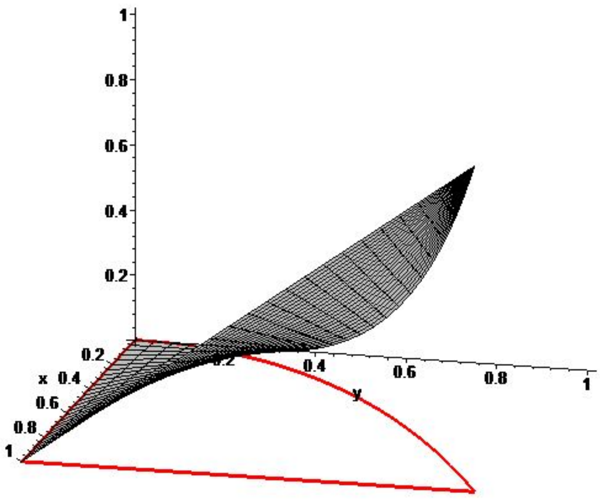
Let Ω be the region in the xy plane bounded by the curve $x = y^2$ and the line $y = 0$ and $x = 1$

$$\int\int_{\Omega} xy\,dA = \int \quad \int_{y^2}^{1} xy\,dx\,dy$$



Let Ω be the region in the xy plane bounded by the curve $x = y^2$ and the line $y = 0$ and $x = 1$

$$\iint_\Omega xy \, dA = \int_0^1 \int_{y^2}^1 xy \, dx \, dy$$

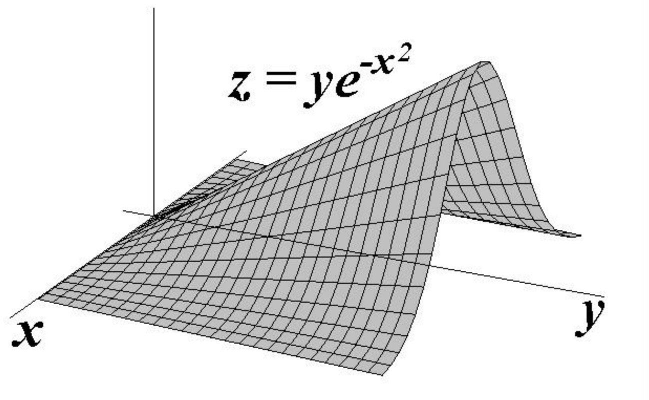


Let Ω be the region in the xy plane bounded by the curve $x = y^2$ and the line $y = 0$ and $x = 1$

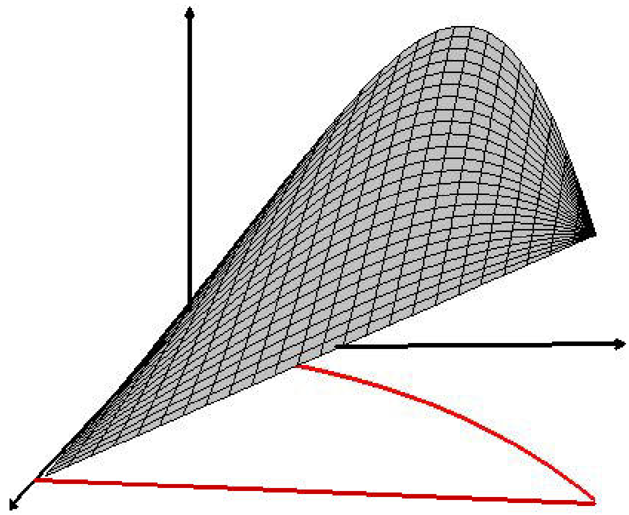
$$
\begin{aligned}
\iint_{\Omega} xy \, dA &= \int_0^1 \int_{y^2}^1 xy \, dx \, dy \\
&= \int_0^1 \left[\frac{1}{2}x^2 y\right]_{x=y^2}^1 dy \\
&= \int_0^1 \left(\frac{1}{2}y - \frac{1}{2}y^5\right) dy \\
&= \frac{1}{6}
\end{aligned}
$$

Let Ω be the region in the xy plane bounded by the curve $x = y^2$ and the line $y = 0$ and $x = 1$ Calculute the volume:

$$\iint_{\Omega} ye^{-x^2}\, dA$$



$$\iint_{\Omega} ye^{-x^2}\, dA = \int_0^1 \int_{y^2}^1 ye^{-x^2}\, dx\, dy$$



How do we integrate $\int e^{-x^2}\, dx$?

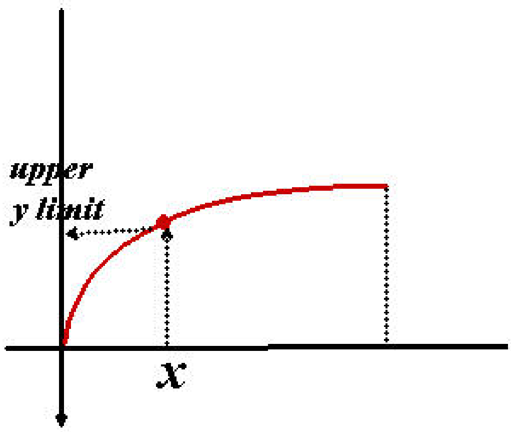
Let $u = x^2$ so $x = u^{1/2}$ and $dx = \frac{1}{2}u^{-1/2}\, du$

$$\int e^{-x^2}\, dx = \int e^{-u} \cdot \frac{1}{2\sqrt{u}}\, du$$

$$\iint_{\Omega} ye^{-x^2}\, dA = \int_0^1 \int_{y^2}^1 ye^{-x^2}\, dx\, dy$$
$$= \int \quad \int_?^? ye^{-x^2}\, dy\, dx$$

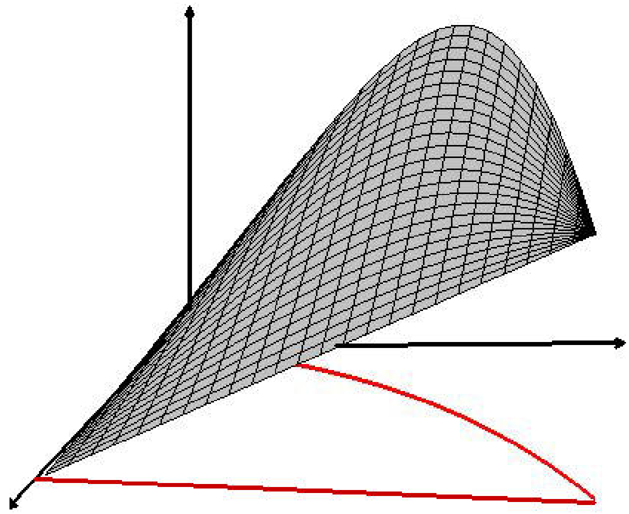
$$\iint_\Omega y e^{-x^2}\, dA = \int_0^1 \int_{y^2}^1 y e^{-x^2}\, dx\, dy$$

$$= \int \quad \int_0^{\sqrt{x}} y e^{-x^2}\, dy\, dx$$



$$\iint_{\Omega} ye^{-x^2}\,dA = \int_0^1 \int_{y^2}^1 ye^{-x^2}\,dx\,dy$$

$$= \int_0^1 \int_0^{\sqrt{x}} ye^{-x^2}\,dy\,dx$$



$$
\begin{aligned}
\iint_\Omega y e^{-x^2}\, dA &= \int_0^1 \int_{y^2}^1 y e^{-x^2}\, dx\, dy \\
&= \int_0^1 \int_0^{\sqrt{x}} y e^{-x^2}\, dy\, dx \\
&= \int_0^1 \left[\frac{1}{2} y^2 e^{-x^2}\right]_{y=0}^{\sqrt{x}} dx \\
&= \int_0^1 \frac{1}{2} x e^{-x^2}\, dx
\end{aligned}
$$

$$
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&= \int_0^1 \int_0^{\sqrt{x}} y e^{-x^2}\, dy\, dx \\
&= \int_0^1 \left[\frac{1}{2} y^2 e^{-x^2}\right]_{y=0}^{\sqrt{x}} dx \\
&= \int_0^1 \frac{1}{2} x e^{-x^2}\, dx \\
&= \frac{1}{4}\left(1 - \frac{1}{e}\right)
\end{aligned}
$$

$$\iint_\Omega ye^{-x^2}\,dA = \int_0^1 \int_{y^2}^1 ye^{-x^2}\,dx\,dy \qquad \longleftarrow \quad (\textbf{hard})$$

$$= \int_0^1 \int_0^{\sqrt{x}} ye^{-x^2}\,dy\,dx \qquad \longleftarrow \quad (\textbf{easy})$$