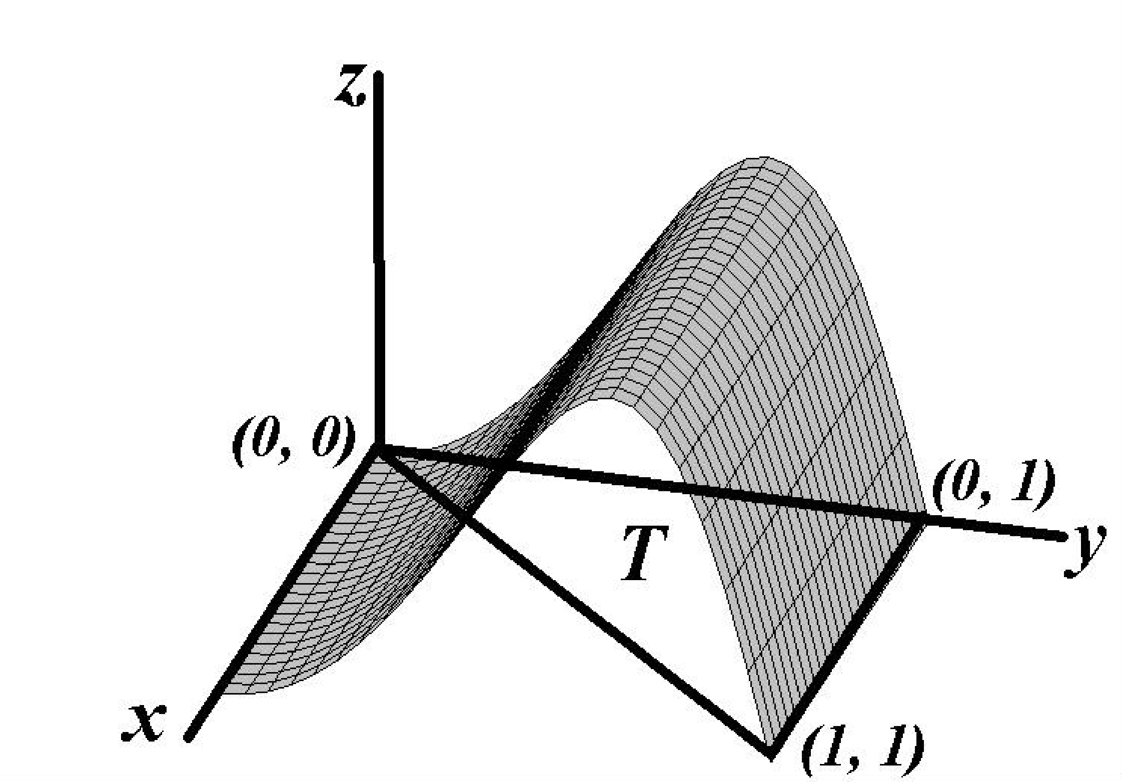
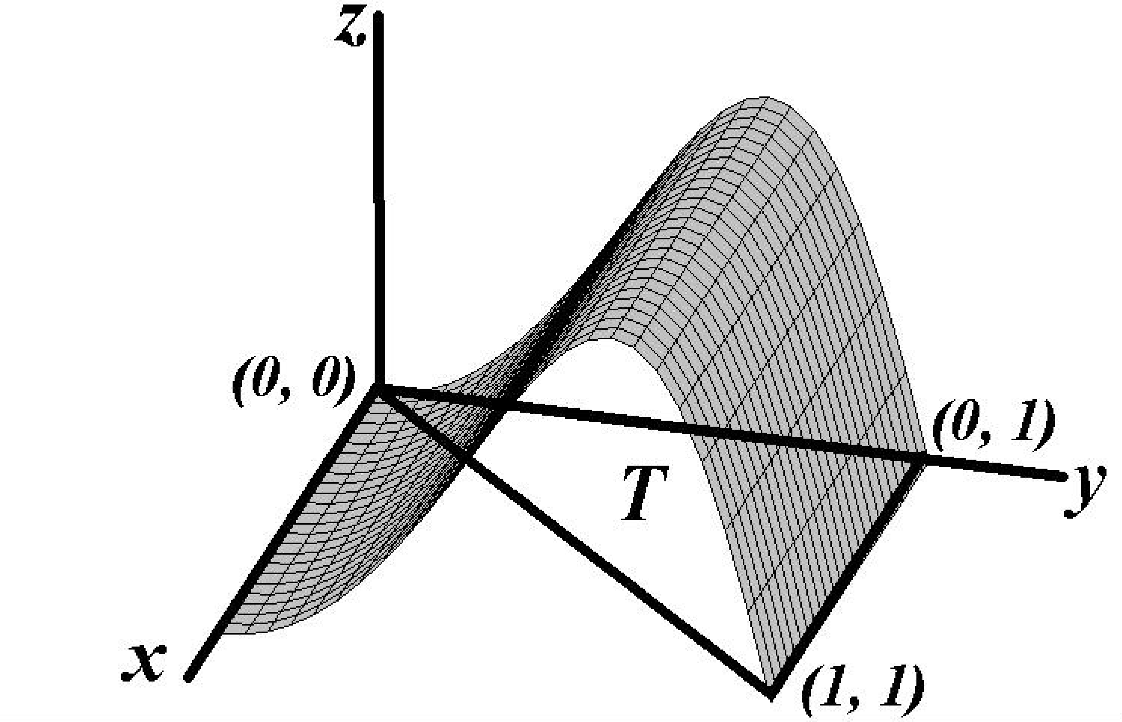
Reversing the Order of Integration



$$\iint_T \sin\left(\pi y^2\right)\, dA = \int \int \sin\left(\pi y^2\right)\, dy\, dx$$



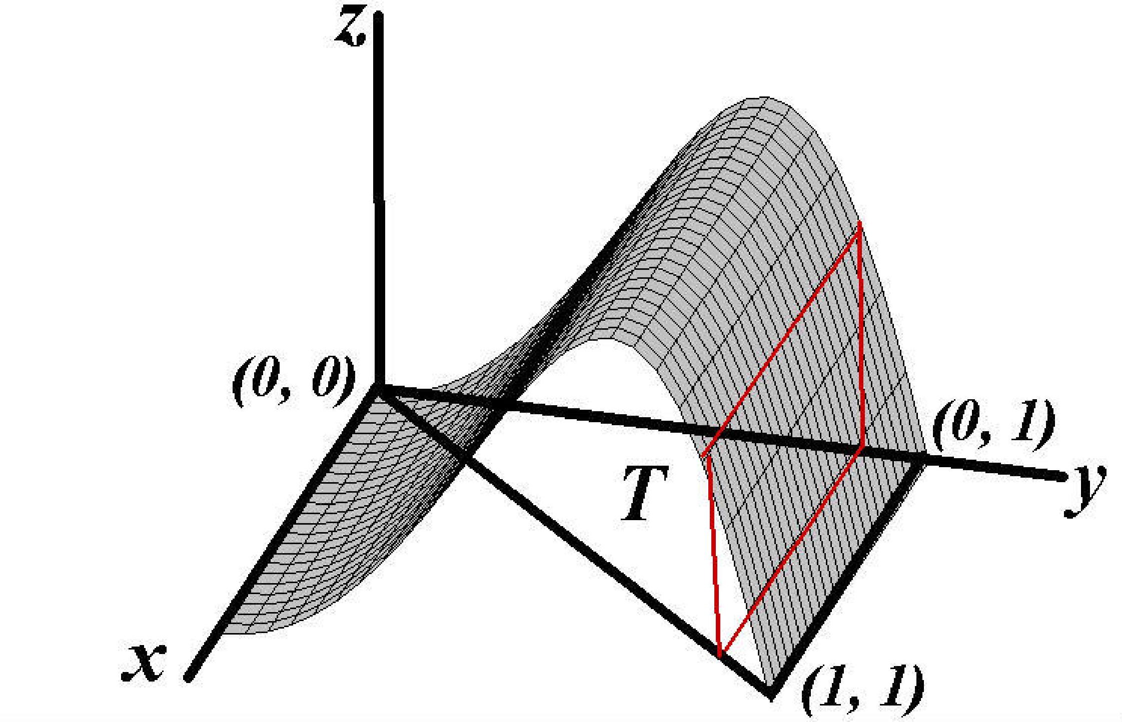
Try variable substitution with $u = \pi y^2$

$$\int \sin\left(\pi y^2\right)\, dy = \int \sin u \cdot \frac{1}{2\sqrt{\pi u}}\, du$$

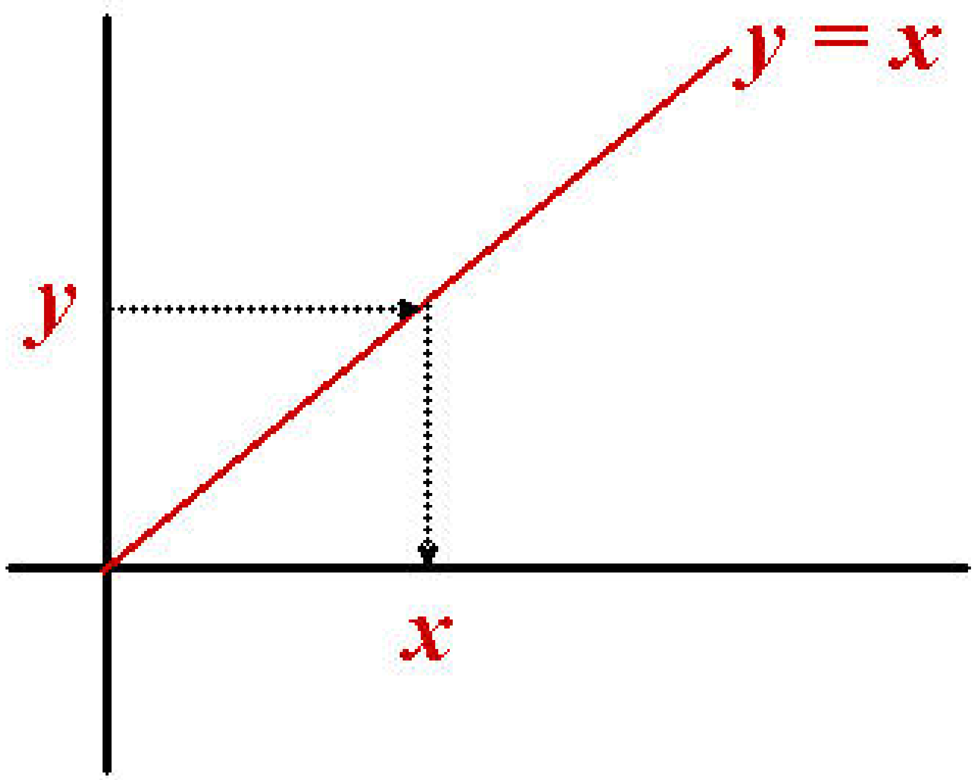
Try integration by parts, $\int u\, dv = uv - \int v\, du$

$$\int \sin\left(\pi y^2\right)\, dy = y \sin\left(\pi y^2\right) - \int 2\pi y^2 \cos\left(\pi y^2\right)\, dy$$

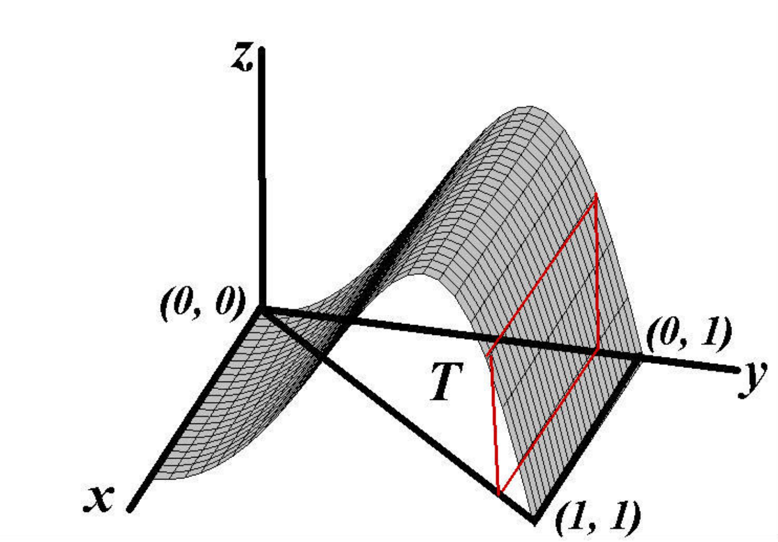
$$\iint_T \sin\left(\pi y^2\right)\, dA = \int \int \sin\left(\pi y^2\right)\, dx\, dy$$



$$\iint_T \sin\left(\pi y^2\right)\, dA = \int \int_0^y \sin\left(\pi y^2\right)\, dx\, dy$$



$$\iint_T \sin\left(\pi y^2\right)\, dA = \int_0^1 \int_0^y \sin\left(\pi y^2\right)\, dx\, dy$$



$$
\begin{aligned}
\iint_T \sin\left(\pi y^2\right)\, dA &= \int_0^1 \int_0^y \sin\left(\pi y^2\right)\, dx\, dy \\
&= \int_0^1 \left[x \sin\left(\pi y^2\right) \right]_{x=0}^{y} dy \\
&= \int_0^1 y \sin\left(\pi y^2\right)\, dy
\end{aligned}
$$

$$
\begin{aligned}
\iint_T \sin\left(\pi y^2\right)\, dA &= \int_0^1 \int_0^y \sin\left(\pi y^2\right)\, dx\, dy \\
&= \int_0^1 \Big[x \sin\left(\pi y^2\right)\Big]_{x=0}^{y} dy \\
&= \int_0^1 y \sin\left(\pi y^2\right)\, dy \\
&= \frac{1}{\pi}
\end{aligned}
$$

$$\int_0^3 \int_0^{\frac{2}{3}\sqrt{9-x^2}} f(x,y)\,dy\,dx = \int_?^? \int_?^? f(x,y)\,dx\,dy$$

$$y = \frac{2}{3}\sqrt{9 - x^2}$$

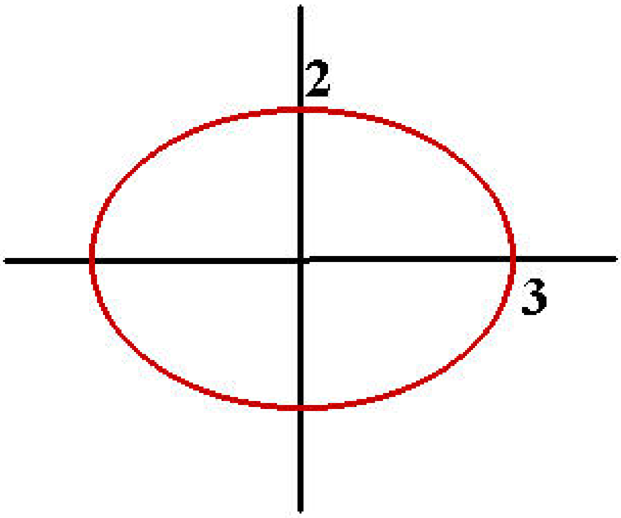
$$y^2 = \frac{4}{9}\left(9 - x^2\right) = 4 - \frac{4}{9}x^2$$

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

This is a special case of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

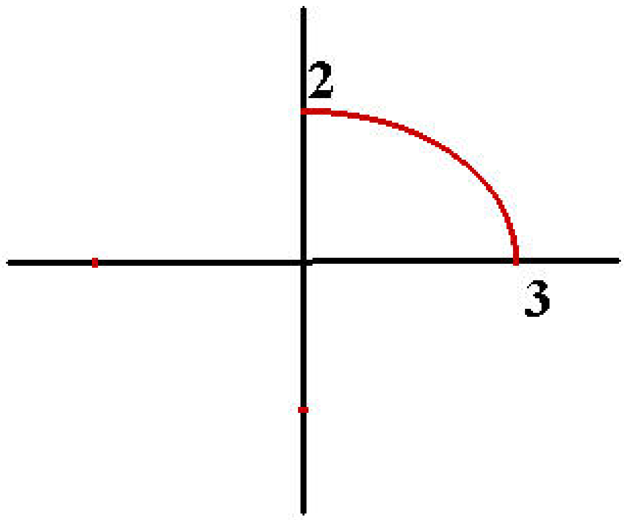
$$\int_0^3 \int_0^{\frac{2}{3}\sqrt{9-x^2}} f(x,y)\,dy\,dx = \int_?^? \int_?^? f(x,y)\,dx\,dy$$

Region of integration: $\frac{x^2}{9} + \frac{y^2}{4} = 1$



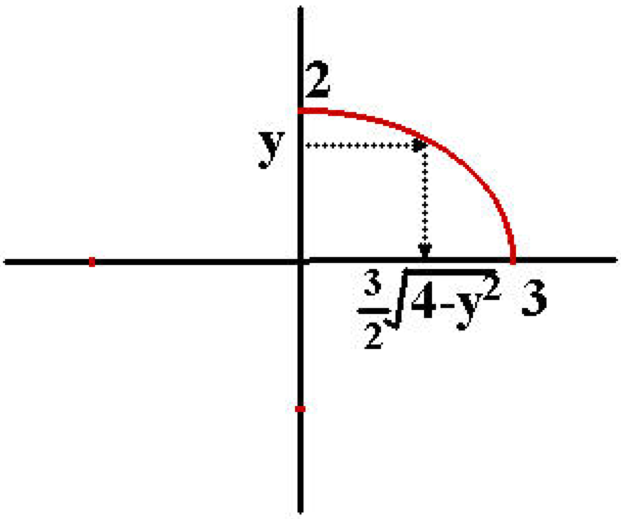
$$\int_0^3 \int_0^{\frac{2}{3}\sqrt{9-x^2}} f(x,y)\,dy\,dx = \int_?^? \int_?^? f(x,y)\,dx\,dy$$

Region of integration: $\frac{x^2}{9} + \frac{y^2}{4} = 1$
with $x \geq 0$ and $y \geq 0$



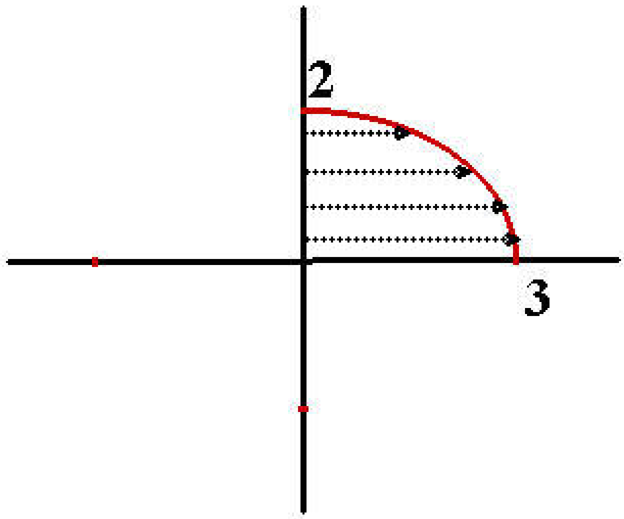
$$\int_0^3 \int_0^{\frac{2}{3}\sqrt{9-x^2}} f(x,y)\,dy\,dx = \int_?^? \int_0^{\frac{3}{2}\sqrt{4-y^2}} f(x,y)\,dx\,dy$$

Region of integration: $\frac{x^2}{9} + \frac{y^2}{4} = 1$
with $x \geq 0$ and $y \geq 0$



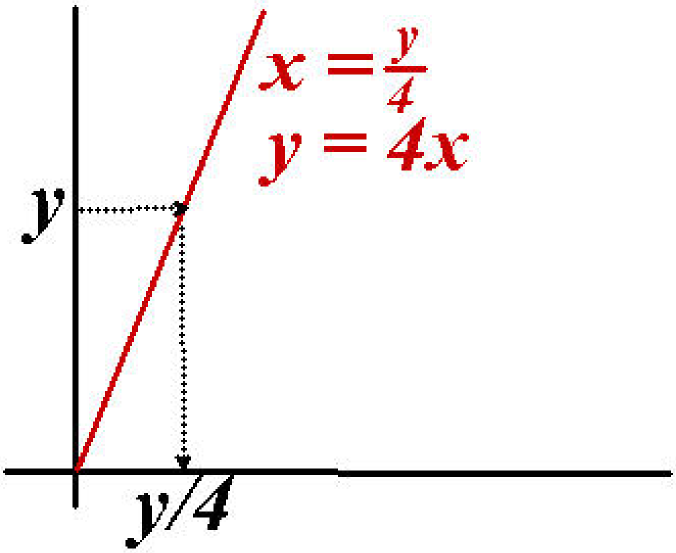
$$\int_0^3 \int_0^{\frac{2}{3}\sqrt{9-x^2}} f(x,y)\,dy\,dx = \int_0^2 \int_0^{\frac{3}{2}\sqrt{4-y^2}} f(x,y)\,dx\,dy$$

Region of integration: $\frac{x^2}{9} + \frac{y^2}{4} = 1$
with $x \geq 0$ and $y \geq 0$

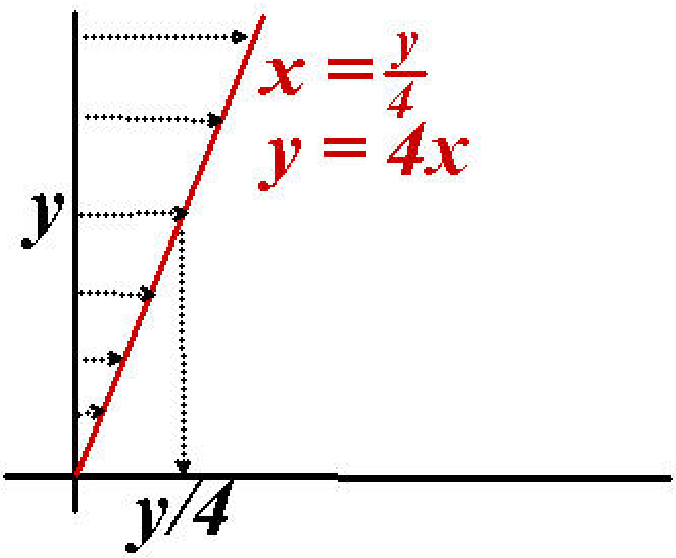


$$\int_0^\infty \int_0^{\frac{y}{4}} f(x,y)\,dx\,dy = \int_?^? \int_?^? f(x,y)\,dy\,dx$$

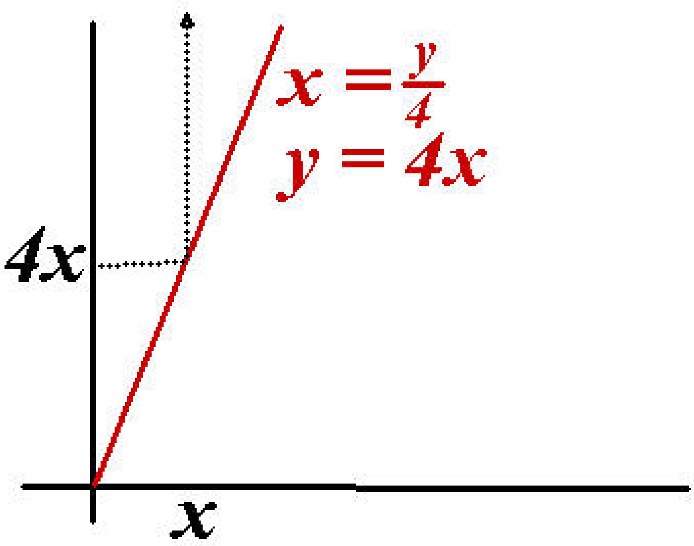
$$\int_0^{\infty}\int_0^{\frac{y}{4}} f(x,y)\,dx\,dy$$



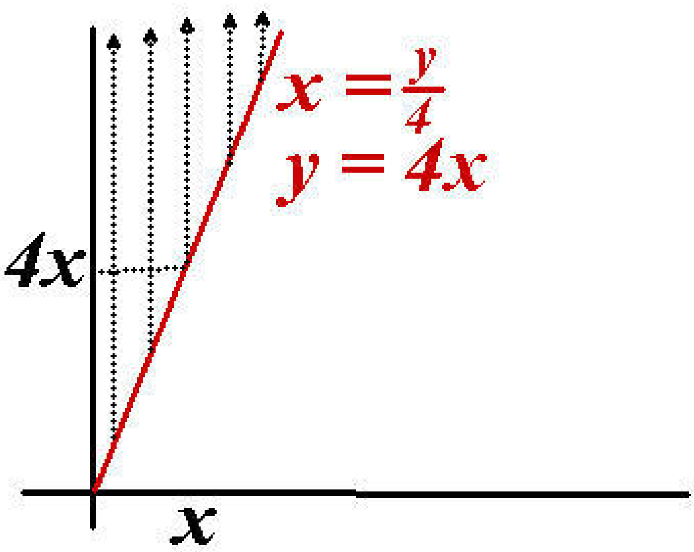
$$\int_0^{\infty}\int_0^{\frac{y}{4}} f(x,y)\,dx\,dy$$



$$\int_0^\infty \int_0^{\frac{y}{4}} f(x,y)\,dx\,dy = \int_?^? \int_{4x}^\infty f(x,y)\,dy\,dx$$

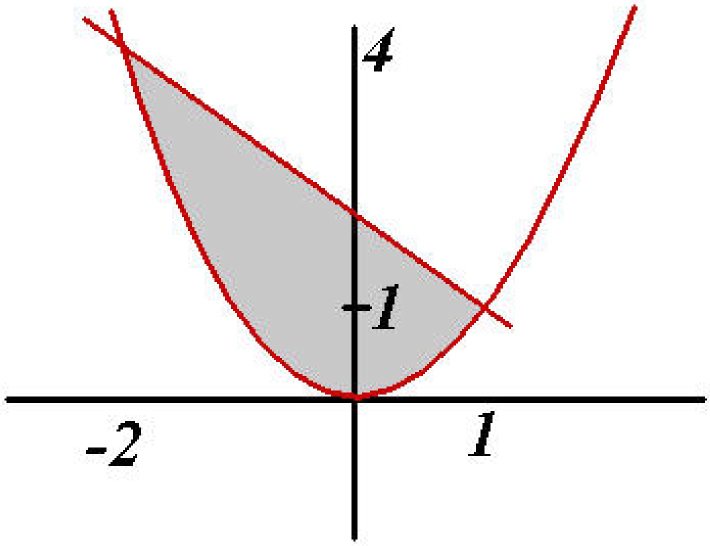


$$\int_0^\infty \int_0^{\frac{y}{4}} f(x,y)\,dx\,dy = \int_0^\infty \int_{4x}^\infty f(x,y)\,dy\,dx$$

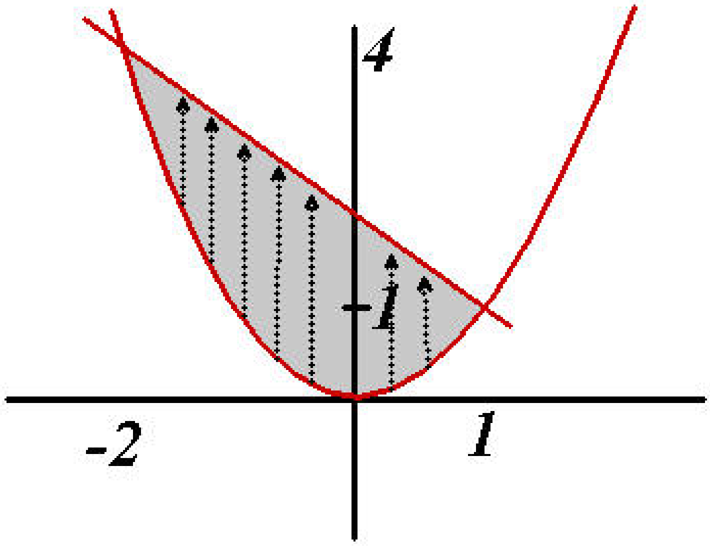


$$\int_{-2}^{1}\int_{x^2}^{2-x} f(x,y)\,dy\,dx = \int_{?}^{?}\int_{?}^{?} f(x,y)\,dx\,dy$$

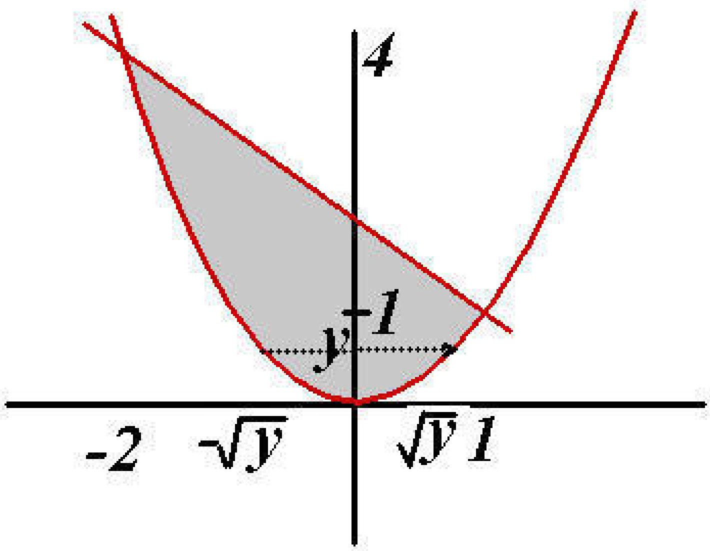
$$\int_{-2}^{1}\int_{x^2}^{2-x} f(x,y)\,dy\,dx = \int_{?}^{?}\int_{?}^{?} f(x,y)\,dx\,dy$$



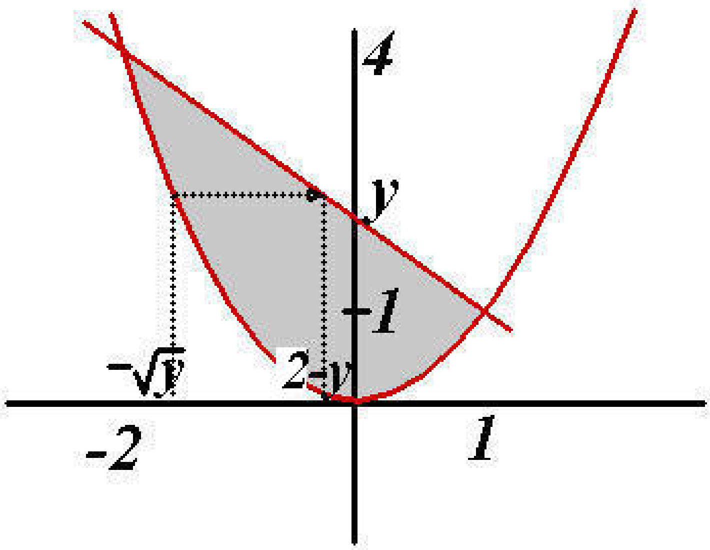
$$\int_{-2}^{1}\int_{x^2}^{2-x} f(x,y)\,dy\,dx = \int_{?}^{?}\int_{?}^{?} f(x,y)\,dx\,dy$$



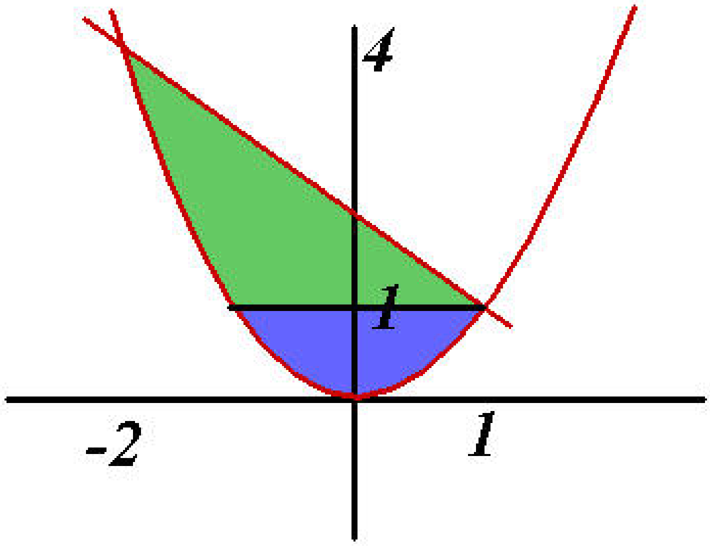
$$\int_{-2}^{1}\int_{x^2}^{2-x} f(x,y)\,dy\,dx = \int_{?}^{?}\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y)\,dx\,dy$$



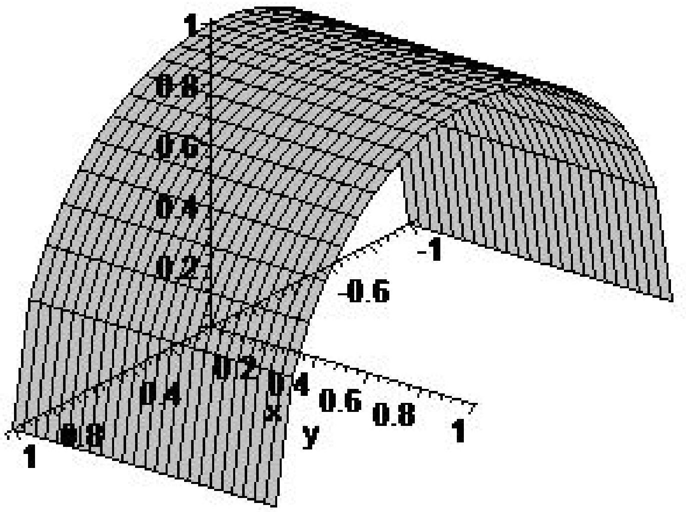
$$\int_{-2}^{1}\int_{x^2}^{2-x} f(x,y)\,dy\,dx = \int_{?}^{?}\int_{-\sqrt{y}}^{2-y} f(x,y)\,dx\,dy$$



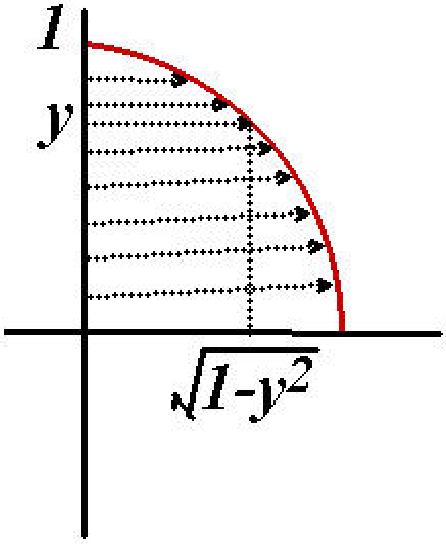
$$\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} f(x,y)\,dx\,dy + \int_1^4 \int_{-\sqrt{y}}^{2-y} f(x,y)\,dx\,dy$$



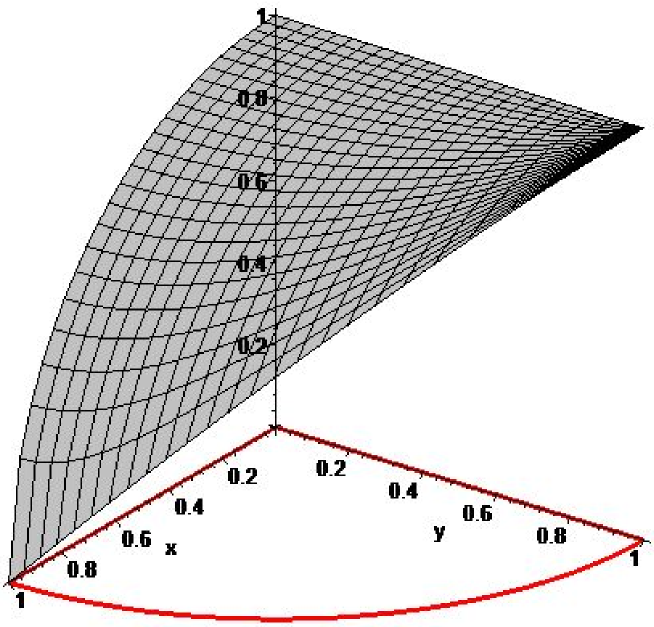
$$\int_0^1 \int_0^{\sqrt{1-y^2}} \sqrt{1-x^2}\, dx\, dy$$



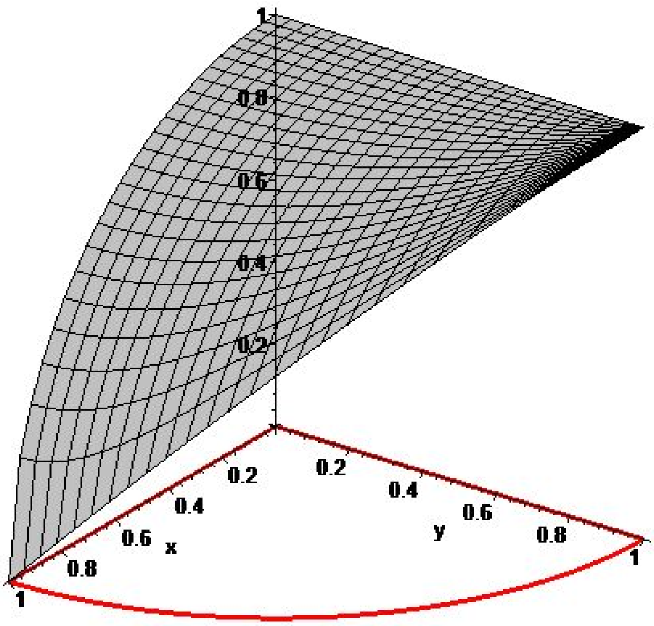
$$\int_0^1 \int_0^{\sqrt{1-y^2}} \sqrt{1-x^2}\, dx\, dy$$



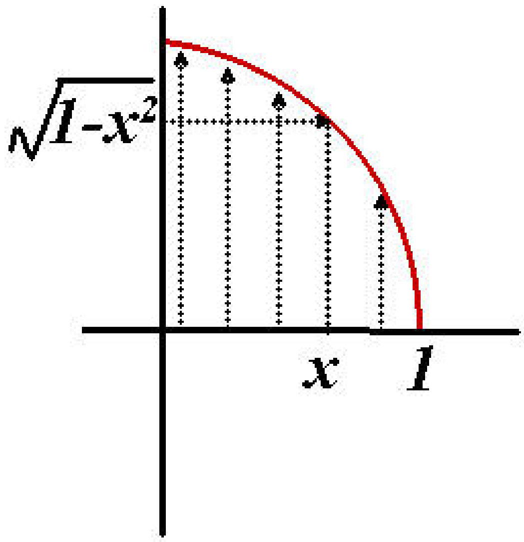
$$\int_0^1 \int_0^{\sqrt{1-y^2}} \sqrt{1-x^2}\, dx\, dy$$



$$\int_0^1 \int_0^{\sqrt{1-y^2}} \sqrt{1-x^2}\,dx\,dy = \int_?^? \int_?^? \sqrt{1-x^2}\,dy\,dx$$



$$\int_0^1 \int_0^{\sqrt{1-y^2}} \sqrt{1-x^2}\, dx\, dy = \int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-x^2}\, dy\, dx$$

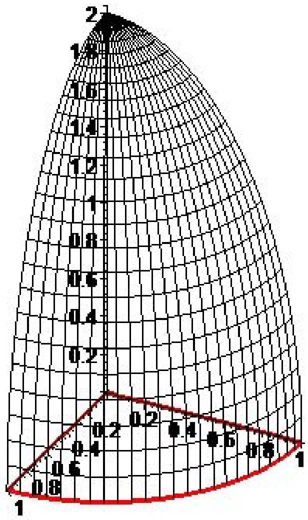


$$\int_0^1 \int_0^{\sqrt{1-y^2}} \sqrt{1-x^2}\,dx\,dy = \int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-x^2}\,dy\,dx$$

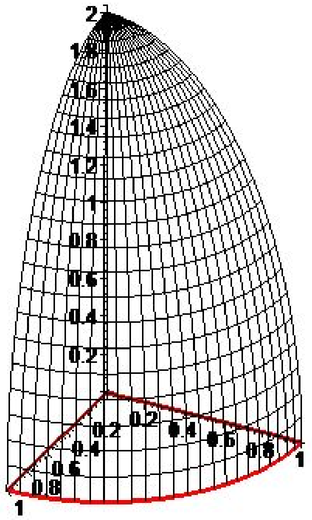
$$= \int_0^1 \left[\sqrt{1-x^2} \cdot y\right]_{y=0}^{\sqrt{1-x^2}} \,dx$$

$$
\begin{aligned}
\int_0^1 \int_0^{\sqrt{1-y^2}} \sqrt{1-x^2}\, dx\, dy = \int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-x^2}\, dy\, dx \\
&= \int_0^1 \left[\sqrt{1-x^2} \cdot y\right]_{y=0}^{\sqrt{1-x^2}} dx \\
&= \int_0^1 \left(1-x^2\right)\, dx \\
&= \frac{2}{3}
\end{aligned}
$$

$$\int_0^1 \int_0^{\sqrt{1-y^2}} 2\sqrt{1-x^2-y^2}\, dx\, dy$$



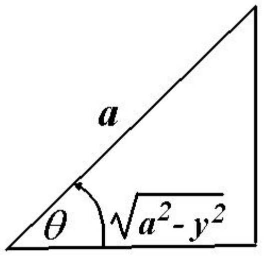
$$\int_0^1 \int_0^{\sqrt{1-x^2}} 2\sqrt{1-x^2-y^2}\,dy\,dx$$



$$\int_0^a \sqrt{a^2 - y^2}\, dy$$

Let $\theta = \sin^{-1} \frac{y}{a}$ so $a \sin \theta = y$ and $dy = a \cos \theta \, d\theta$

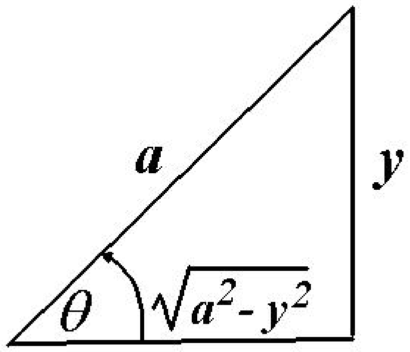
$\cos \theta = \frac{\sqrt{a^2 - y^2}}{a}$ so $\sqrt{a^2 - y^2} = a \cos \theta$



$$\int_0^a \sqrt{a^2 - y^2}\, dy = \int_0^{\pi/2} a^2 \cos^2\theta\, d\theta$$

Let $\theta = \sin^{-1}\frac{y}{a}$ so $a\sin\theta = y$ and $dy = a\cos\theta\, d\theta$

$\cos\theta = \frac{\sqrt{a^2-y^2}}{a}$ so $\sqrt{a^2 - y^2} = a\cos\theta$



$$\begin{aligned}\int_0^a \sqrt{a^2 - y^2}\, dy &= \int_0^{\pi/2} a^2 \cos^2\theta \, d\theta \\ &= \int_0^{\pi/2} a^2 \cdot \frac{1}{2}(1 + \cos 2\theta)\, d\theta \\ &= \frac{a^2}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_{\theta=0}^{\pi/2} \\ &= \frac{a^2\pi}{4}\end{aligned}$$

$$\int_0^a \sqrt{a^2 - y^2}\, dy = \frac{\pi}{4} a^2$$

$$\begin{aligned}\int_0^1 \int_0^{\sqrt{1-x^2}} 2\sqrt{1 - x^2 - y^2}\, dy\, dx &= \int_0^1 \frac{\pi}{2} \left(1 - x^2\right)\, dx \\ &= \frac{\pi}{3}\end{aligned}$$

$$\iint_R f(x,y)\,dA = \iint_R f(r\cos\theta, r\sin\theta)\,r\,d\theta\,dr$$