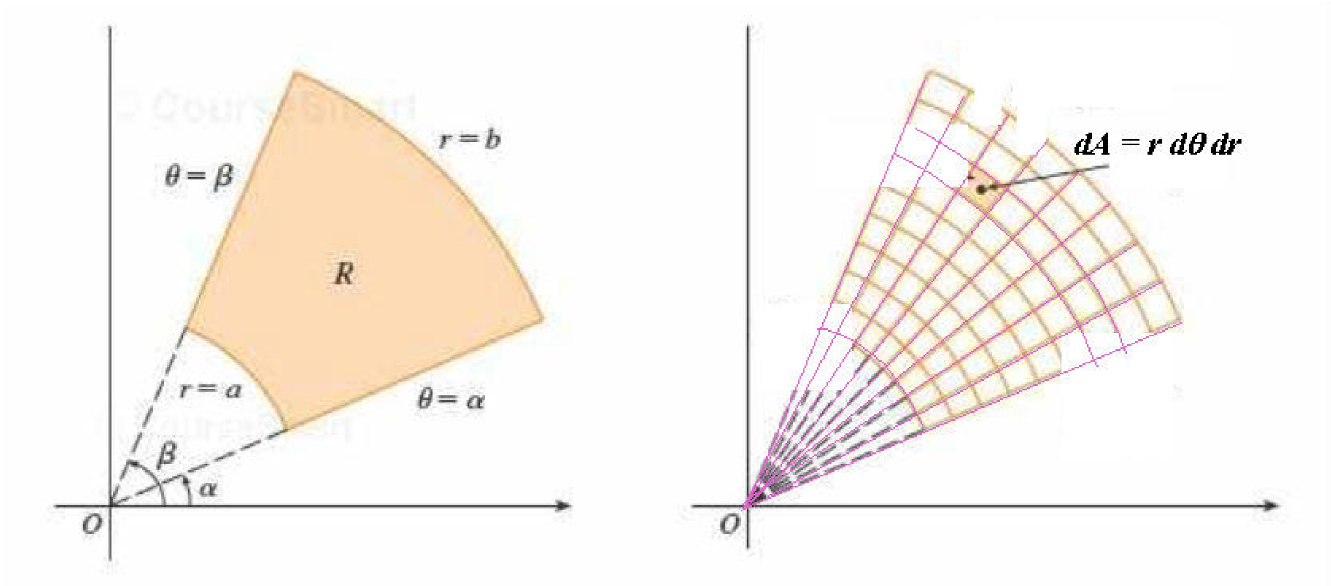
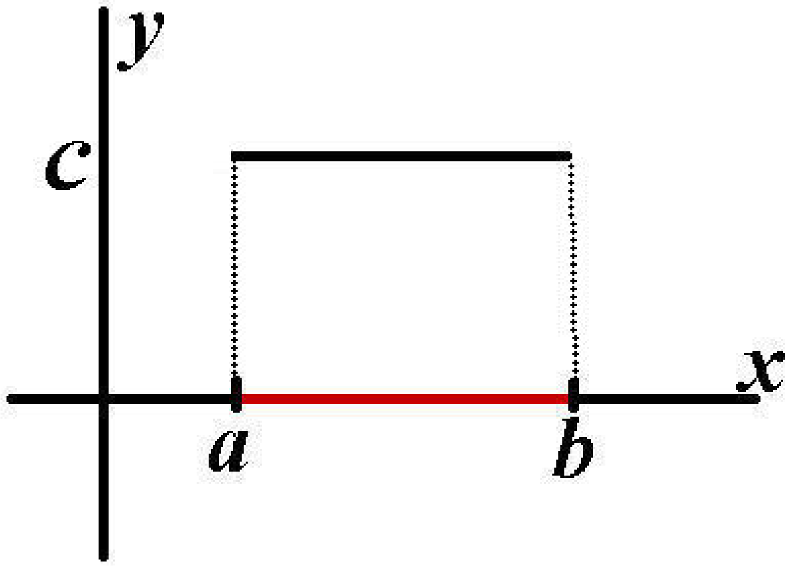
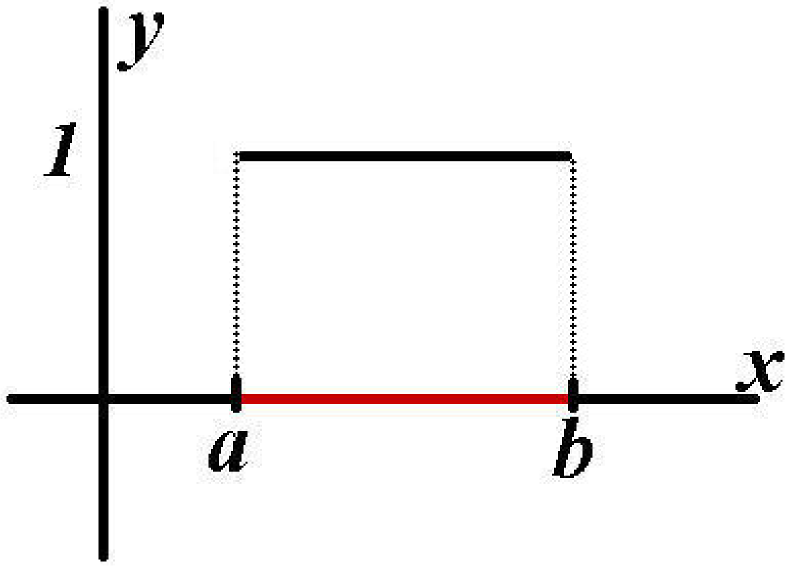
Area as a Double Integral



$$\int_a^b c\,dx = c\,(b-a)$$



$$\int_a^b 1\,dx = b - a$$

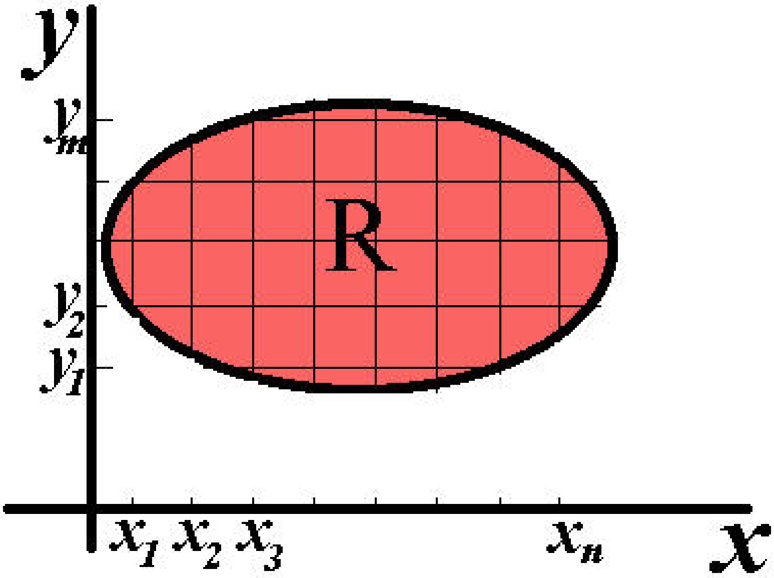


$$\lim_{n,m\to\infty} \sum_{i=1}^{n} \sum_{j=1}^{m} f(x_i,\ y_j)\,\Delta x\,\Delta y = \iint_R f(x,\ y)\,dx\,dy$$

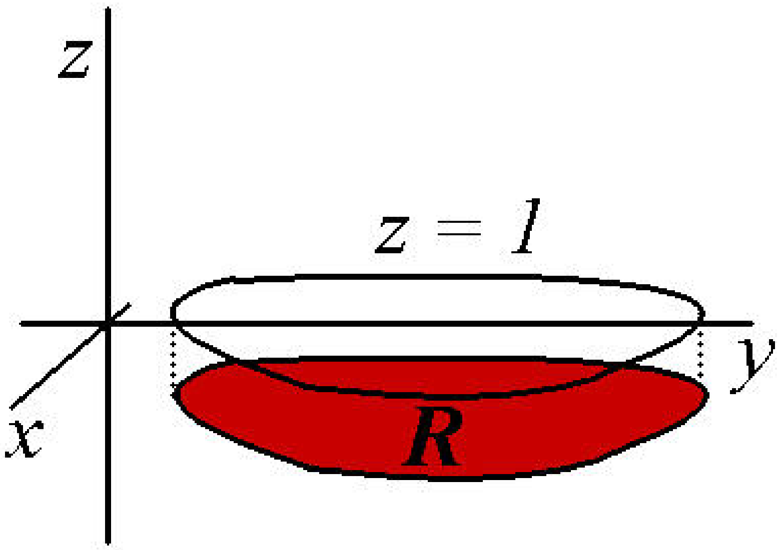
Therefore, if $f(x, y) = 1$ then:

$$\lim_{n,m\to\infty} \sum_{i=1}^{n} \sum_{j=1}^{m} 1\,\Delta x\,\Delta y = \iint_R 1\,dA$$

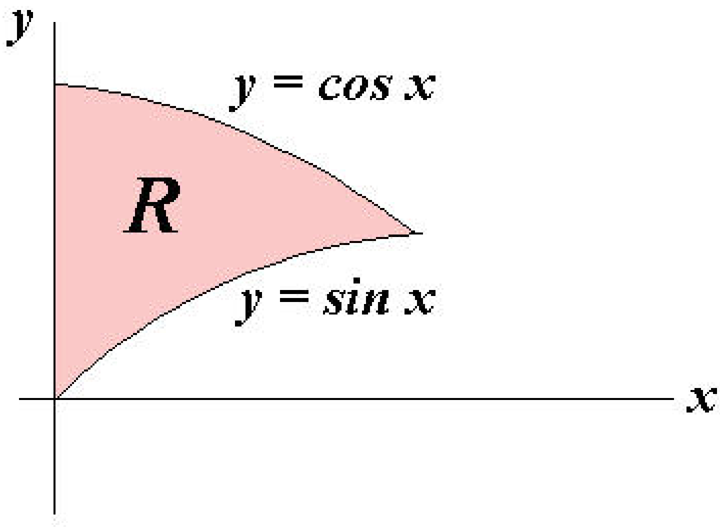
$$\lim_{n,m\to\infty}\sum_{i=1}^{n}\sum_{j=1}^{m} 1\,\Delta x\,\Delta y = \iint_R 1\,dA = \text{Area}(R)$$



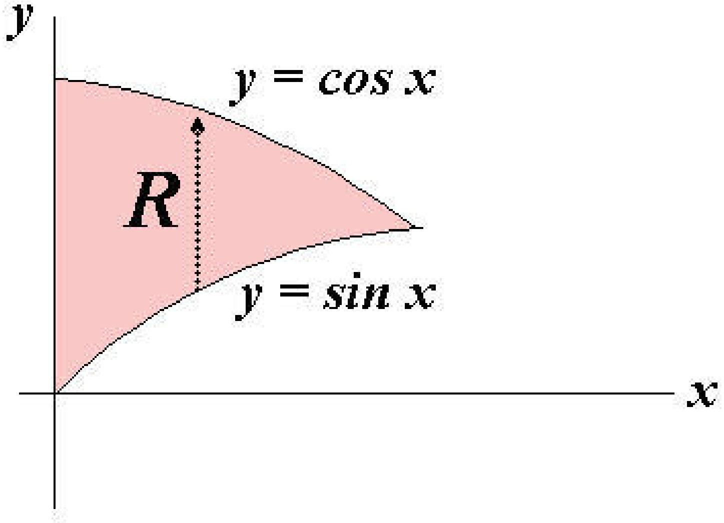
$$\iint_R 1\,dA = \text{Area}(R)$$



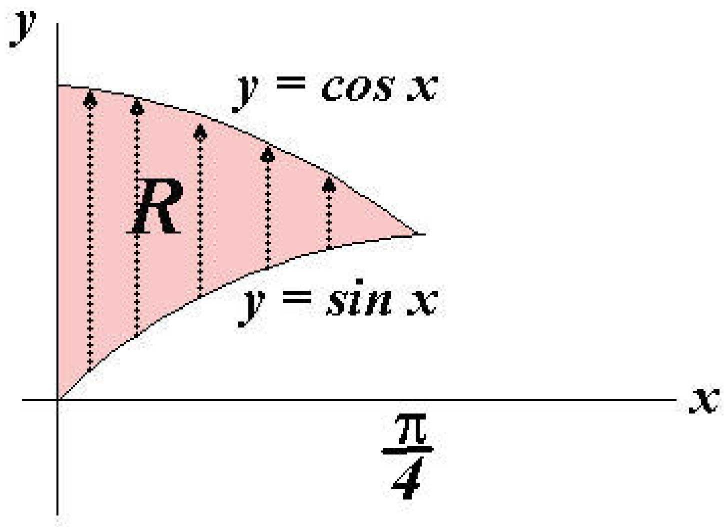
$$\text{Area}(R) = \iint_R 1 \, dA$$



$$\text{Area}(R) = \int\int_{\sin x}^{\cos x} 1\, dy\, dx$$



$$\text{Area}(R) = \int_0^{\pi/4} \int_{\sin x}^{\cos x} 1\, dy\, dx$$



$$
\begin{aligned}
\text{Area}(R) &= \int_0^{\pi/4} \int_{\sin x}^{\cos x} 1 \, dy \, dx \\
&= \int_0^{\pi/4} \Big[\, y \, \Big]_{y=\sin x}^{\cos x} dx \\
&= \int_0^{\pi/4} (\cos x - \sin x) \, dx
\end{aligned}
$$

$$
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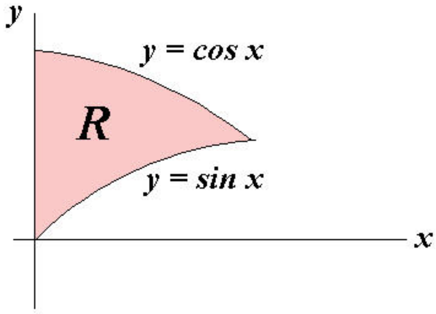
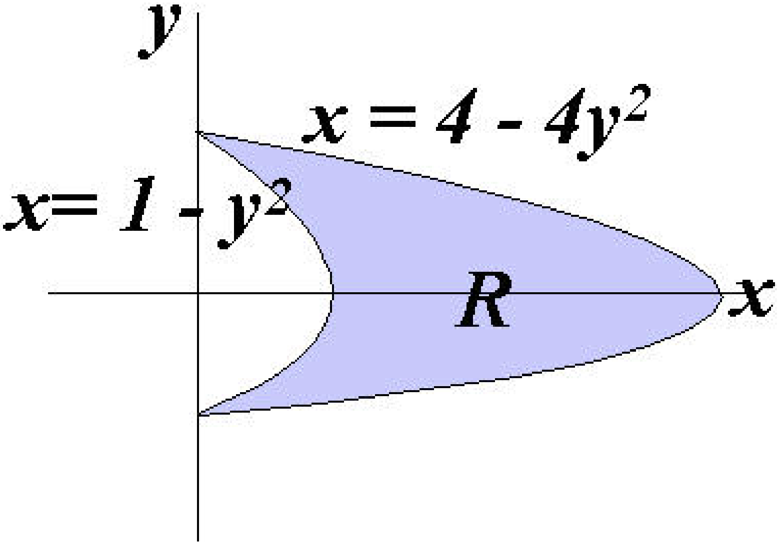
\text{Area}(R) &= \int_0^{\pi/4} \int_{\sin x}^{\cos x} 1 \, dy \, dx \\
&= \int_0^{\pi/4} \Big[\, y \, \Big]_{y=\sin x}^{\cos x} dx \\
&= \int_0^{\pi/4} (\cos x - \sin x) \, dx \\
&= \sqrt{2} - 1
\end{aligned}
$$

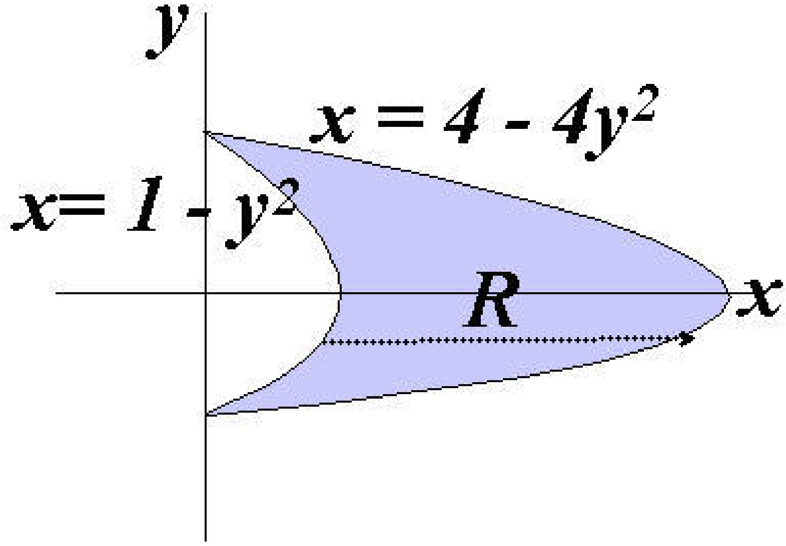
$$
\begin{aligned}
\text{Area}(R) &= \int_0^{\pi/4} \int_{\sin x}^{\cos x} 1 \, dy \, dx \\
&= \int_0^{\pi/4} (\cos x - \sin x) \, dx \\
&= \int_0^{\pi/4} \cos x \, dx - \int_0^{\pi/4} \sin x \, dx
\end{aligned}
$$



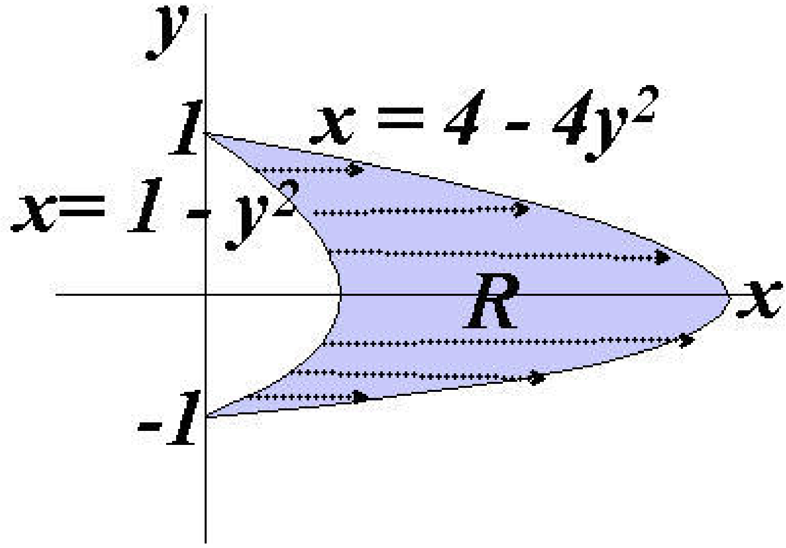
$$\text{Area}(R) = \iint_R 1 \, dA$$



$$\text{Area}(R) = \iint_R 1\,dA = \int_?^? \int_{1-y^2}^{4-4y^2} 1\,dx\,dy$$



$$\text{Area}(R) = \iint_R 1\,dA = \int_{-1}^{1}\int_{1-y^2}^{4-4y^2} 1\,dx\,dy$$

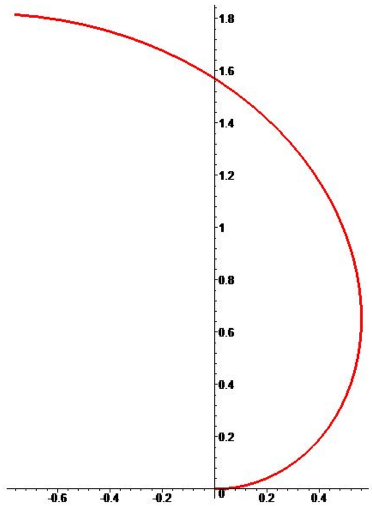


$$\begin{aligned}
\iint_R 1\, dA &= \int_{-1}^{1} \int_{1-y^2}^{4-4y^2} 1\, dx\, dy \\
&= \int_{-1}^{1} \Big[\ x\ \Big]_{x=1-y^2}^{4-4y^2}\, dy \\
&= \int_{-1}^{1} (3 - 3y^2)\, dy
\end{aligned}$$

$$\begin{aligned}
\iint_R 1\,dA &= \int_{-1}^{1}\int_{1-y^2}^{4-4y^2} 1\,dx\,dy \\
&= \int_{-1}^{1} \Big[\ x\ \Big]_{x=1-y^2}^{4-4y^2}\,dy \\
&= \int_{-1}^{1} (3-3y^2)\,dy \\
&= 4
\end{aligned}$$

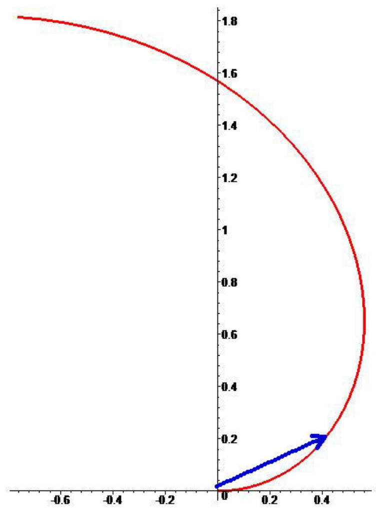
Polar Coordinates:

$$r = \theta$$



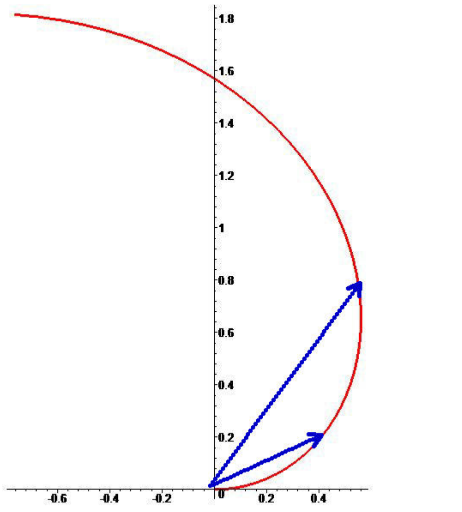
Polar Coordinates:

$$r = \theta \text{ where } \theta = \pi/6$$



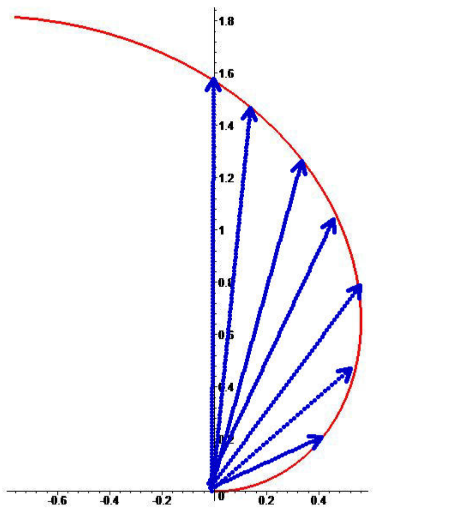
Polar Coordinates:

$$r = \theta \text{ where } \theta = \pi/4$$

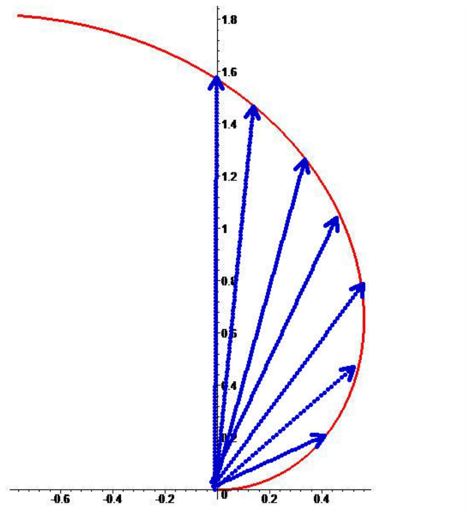


Polar Coordinates:

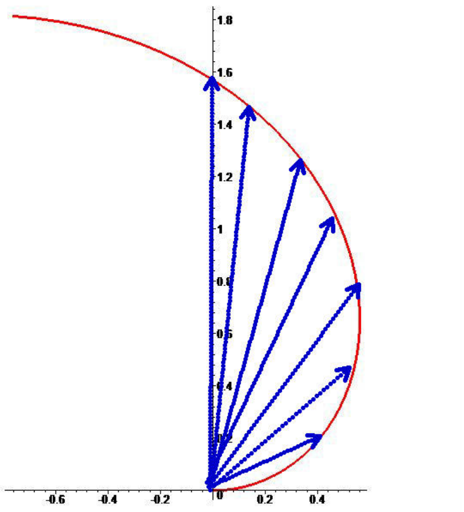
$$r = \theta \text{ where } 0 \leq \theta \leq \pi/2$$



$$\text{Area} = \iint 1 \, dA = \iint r \, dr \, d\theta$$

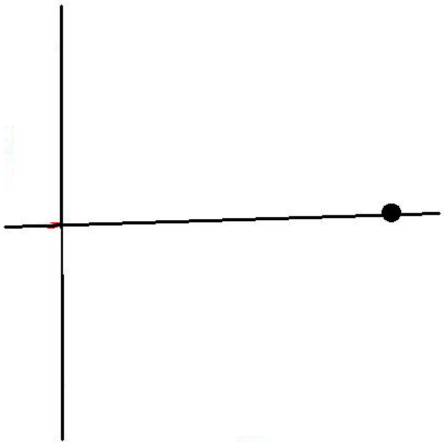


$$\text{Area} = \iint 1\, dA = \int_0^{\pi/2} \int_0^{\theta} r\, dr\, d\theta$$



$$
\begin{aligned}
\text{Area} &= \int_0^{\pi/2} \int_0^{\theta} r \, dr \, d\theta \\
&= \int_0^{\pi/2} \left[\frac{1}{2} r^2 \right]_{r=0}^{\theta} d\theta \\
&= \int_0^{\pi/2} \frac{1}{2} \theta^2 \, d\theta \\
&= \left[\frac{1}{6} \theta^3 \right]_0^{\pi/2} \\
&= \frac{\pi^3}{48}
\end{aligned}
$$

Let $\mathcal{D}$ be the region bounded by $r = 1 + \cos\theta$.
Calculate $\iint_{\mathcal{D}} 1\, dA$.



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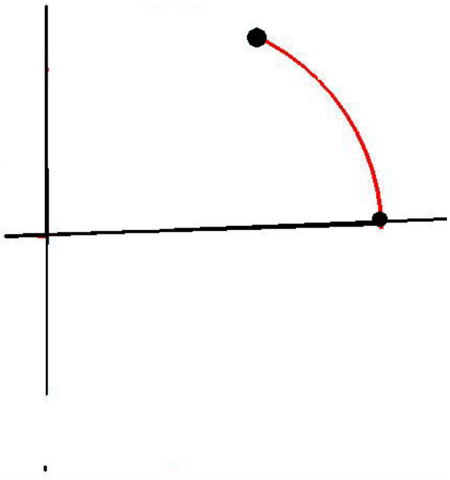

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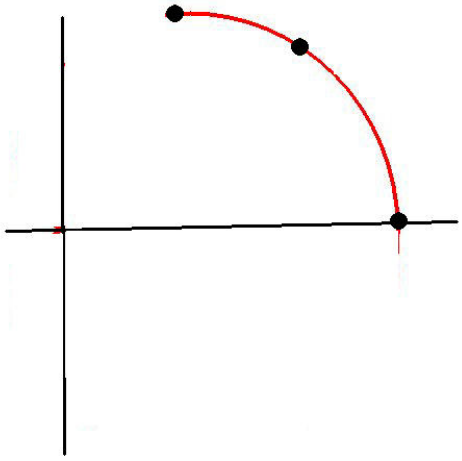
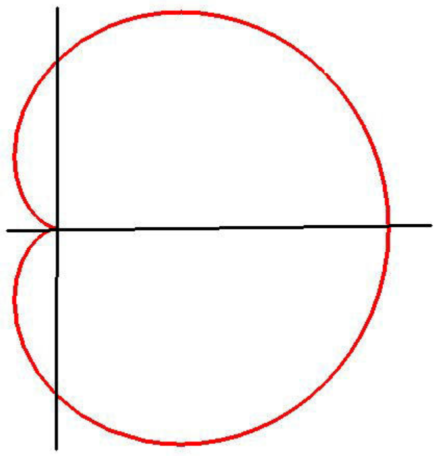
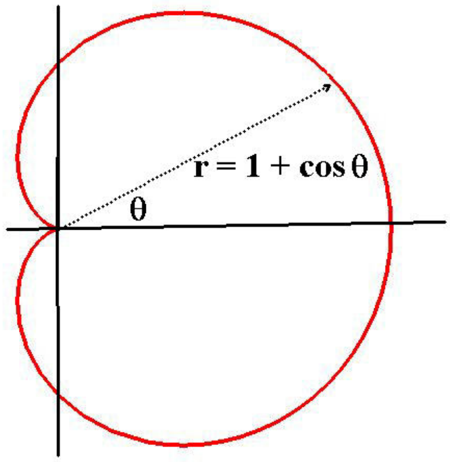


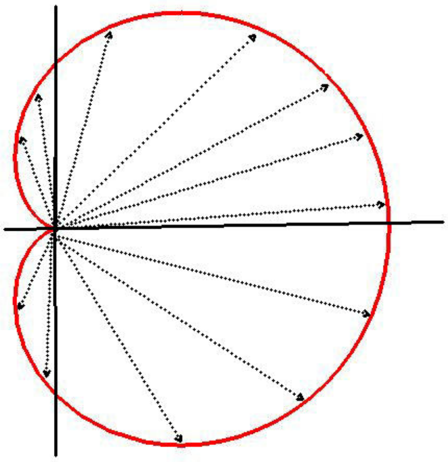
Let $\mathcal{D}$ be the region bounded by $r = 1 + \cos\theta$.
Calculate $\iint_{\mathcal{D}} 1\, dA$.



$$\iint_{\mathcal{D}} 1 \, dA = \int_{?}^{?} \int_{0}^{1+\cos\theta} 1 \, r \, dr \, d\theta$$



$$\iint_{\mathcal{D}} 1\, dA = \int_0^{2\pi} \int_0^{1+\cos\theta} 1\, r\, dr\, d\theta$$



$$\begin{aligned}
\iint_{\mathcal{D}} 1\, dA &= \int_0^{2\pi} \int_0^{1+\cos\theta} 1\, r\, dr\, d\theta \\
&= \int_0^{2\pi} \left[\frac{1}{2} r^2\right]_{r=0}^{1+\cos\theta} d\theta \\
&= \int_0^{2\pi} \frac{1}{2}(1+\cos\theta)^2\, d\theta
\end{aligned}$$

$$\begin{aligned}\iint_{\mathcal{D}} 1\, dA &= \int_0^{2\pi} \int_0^{1+\cos\theta} 1\, r\, dr\, d\theta \\ &= \int_0^{2\pi} \left[\frac{1}{2} r^2\right]_{r=0}^{1+\cos\theta} d\theta \\ &= \int_0^{2\pi} \frac{1}{2}(1+\cos\theta)^2\, d\theta \\ &= \frac{3\pi}{2}\end{aligned}$$