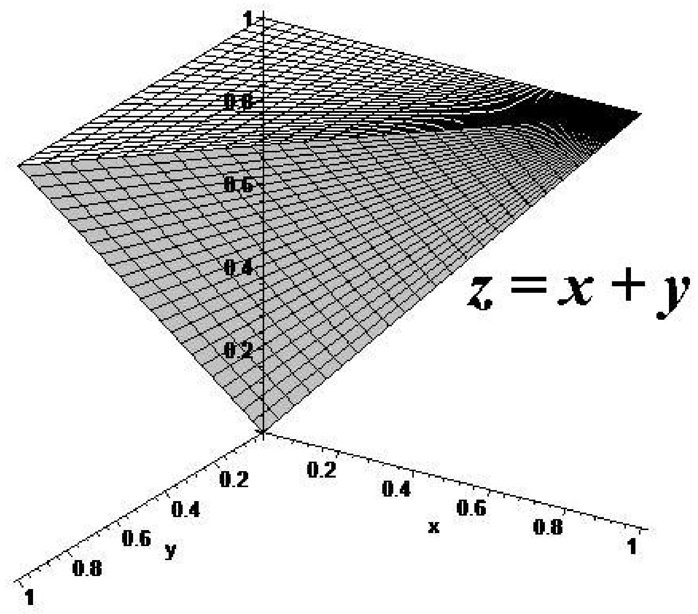
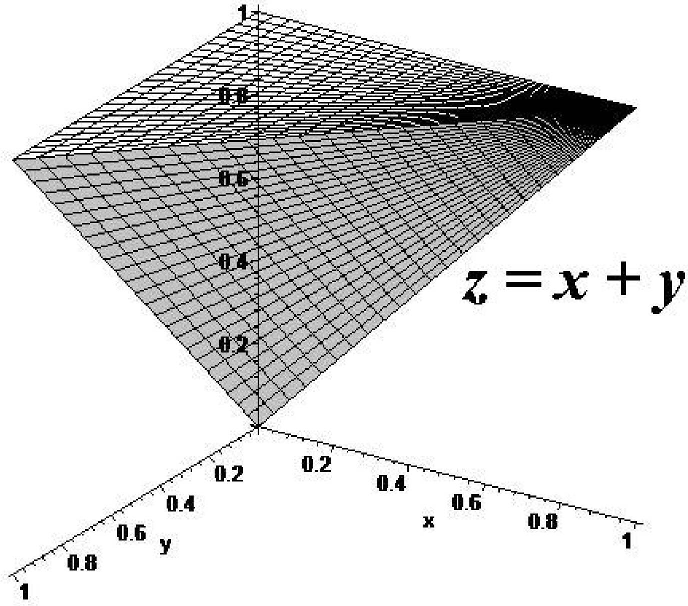
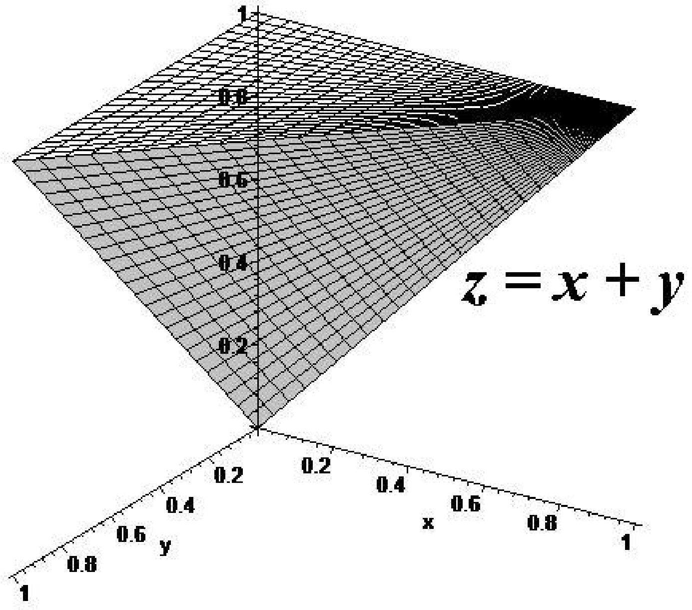
Triple Integrals - Additional Problems



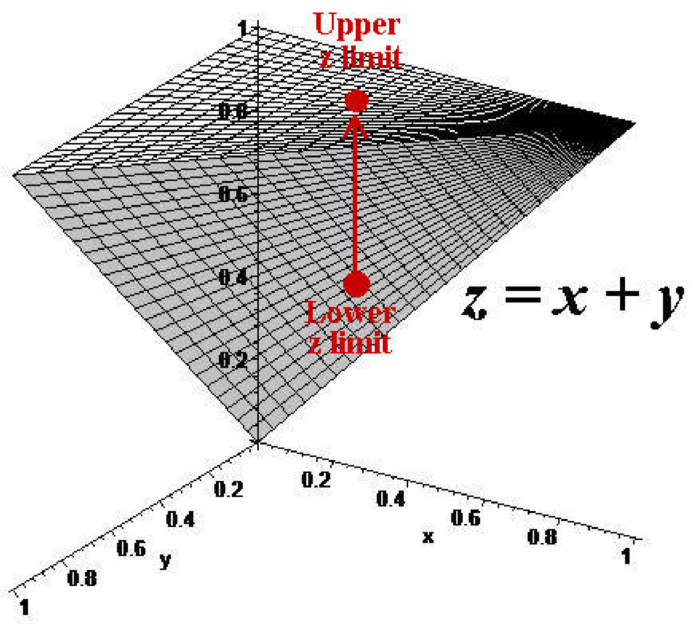
$$\text{Vol}(T) = \iiint_T 1 \, dV$$



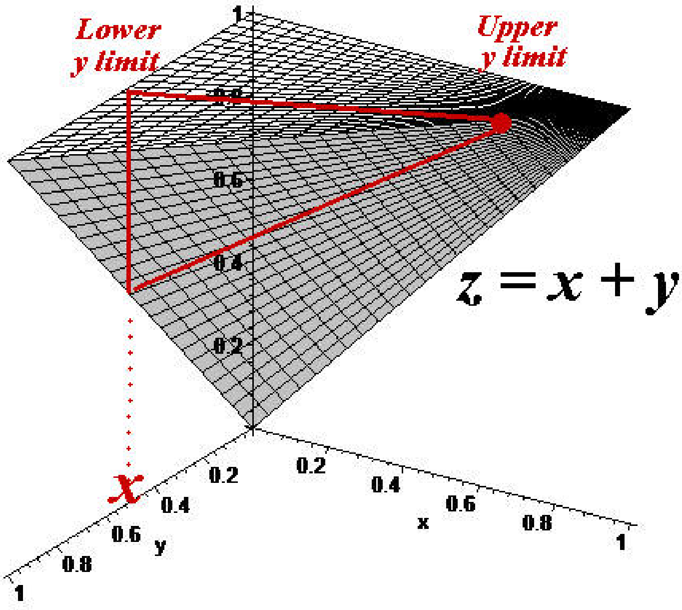
$$\text{Vol}(T) = \int_?^? \int_?^? \int_?^? 1 \, dz \, dy \, dx$$



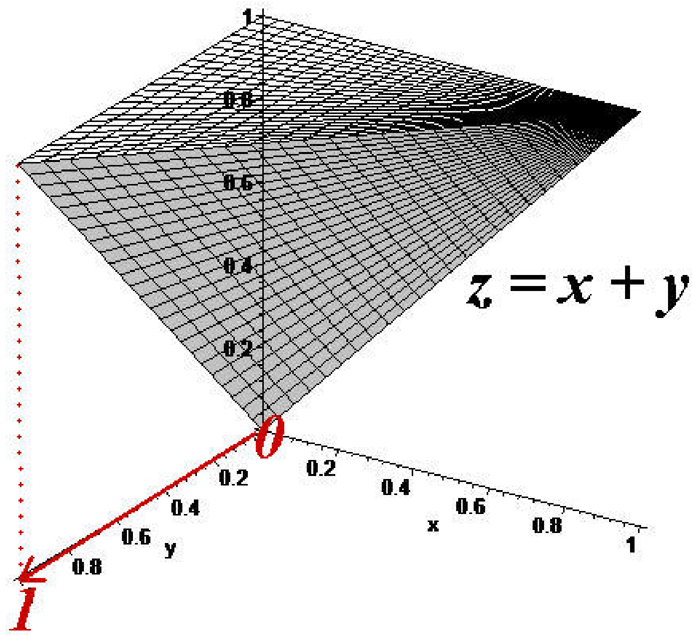
$$\mathrm{Vol}(T) = \int_{?}^{?}\int_{?}^{?}\int_{x+y}^{1} dz\,dy\,dx$$



$$\mathrm{Vol}(T) = \int_{?}^{?} \int_{0}^{1-x} \int_{x+y}^{1} dz\, dy\, dx$$

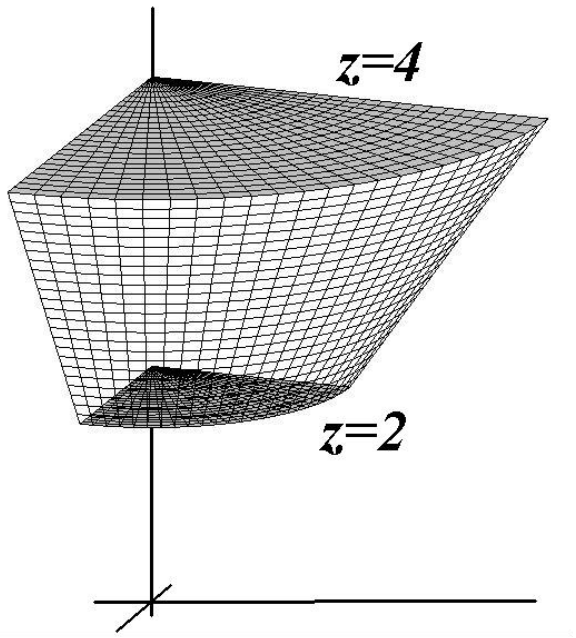


$$\mathrm{Vol}(T) = \int_0^1 \int_0^{1-x} \int_{x+y}^1 dz\, dy\, dx$$

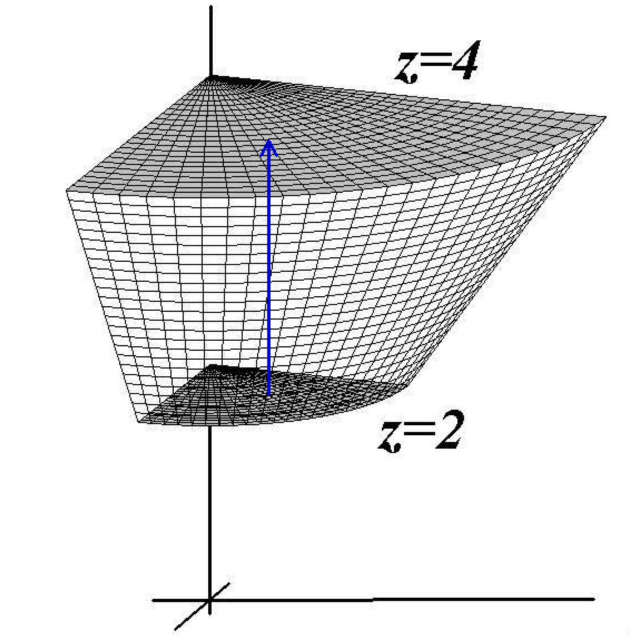


$$
\begin{aligned}
\mathrm{Vol}(T) &= \int_0^1 \int_0^{1-x} \int_{x+y}^1 \, dz\, dy\, dx \\
&= \int_0^1 \int_0^{1-x} (1 - x - y)\, dy\, dx \\
&= \int_0^1 \left(\frac{1}{2} - x + \frac{1}{2}x^2 \right) \, dx \\
&= \frac{1}{6}
\end{aligned}
$$

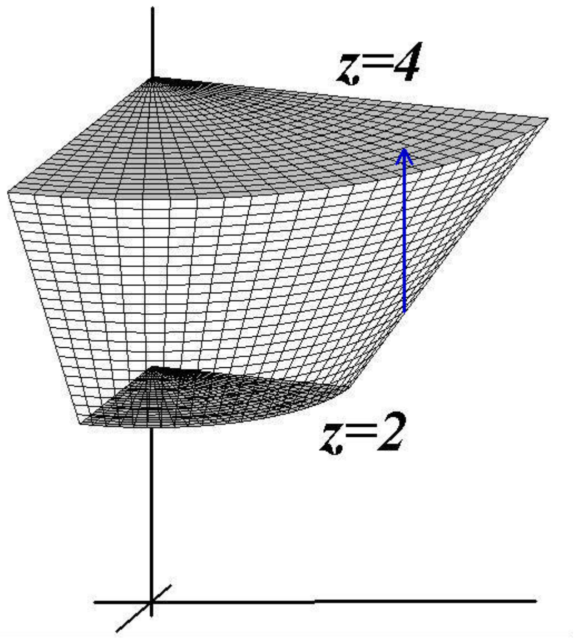
Let Q be the region that is inside the cone $z = \sqrt{x^2 + y^2}$ for $x \geq 0$ and $y \geq 0$ and bounded by the planes $z = 2$ and $z = 4$. Find the volume of Q



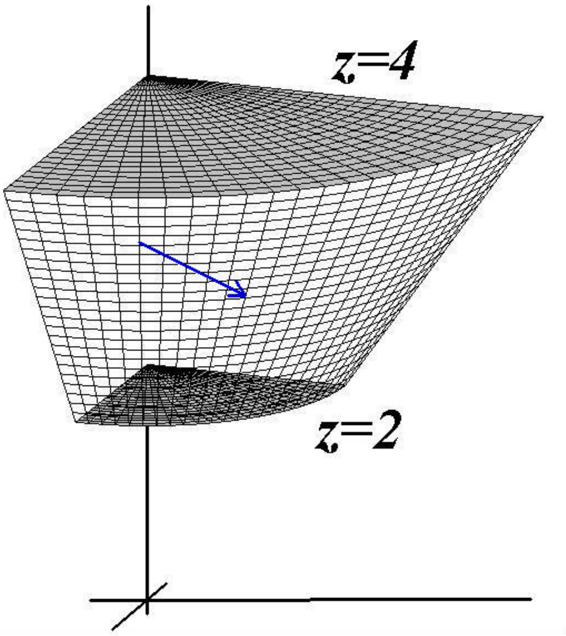
$$\mathrm{Vol}(Q) = \int_{?}^{?}\int_{?}^{?}\int_{?}^{?} 1\, r\, dz\, d\theta\, dr$$



$$\mathrm{Vol}(Q) = \int_{?}^{?} \int_{?}^{?} \int_{?}^{?} 1 \, r \, dz \, d\theta \, dr$$



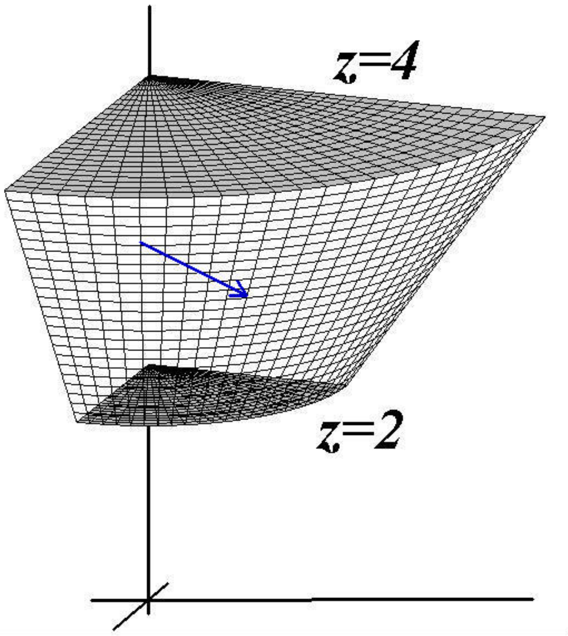
$$\mathrm{Vol}(Q) = \int_?^? \int_?^? \int_?^? 1\, r\, dr\, d\theta\, dz$$



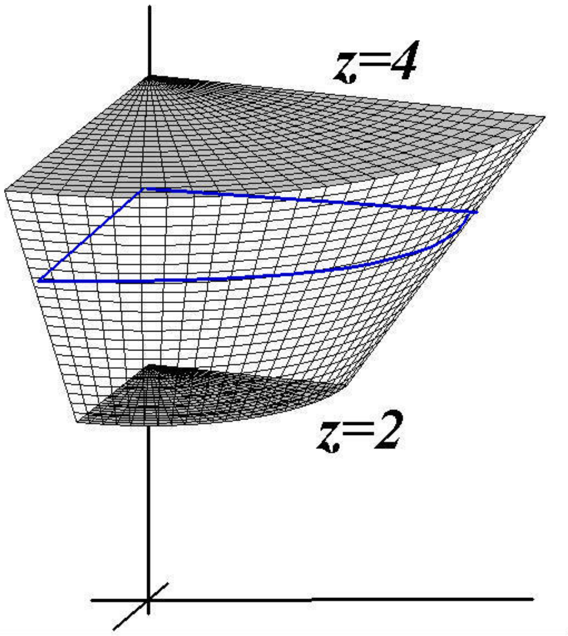
On the surface of the cone,

$$z = \sqrt{x^2 + y^2} = \sqrt{(r\cos\theta)^2 + (r\sin\theta)^2} = r$$

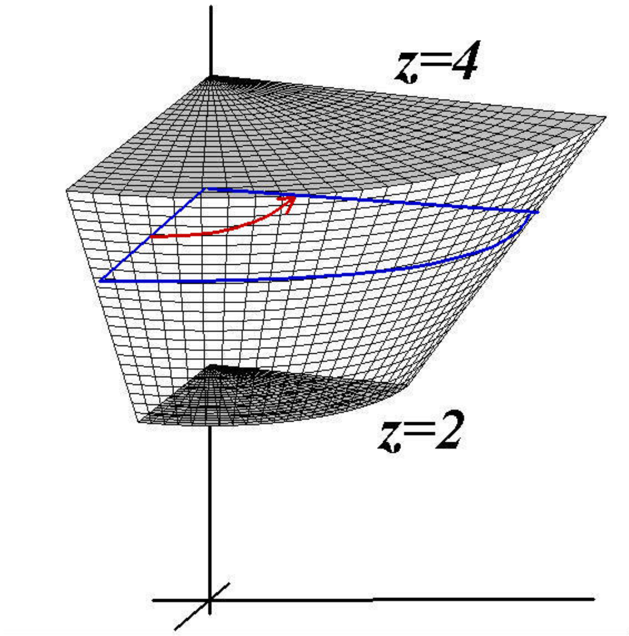
$$\mathrm{Vol}(Q) = \int_{?}^{?} \int_{?}^{?} \int_{0}^{z} r \, dr \, d\theta \, dz$$



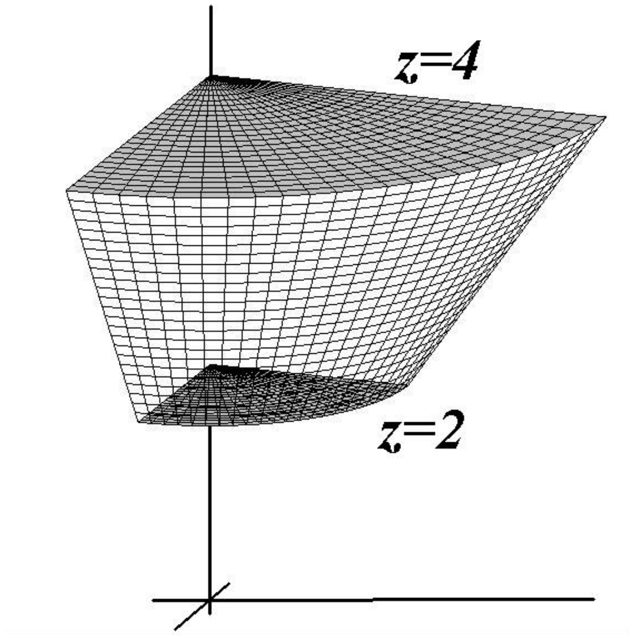
$$\mathrm{Vol}(Q) = \int_{?}^{?} \int_{?}^{?} \int_{0}^{z} r\, dr\, d\theta\, dz$$



$$\text{Vol}(Q) = \int_{?}^{?} \int_{0}^{\pi/2} \int_{0}^{z} r \, dr \, d\theta \, dz$$



$$\mathrm{Vol}(Q) = \int_2^4 \int_0^{\pi/2} \int_0^z r\,dr\,d\theta\,dz$$

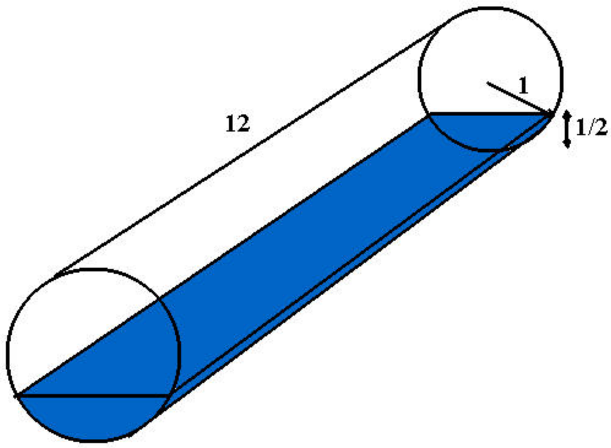


$$
\begin{aligned}
\mathrm{Vol}(Q) &= \int_2^4 \int_0^{\pi/2} \int_0^z r\, dr\, d\theta\, dz \\
&= \int_2^4 \int_0^{\pi/2} \left[\frac{1}{2} r^2\right]_{r=0}^{z} d\theta\, dz \\
&= \int_2^4 \int_0^{\pi/2} \frac{1}{2} z^2\, d\theta\, dz
\end{aligned}
$$

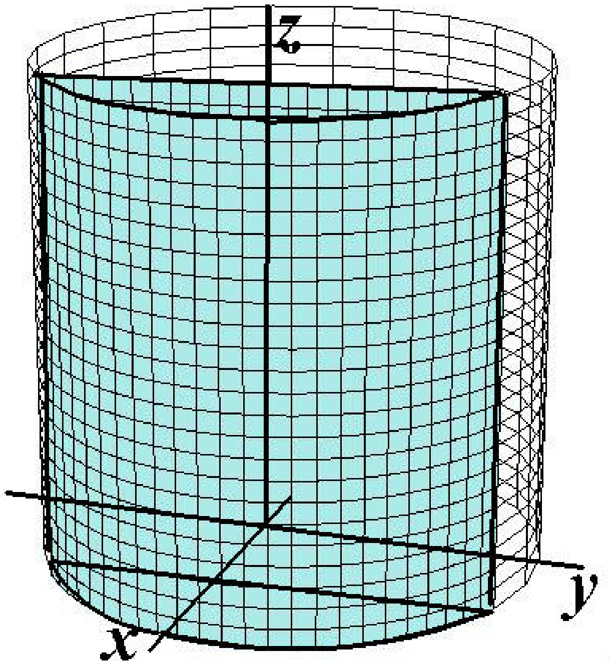
$$\begin{aligned}
\mathrm{Vol}(Q) &= \int_2^4 \int_0^{\pi/2} \int_0^z r\,dr\,d\theta\,dz \\
&= \int_2^4 \int_0^{\pi/2} \left[\frac{1}{2}r^2\right]_{r=0}^{z} d\theta\,dz \\
&= \int_2^4 \int_0^{\pi/2} \frac{1}{2}z^2\,d\theta\,dz \\
&= \int_2^4 \frac{\pi}{4}z^2\,dz
\end{aligned}$$

$$
\begin{aligned}
\mathrm{Vol}(Q) &= \int_2^4 \int_0^{\pi/2} \int_0^z r\,dr\,d\theta\,dz \\
&= \int_2^4 \int_0^{\pi/2} \left[\frac{1}{2}r^2\right]_{r=0}^{z} d\theta\,dz \\
&= \int_2^4 \int_0^{\pi/2} \frac{1}{2}z^2\,d\theta\,dz \\
&= \int_2^4 \frac{\pi}{4}z^2\,dz \\
&= \frac{14\pi}{3}
\end{aligned}
$$

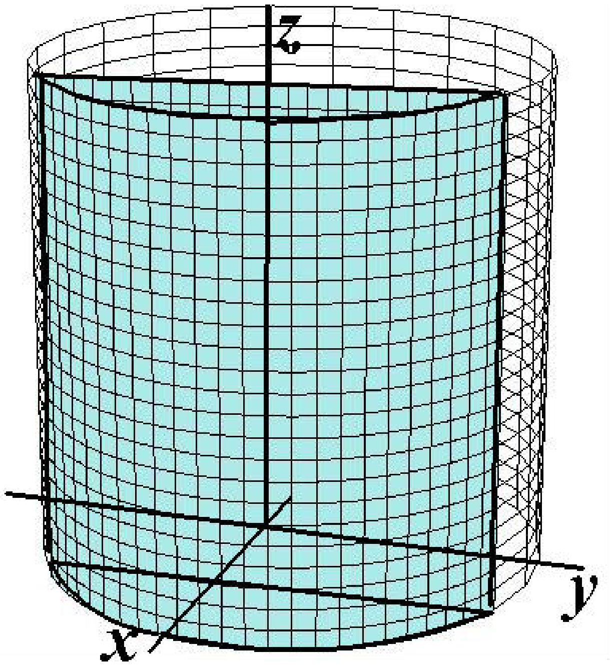
A cylindrical fuel tank is 12 feet long and has a radius of 1 foot. There is water in the bottom of the tank reaching $\frac{1}{2}$ above the lowest point. The water must be drained before we can add fuel. What is the volume of the water?



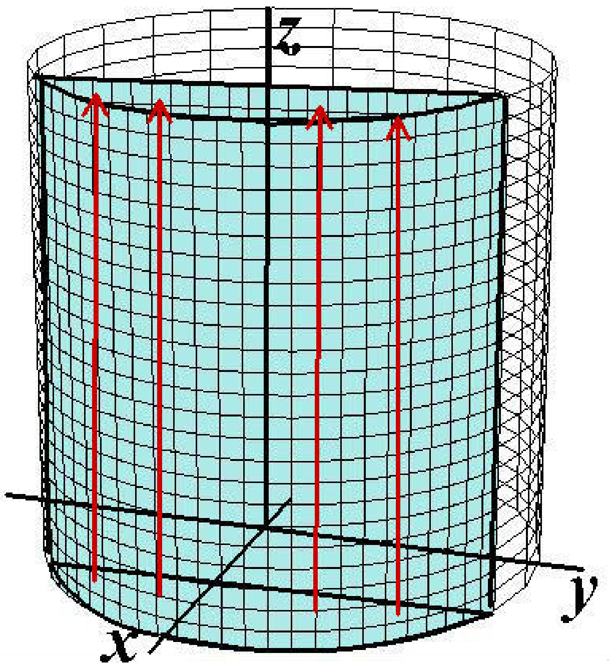
Turn the diagram vertically to agree with the coordinate system we have been using.



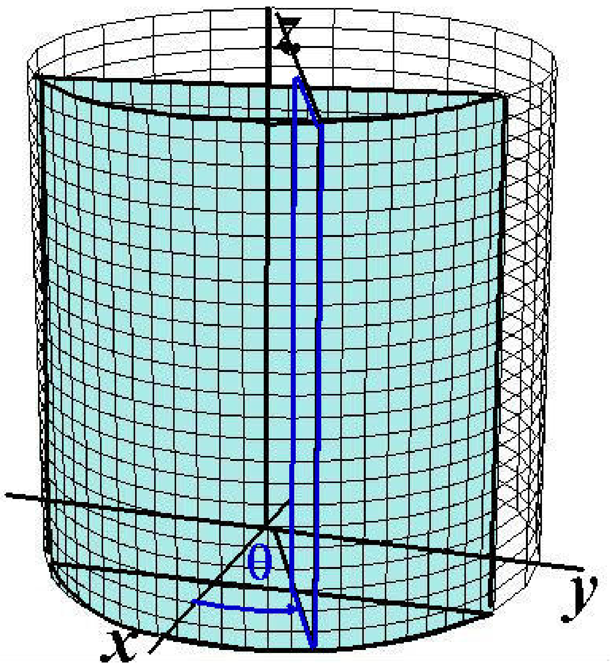
$$V = \iiint r\,dz\,dr\,d\theta$$



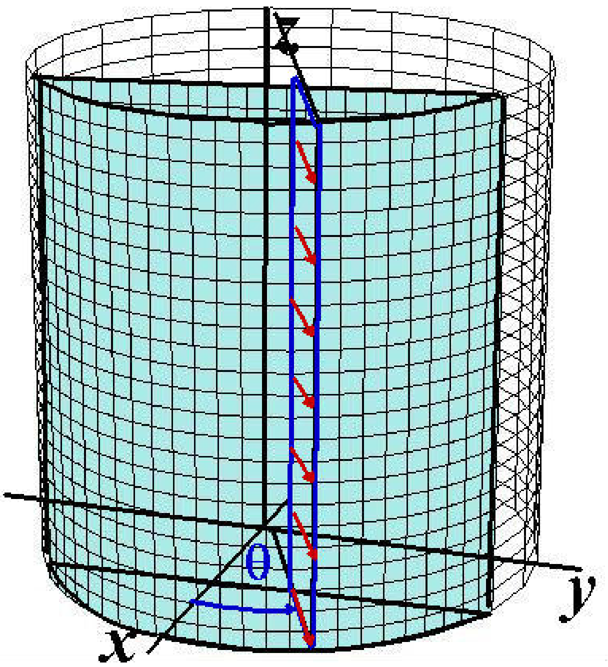
$$V = \iiint_0^{12} r\,dz\,dr\,d\theta$$



$$V = \iint\int_0^{12} r \, dz \, dr \, d\theta$$

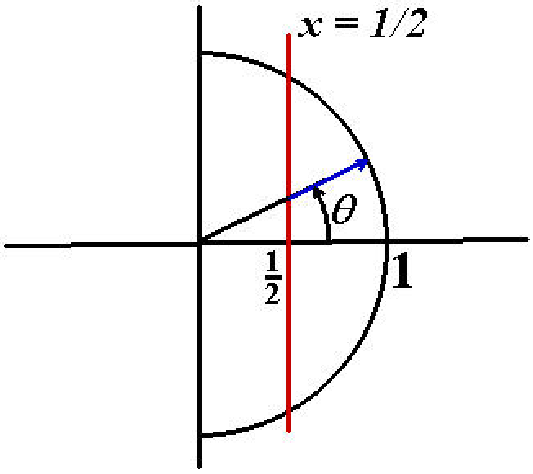


$$V = \int\!\!\int\!\!\int_0^{12} r\,dz\,dr\,d\theta$$

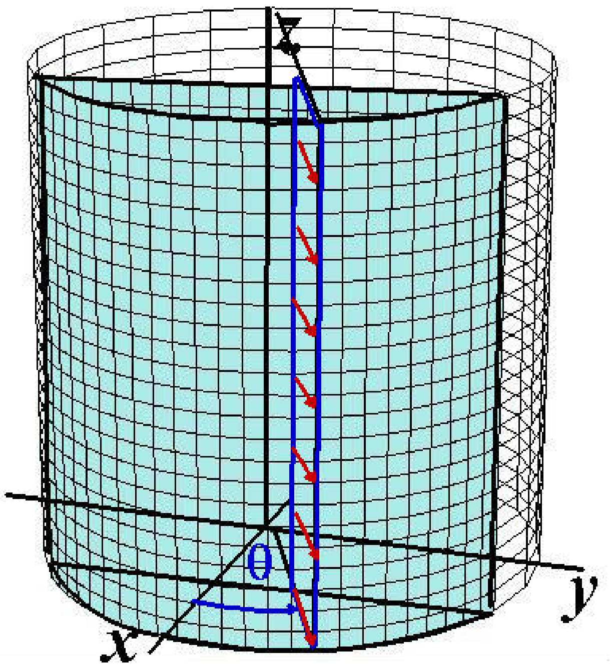


r starts at the line $x = \frac{1}{2}$ so $r\cos\theta = \frac{1}{2}$

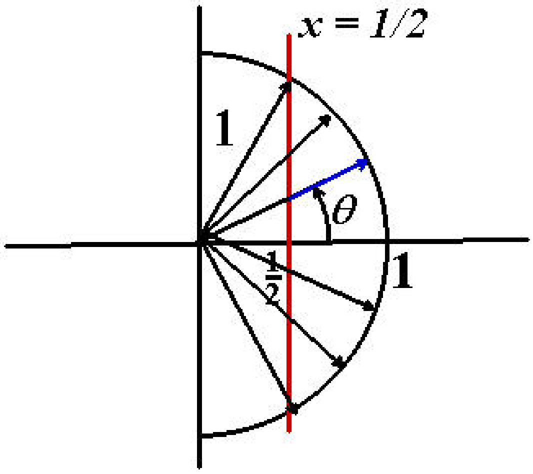
$$r = \frac{1}{2}\sec\theta$$



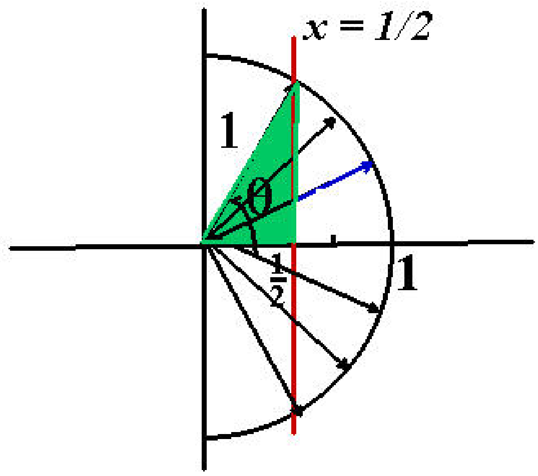
$$V = \iint_{\frac{1}{2}\sec\theta}^{1} \int_{0}^{12} r \, dz \, dr \, d\theta$$



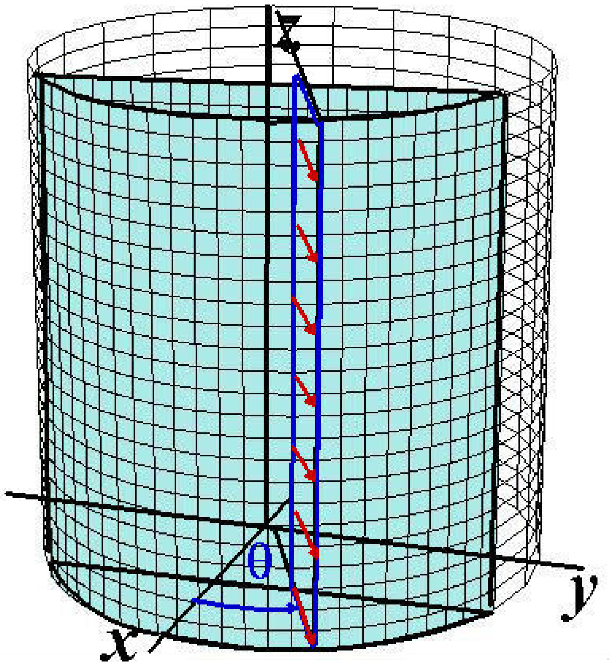
Vary θ



At the final value of θ, $\cos\theta = \frac{1}{2}$ so $\theta = \frac{\pi}{3}$



$$V = \int_{-\pi/3}^{\pi/3} \int_{\frac{1}{2}\sec\theta}^{1} \int_{0}^{12} r\,dz\,dr\,d\theta$$

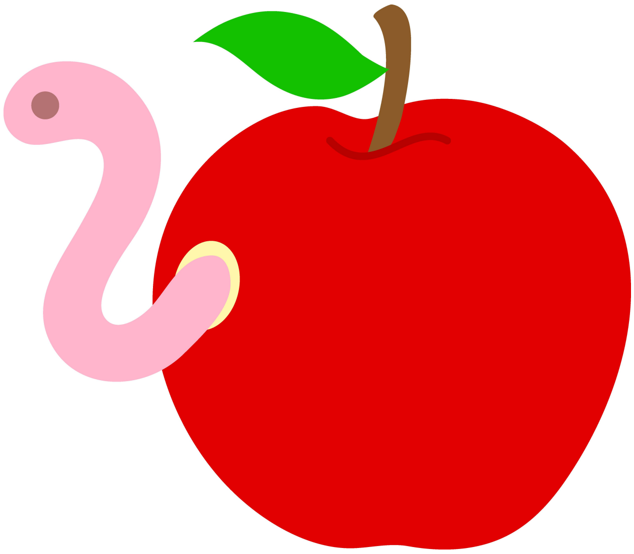


$$
\begin{aligned}
V &= \int_{-\pi/3}^{\pi/3} \int_{\frac{1}{2}\sec\theta}^{1} \int_{0}^{12} r \, dz \, dr \, d\theta \\
&= \int_{-\pi/3}^{\pi/3} \int_{\frac{1}{2}\sec\theta}^{1} 12r \, dr \, d\theta
\end{aligned}
$$

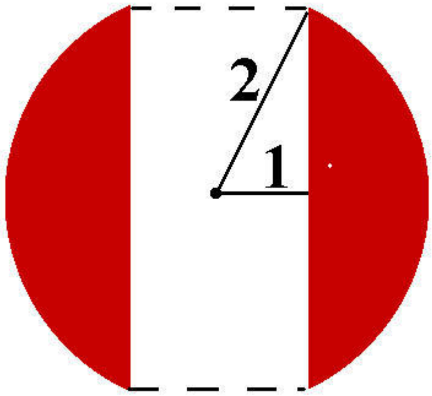
$$
\begin{aligned}
V &= \int_{-\pi/3}^{\pi/3} \int_{\frac{1}{2}\sec\theta}^{1} \int_{0}^{12} r \, dz \, dr \, d\theta \\
&= \int_{-\pi/3}^{\pi/3} \int_{\frac{1}{2}\sec\theta}^{1} 12r \, dr \, d\theta \\
&= \int_{-\pi/3}^{\pi/3} \left(6 - \frac{3}{2}\sec^2\theta \right) \, d\theta
\end{aligned}
$$

$$
\begin{aligned}
V &= \int_{-\pi/3}^{\pi/3}\int_{\frac{1}{2}\sec\theta}^{1}\int_{0}^{12} r\,dz\,dr\,d\theta \\
&= \int_{-\pi/3}^{\pi/3}\int_{\frac{1}{2}\sec\theta}^{1} 12r\,dr\,d\theta \\
&= \int_{-\pi/3}^{\pi/3}\left(6-\frac{3}{2}\sec^2\theta\right)\,d\theta \\
&= 4\pi-3\sqrt{3}
\end{aligned}
$$

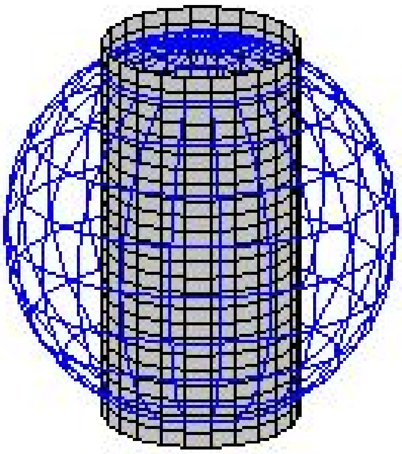
A worm eats through an apple



Sphere radius = 2 inches. Hole radius = 1 inch.
How much volume is left?

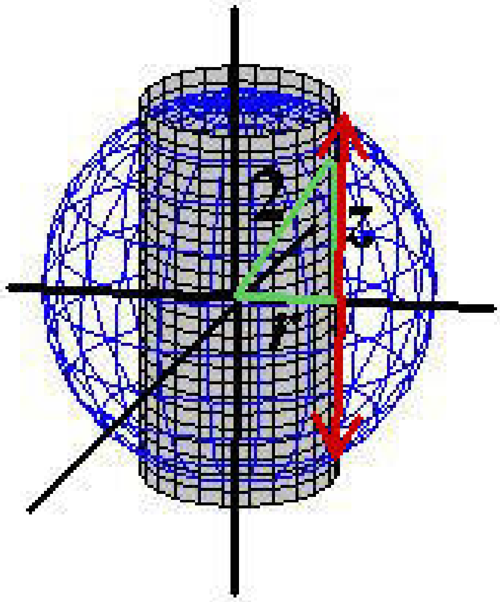


$$V = \int\!\!\int\!\!\int r\, dz\, d\theta\, dr$$

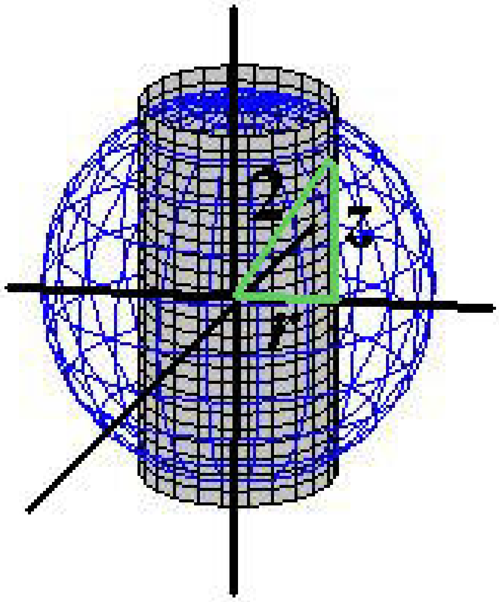


$$V = \iiint r\,dz\,d\theta\,dr$$

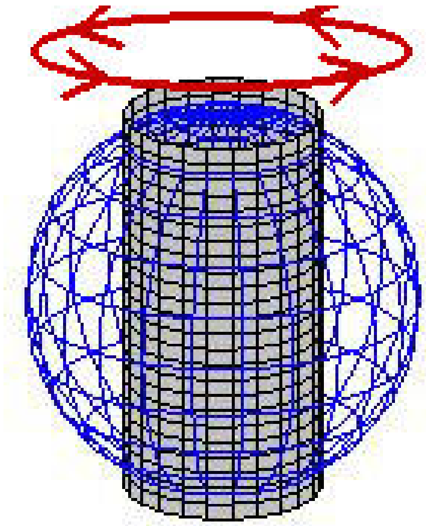
$$r^2 + z^2 = 4 \quad \text{so} \quad z = \pm\sqrt{4 - r^2}$$



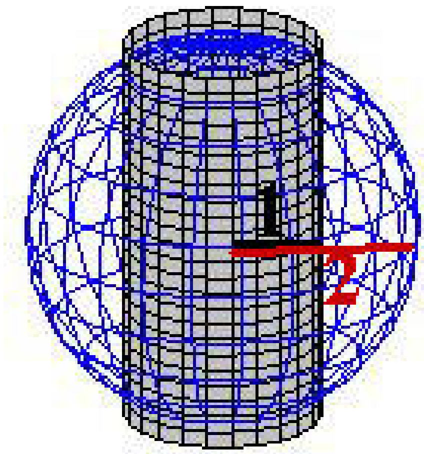
$$V = \int\int\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r\, dz\, d\theta\, dr$$



$$V = \iint_0^{2\pi} \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r\,dz\,d\theta\,dr$$



$$V = \int_1^2 \int_0^{2\pi} \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r \, dz \, d\theta \, dr$$



$$
\begin{aligned}
V &= \int_1^2 \int_0^{2\pi} \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r \, dz \, d\theta \, dr \\
&= \int_1^2 \int_0^{2\pi} 2r\sqrt{4-r^2} \, d\theta \, dr
\end{aligned}
$$

$$
\begin{aligned}
V &= \int_1^2 \int_0^{2\pi} \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r\, dz\, d\theta\, dr \\
&= \int_1^2 \int_0^{2\pi} 2r\sqrt{4-r^2}\, d\theta\, dr \\
&= \int_1^2 4\pi r\sqrt{4-r^2}\, dr
\end{aligned}
$$

$$
\begin{aligned}
V &= \int_1^2 \int_0^{2\pi} \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r \, dz \, d\theta \, dr \\
&= \int_1^2 \int_0^{2\pi} 2r\sqrt{4-r^2} \, d\theta \, dr \\
&= \int_1^2 4\pi r\sqrt{4-r^2} \, dr \\
&= 4\pi\sqrt{3}
\end{aligned}
$$