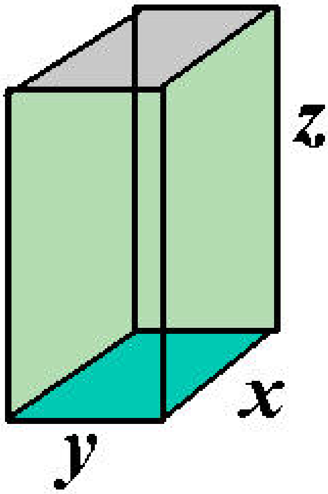
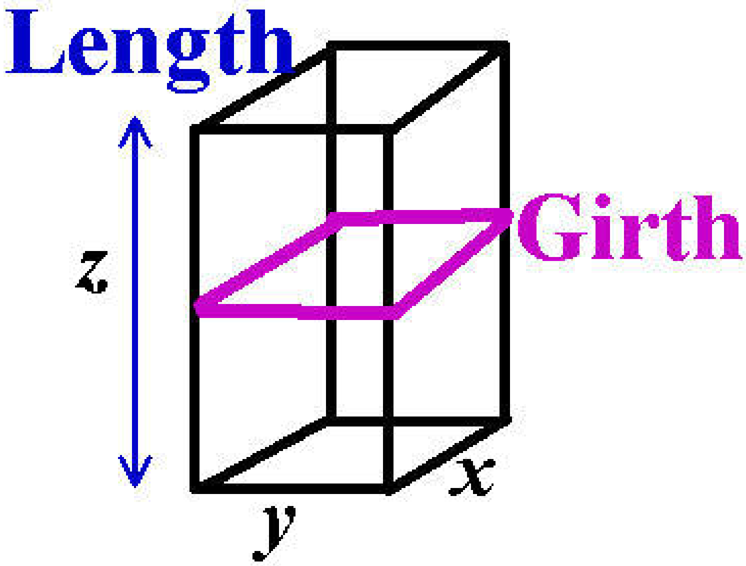
Max-min examples



Card	• Min: 3 ½" x 5 ½" • Max: 4 ¼" x 6"	Must be a flat surface
Letter	• Min: 3 ½" x 5 ½" • Max: 6 ⅛" x 11 ½"	Maximum weight is 3.5 oz.
Flat	• Min: 6 ⅛" x 11 ½" • Max: 12" x 15"	Maximum weight of 13 oz.
Parcel	Maximum size is 108" in combined length and girth (distance around the thickest part). First-Class™ packages cannot exceed 22" x 18" x 15".	Contents for Priority Mail Express, Priority Mail, or Media Mail® must weigh less than 70 lbs. First-Class™ packages must weigh less than 16 oz.

The U. S. Postal Service will not accept a rectangular box if the sum of its length and girth is more than 108 inches.



Maximize $V = xyz$

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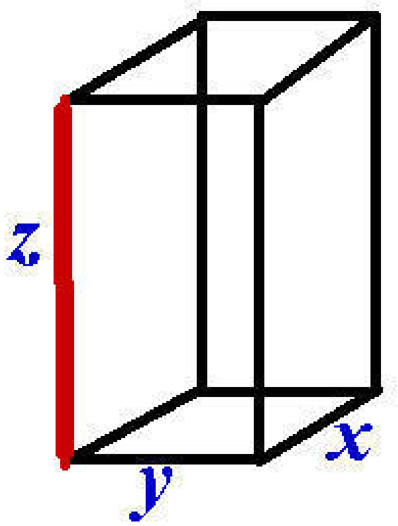

Dimensions : z

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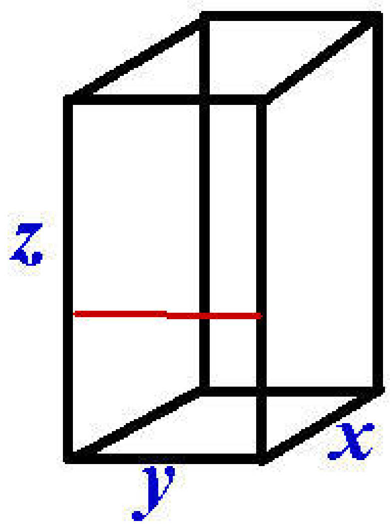
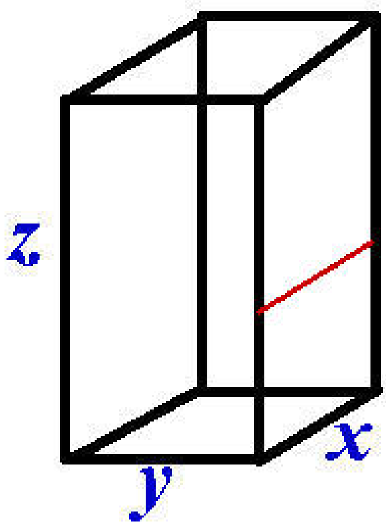


Dimensions : $z + y$

The U. S. Postal Service will not accept a rectangular box if the sum of its length and girth is more than 108 inches.



Dimensions : $z + y + x$

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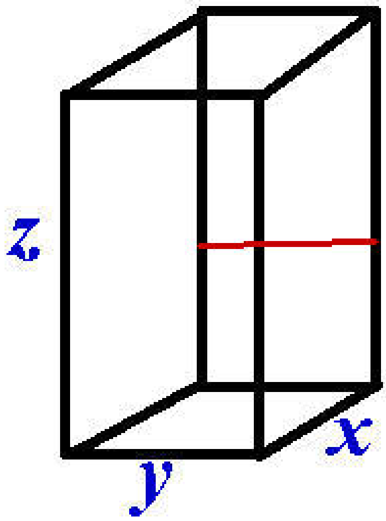
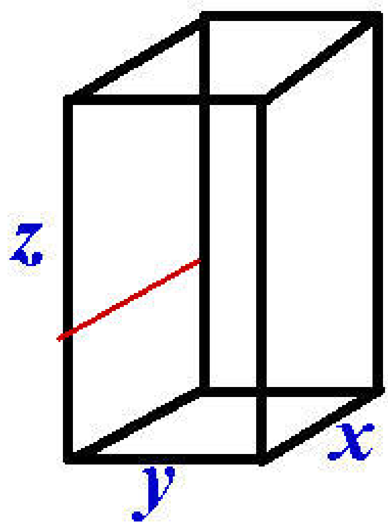


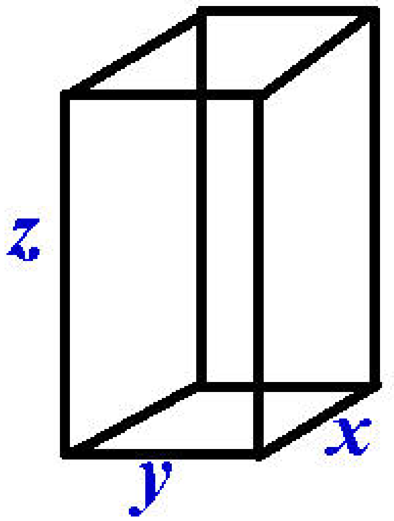
$$\text{Dimensions} : z + y + x + y$$

The U. S. Postal Service will not accept a rectangular box if the sum of its length and girth is more than 108 inches.



Dimensions : $z + y + x + y + x$

The U. S. Postal Service will not accept a rectangular box if the sum of its length and girth is more than 108 inches.



Constraint: $z + 2y + 2x = 108$

If $z + 2y + 2x = 108$ then $z = 108 - 2x - 2y$

$$V = xyz = xy(108 - 2x - 2y)$$

$$V = xy(108 - 2x - 2y) = 108xy - 2x^2y - 2xy^2$$

$$\frac{\partial V}{\partial x} = 108y - 4xy - 2y^2 \text{ and } \frac{\partial V}{\partial y} = 108x - 2x^2 - 4xy$$

Solve for x and y so that $\frac{\partial V}{\partial x} = 0$ and $\frac{\partial V}{\partial y} = 0$

$$108y - 4xy - 2y^2 = 0 \qquad 108x - 2x^2 - 4xy = 0$$

$$108y - 4xy - 2y^2 = 0 \qquad 108x - 2x^2 - 4xy = 0$$

$$54 - 2x - y = 0 \qquad 54 - x - 2y = 0$$

Solve for x and y

$$108y - 4xy - 2y^2 = 0 \qquad 108x - 2x^2 - 4xy = 0$$

$$54 - 2x - y = 0 \qquad 54 - x - 2y = 0$$

Solve for x and y

$x = 18 \qquad y = 18 \qquad z = 108 - 2x - 2y = 36$ inches

$$\frac{\partial V}{\partial x} = 108y - 4xy - 2y^2 \qquad \frac{\partial V}{\partial y} = 108x - 2x^2 - 4xy$$

$$\frac{\partial^2 V}{\partial x^2} = -4y \qquad \frac{\partial^2 V}{\partial y^2} = -4x$$

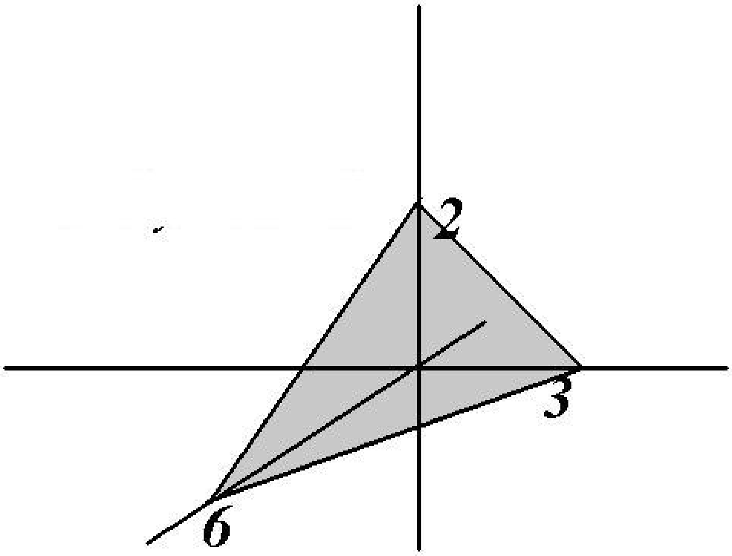
$$\frac{\partial V}{\partial x} = 108y - 4xy - 2y^2 \qquad \frac{\partial V}{\partial y} = 108x - 2x^2 - 4xy$$

$$V_{xx} = -4y \qquad V_{yy} = -4x \qquad V_{xy} = 108 - 4x - 4y$$

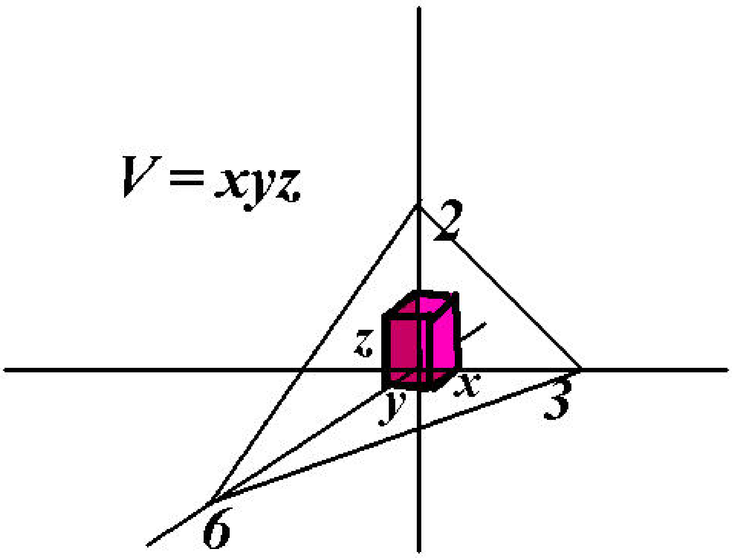
If $x = 18$ and $y = 18$, the Hessian is:

$$\begin{aligned} \mathcal{H} &= V_{xx}V_{yy} - V_{xy}^2 = (-4)(18)(-4)(18) - (-36)^2 \\ &= 2^4 3^5 > 0 \end{aligned}$$

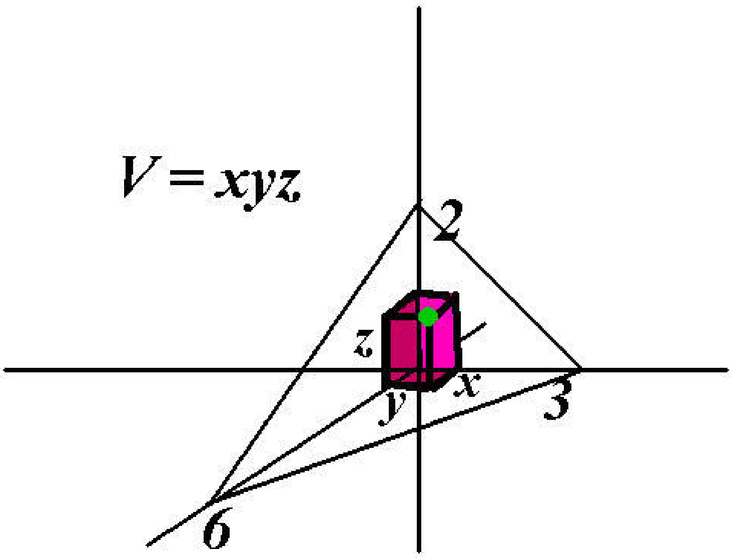
Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane: $x + 2y + 3z = 6$



Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane: $x + 2y + 3z = 6$



Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane: $x + 2y + 3z = 6$



If $x + 2y + 3z = 6$ then $z = 2 - \frac{1}{3}x - \frac{2}{3}y$

$$V = xyz = xy\left(2 - \frac{1}{3}x - \frac{2}{3}y\right)$$

$$V = 2xy - \frac{1}{3}x^2y - \frac{2}{3}xy^2$$

$$V = 2xy - \frac{1}{3}x^2y - \frac{2}{3}xy^2$$

$$\frac{\partial V}{\partial x} = 2y - \frac{2}{3}xy - \frac{2}{3}y^2$$

$$\frac{\partial V}{\partial y} = 2x - \frac{1}{3}x^2 - \frac{4}{3}xy$$

$$V = 2xy - \frac{1}{3}x^2y - \frac{2}{3}xy^2$$

$$\frac{\partial V}{\partial x} = 2y - \frac{2}{3}xy - \frac{2}{3}y^2 = \frac{2}{3}y\,(3 - x - y) = 0$$

$$\frac{\partial V}{\partial y} = 2x - \frac{1}{3}x^2 - \frac{4}{3}xy = \frac{1}{3}x(6 - x - 4y) = 0$$

$$V = 2xy - \frac{1}{3}x^2y - \frac{2}{3}xy^2$$

$$\frac{\partial V}{\partial x} = 2y - \frac{2}{3}xy - \frac{2}{3}y^2 = \frac{2}{3}y\,(3 - x - y) = 0$$

$$\frac{\partial V}{\partial y} = 2x - \frac{1}{3}x^2 - \frac{4}{3}xy = \frac{1}{3}x(6 - x - 4y) = 0$$

$$3 - x - y = 0 \qquad 6 - x - 4y = 0$$

Solve for x and y:

$$x = 2 \qquad y = 1 \qquad z = 2 - \frac{1}{3}x - \frac{2}{3}y = \frac{2}{3}$$

$$V_x = 2y - \frac{2}{3}xy - \frac{2}{3}y^2 \qquad V_y = 2x - \frac{1}{3}x^2 - \frac{4}{3}xy$$

$$V_{xx} = -\frac{2}{3}y \qquad V_{yy} = -\frac{4}{3}x \qquad V_{xy} = 2 - \frac{2}{3}x - \frac{4}{3}y$$

$$V_x = 2y - \frac{2}{3}xy - \frac{2}{3}y^2 \qquad V_y = 2x - \frac{1}{3}x^2 - \frac{4}{3}xy$$

$$V_{xx} = -\frac{2}{3}y \qquad V_{yy} = -\frac{4}{3}x \qquad V_{xy} = 2 - \frac{2}{3}x - \frac{4}{3}y$$

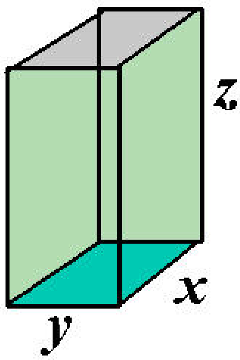
$$\mathcal{H} = V_{xx}V_{yy} - V_{xy}^2 = \frac{8}{9}xy - \left(2 - \frac{2}{3}x - \frac{4}{3}y\right)^2$$

At $x = 2$ and $y = 1$, $\mathcal{H} = \frac{4}{3} > 0$
and $V_{max} = 2xy - \frac{1}{3}x^2y - \frac{2}{3}xy^2 = \frac{4}{3}$

A box with no lid is to be assembled from rectangular pieces of metal and is to have a volume of 9 cubic meters.

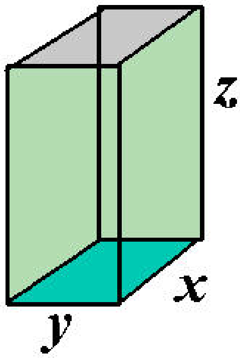
The base costs 2 dollars per square meter.
The 4 sides cost 3 dollars per square meter.

Find the dimensions of the most economical box.



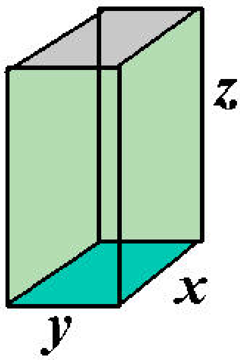
The base costs 2 dollars per square meter.
The 4 sides cost 3 dollars per square meter.

$$\text{Total Cost} = \begin{pmatrix} \text{cost of} \\ \text{sides} \end{pmatrix} + \begin{pmatrix} \text{cost of} \\ \text{bottom} \end{pmatrix}$$



$$\text{Total Cost} = \begin{pmatrix} \text{cost of} \\ \text{sides} \end{pmatrix} + \begin{pmatrix} \text{cost of} \\ \text{bottom} \end{pmatrix}$$

$$T = 3(yz + yz + xz + xz) + 2(xy)$$



$$\text{Total Cost} = \begin{pmatrix} \text{cost of} \\ \text{sides} \end{pmatrix} + \begin{pmatrix} \text{cost of} \\ \text{bottom} \end{pmatrix}$$

$$\begin{aligned} T &= 3(yz + yz + xz + xz) + 2(xy) \\ &= 6z(y + x) + 2xy \end{aligned}$$

If $V = xyz = 9$ then $z = \frac{9}{xy}$

$$\text{Total Cost} = \begin{pmatrix} \text{cost of} \\ \text{sides} \end{pmatrix} + \begin{pmatrix} \text{cost of} \\ \text{bottom} \end{pmatrix}$$

$$\begin{aligned} T &= 3(yz + yz + xz + xz) + 2(xy) \\ &= 6z(y + x) + 2xy \end{aligned}$$

If $V = xyz = 9$ then $z = \frac{9}{xy}$

$$T = 6 \cdot \frac{9}{xy}(y + x) + 2xy$$

$$T = \frac{54}{x} + \frac{54}{y} + 2xy$$

$$T = \frac{54}{x} + \frac{54}{y} + 2xy$$

$$\frac{\partial T}{\partial x} = -\frac{54}{x^2} + 2y \qquad \frac{\partial T}{\partial y} = -\frac{54}{y^2} + 2x$$

$$T = \frac{54}{x} + \frac{54}{y} + 2xy$$

$$\frac{\partial T}{\partial x} = -\frac{54}{x^2} + 2y = 0 \qquad \frac{\partial T}{\partial y} = -\frac{54}{y^2} + 2x = 0$$

$$x = 3 \qquad y = 3 \qquad z = \frac{9}{xy} = 1$$

$$T = \frac{54}{x} + \frac{54}{y} + 2xy$$

$$\frac{\partial T}{\partial x} = -\frac{54}{x^2} + 2y = 0 \qquad \frac{\partial T}{\partial y} = -\frac{54}{y^2} + 2x = 0$$

$$x = 3 \qquad y = 3 \qquad z = \frac{9}{xy} = 1$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{108}{x^3} > 0 \qquad \frac{\partial^2 T}{\partial y^2} = \frac{108}{y^3} > 0$$

$$T = \frac{54}{x} + \frac{54}{y} + 2xy$$

$$\frac{\partial T}{\partial x} = -\frac{54}{x^2} + 2y = 0 \qquad \frac{\partial T}{\partial y} = -\frac{54}{y^2} + 2x = 0$$

$$x = 3 \qquad y = 3 \qquad z = \frac{9}{xy} = 1$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{108}{x^3} > 0 \qquad \frac{\partial^2 T}{\partial y^2} = \frac{108}{y^3} > 0$$

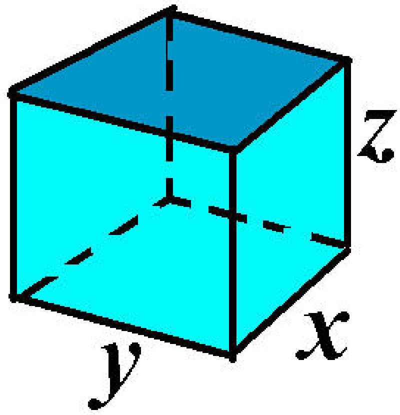
$$\frac{\partial^2 T}{\partial x \partial y} = 2$$

$$T_{xx} = \frac{108}{x^3} \qquad T_{yy} = \frac{108}{y^3} \qquad T_{xy} = 2$$

At $x = 3$ and $y = 3$ the Hessian is:

$$\mathcal{H} = T_{xx}T_{yy} - T_{xy}^2 = \frac{108}{3^3} \cdot \frac{108}{3^3} - 2^2 = 12 > 0$$

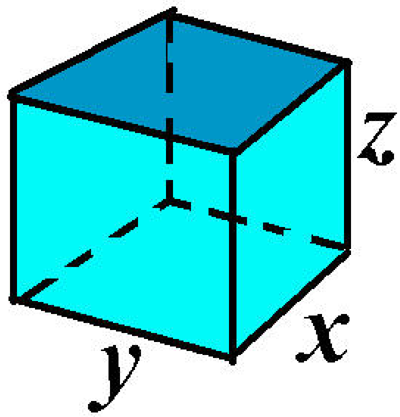
Let's assume a house is a rectangular solid



The glass walls admit heat at 3 units/min per ft^2
The roof admits heat at 2 units/min per ft^2
The floor admits heat at 1 unit/min per ft^2

The glass walls admit heat at 3 units/min per ft^2
The roof admits heat at 2 units/min per ft^2
The floor admits heat at 1 unit/min per ft^2
Total heat per minute is:
Through Walls $= 3yz + 3yz + 3xz + 3xz = 6yz + 6xz$
Through Roof $= 2xy$ Through Floor $= xy$

$$\text{Total} = 6yz + 6xz + 3xy$$



$$xyz = V \quad \text{so} \quad z = \frac{V}{xy}$$

Let us assume that V is 32,000 cubic feet.

$$\begin{aligned} \text{Total heat per min} &= 6yz + 6xz + 3xy \\ &= \frac{6V}{x} + \frac{6V}{y} + 3xy \end{aligned}$$

Find the dimensions of the house that minimize the amount of heat admitted to the house per minute.

$$f(x, y) = \frac{6V}{x} + \frac{6V}{y} + 3xy$$

$$f(x,y) = \frac{6V}{x} + \frac{6V}{y} + 3xy$$

$$f_x = -\frac{6V}{x^2} + 3y \qquad f_y = -\frac{6V}{y^2} + 3x$$

Set f_x and f_y equal to 0

$$f(x,y) = \frac{6V}{x} + \frac{6V}{y} + 3xy$$

$$f_x = -\frac{6V}{x^2} + 3y \qquad f_y = -\frac{6V}{y^2} + 3x$$

If $f_x = 0$ then $6V = 3x^2y$
If $f_y = 0$ then $6V = 3xy^2$

$$f(x,y) = \frac{6V}{x} + \frac{6V}{y} + 3xy$$

$$f_x = -\frac{6V}{x^2} + 3y \qquad f_y = -\frac{6V}{y^2} + 3x$$

If $f_x = 0$ then $6V = 3x^2y$
If $f_y = 0$ then $6V = 3xy^2$

$$3x^2y = 3xy^2$$

$$x = y$$

$$6V = 3x^2 y = 3x^2 \cdot x$$

$$6V = 3x^3$$

$$x = (2V)^{1/3} \qquad y = (2V)^{1/3}$$

$$6V = 3x^2 y = 3x^2 \cdot x$$

$$6V = 3x^3$$

$$x = (2V)^{1/3} \qquad y = (2V)^{1/3}$$

If $V = 32,000$ then:

$$x = 40 \qquad y = 40$$

$$6V = 3x^2y = 3x^2 \cdot x$$

$$6V = 3x^3$$

$$x = (2V)^{1/3} \qquad y = (2V)^{1/3}$$

If $V = 32,000$ then:

$$x = 40 \qquad y = 40$$

If $\frac{\partial f}{\partial x} = -\frac{6V}{x^2} + 3y$ then:

$$\frac{\partial^2 f}{\partial x^2} = \frac{12V}{x^3} > 0$$

$$f_x = -\frac{6V}{x^2} + 3y \qquad f_y = -\frac{6V}{y^2} + 3x$$

$$f_{xx} = \frac{12V}{x^3} \qquad f_{yy} = \frac{12V}{y^3} \qquad f_{xy} = 3$$

$$\mathcal{H} = f_{xx}f_{yy} - f_{xy}^2 = \frac{144V^2}{x^3y^3} - 9 = 27$$