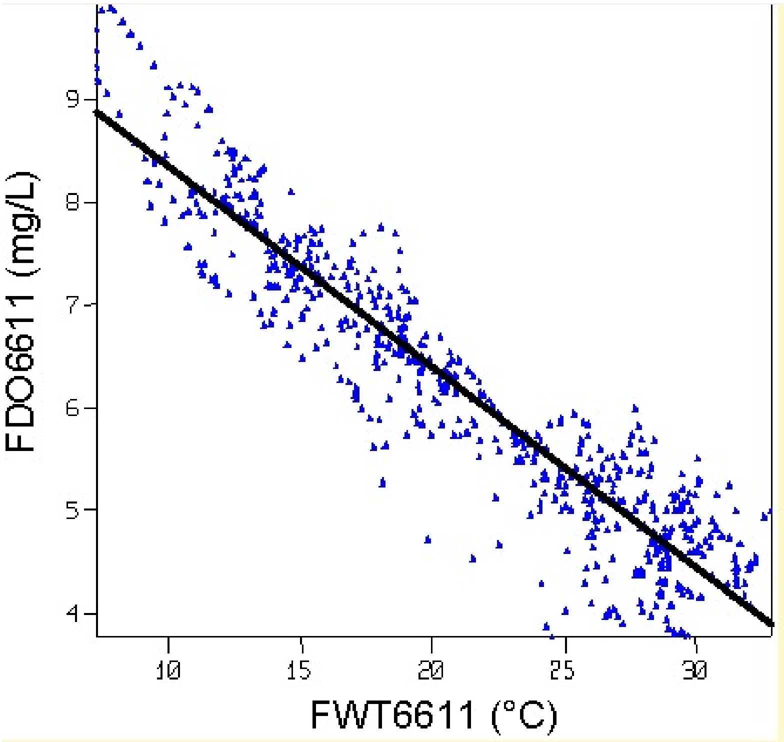
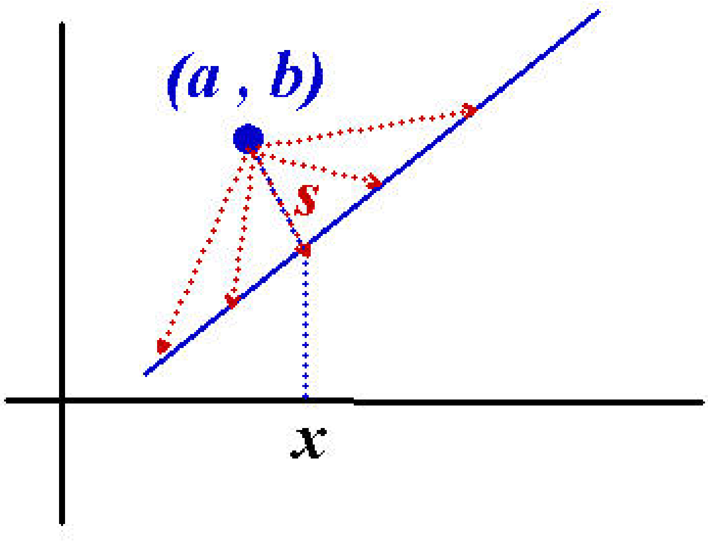
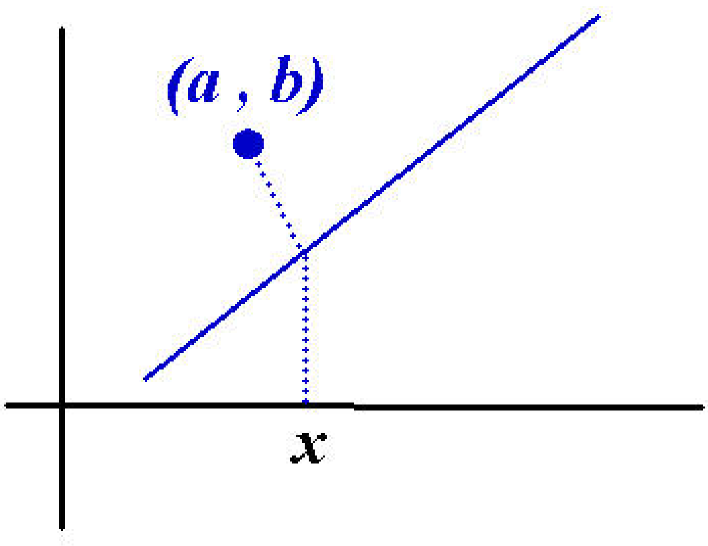
More max-min examples



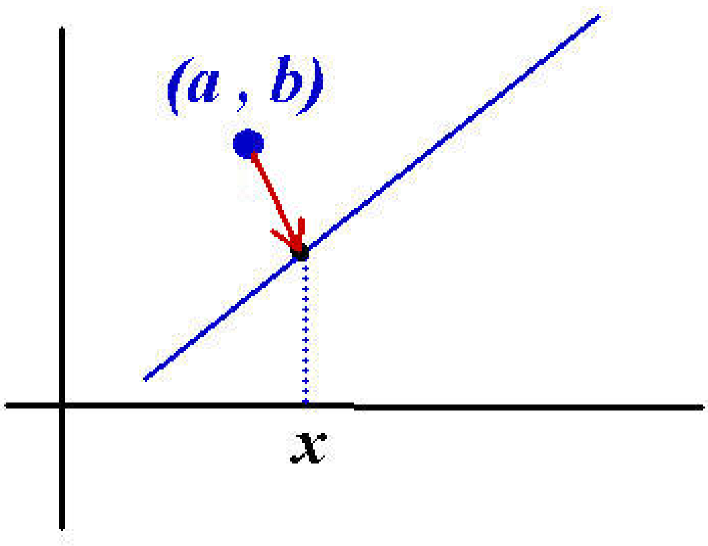
Distance from a point $(a,\ b)$ to a line



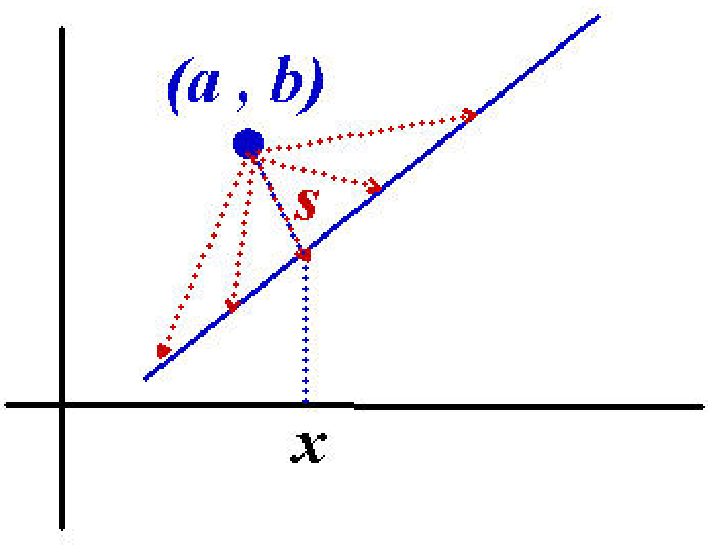
Distance from a point $(a,\ b)$ to a line



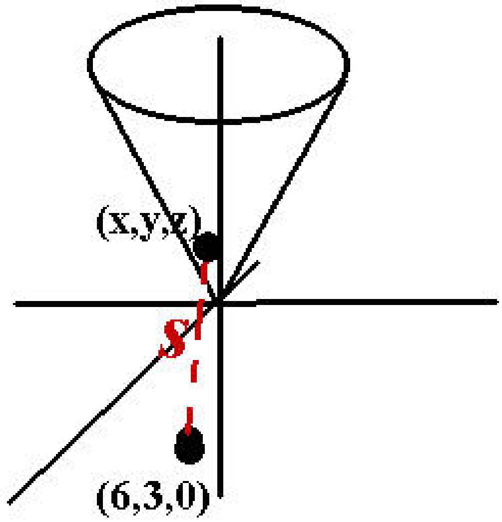
Distance from a point $(a,\ b)$ to a line



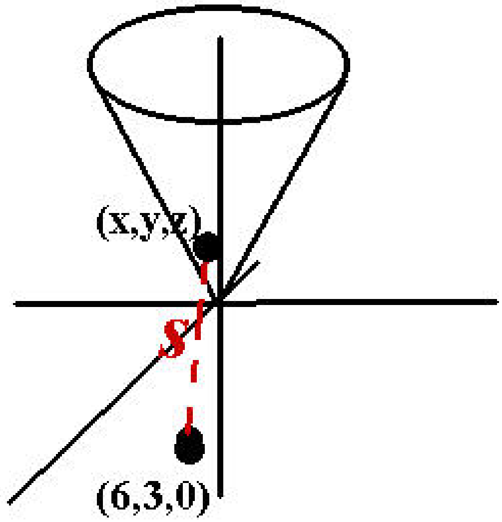
Minimize $s = \sqrt{(a-x)^2 + (b-y)^2}$



Find the point on the cone $z = \sqrt{2x^2 + 2y^2}$ that is closest to $(6,\ 3,\ 0)$



Find the point on the cone $z = \sqrt{2x^2 + 2y^2}$ that is closest to $(6,\ 3,\ 0)$



Minimize distance: $s = \sqrt{(x-6)^2 + (y-3)^2 + z^2}$

$$z = \sqrt{2x^2 + 2y^2}$$

$$z^2 = 2x^2 + 2y^2$$

Define $G(x, y, z) = 2x^2 + 2y^2 - z^2$

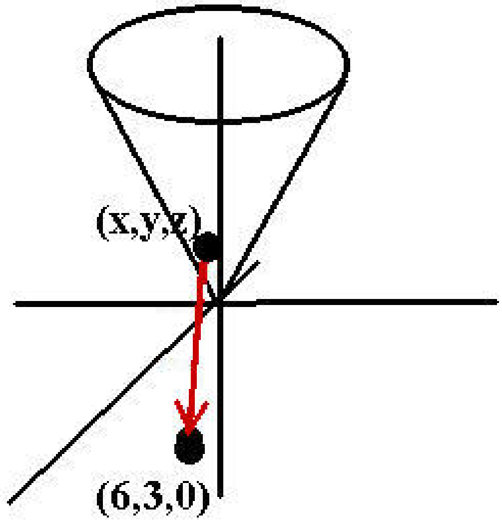
The cone consists of the points (x, y, z) for which $G(x, y, z) = 0$

If a surface is described by an equation of the form $G(x, y, z) = C$ then $\nabla G = \left\langle \frac{\partial G}{\partial x}, \ \frac{\partial G}{\partial y}, \ \frac{\partial G}{\partial z} \right\rangle$ is perpendicular to this surface.

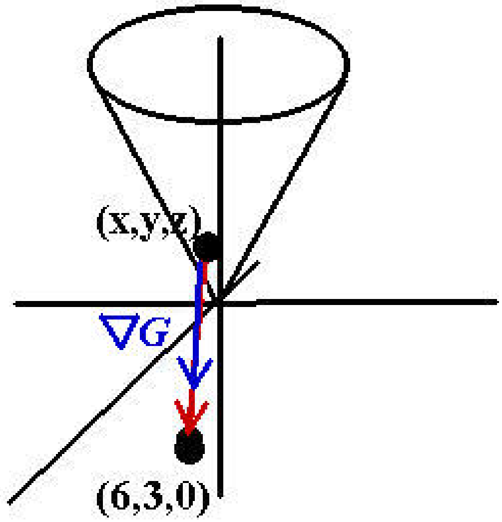
$$G(x, y, z) = 2x^2 + 2y^2 - z^2$$

$$\nabla G = \langle 4x, \ 4y, \ -2z \rangle$$

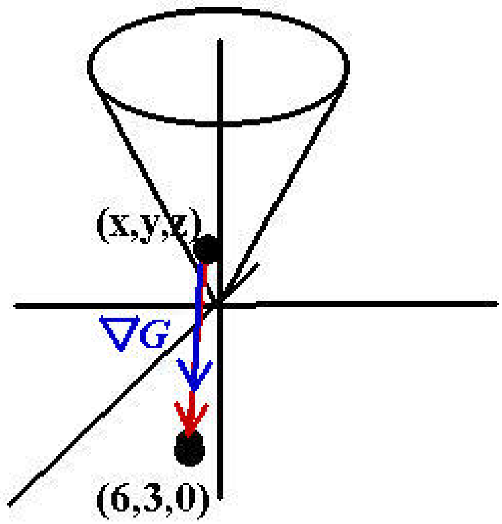
If (x, y, z) is the point on the cone that is closest to $(6,\ 3,\ 0)$ then $\langle 6 - x,\ 3 - y,\ 0 - z \rangle$ is perpendicular to the cone.



At this point, ∇G is also perpendicular to the cone



$$\langle 6-x,\ 3-y,\ 0-z\rangle = k\nabla G$$



$$\langle 6 - x,\ 3 - y,\ 0 - z\rangle = k\nabla G$$

$$\langle 6 - x,\ 3 - y,\ 0 - z\rangle = k\langle 4x,\ 4y,\ -2z\rangle$$

$$\langle 6-x,\ 3-y,\ 0-z\rangle = k\nabla G$$

$$\langle 6-x,\ 3-y,\ 0-z\rangle = \langle 4kx,\ 4ky,\ -2kz\rangle$$

$$6-x = 4kx \qquad 3-y = 4ky \qquad -z = -2kz$$

$-z = -2kz$ implies that $k = \frac{1}{2}$

$$\langle 6-x,\ 3-y,\ 0-z\rangle = k\nabla G$$

$$\langle 6-x,\ 3-y,\ 0-z\rangle = \langle 4kx,\ 4ky,\ -2kz\rangle$$

$$6-x=4kx \qquad 3-y=4ky \qquad -z=-2kz$$

$-z=-2kz$ implies that $k=\frac{1}{2}$

$$6-x=2x \qquad 3-y=2y$$

$$\langle 6-x,\ 3-y,\ 0-z\rangle = k\nabla G$$

$$\langle 6-x,\ 3-y,\ 0-z\rangle = \langle 4kx,\ 4ky,\ -2kz\rangle$$

$$6-x = 4kx \qquad 3-y = 4ky \qquad -z = -2kz$$

$-z = -2kz$ implies that $k = \frac{1}{2}$

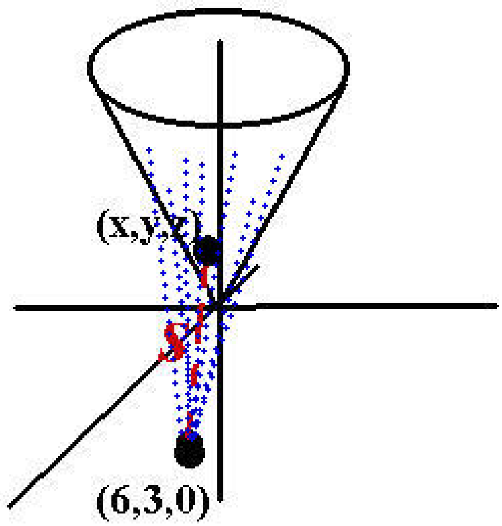
$$6-x = 2x \qquad 3-y = 2y$$

$$x = 2 \qquad y = 1$$

$$z = \sqrt{2x^2+2y^2} = \sqrt{10}$$

$$z = \sqrt{2x^2 + 2y^2}$$

$$\begin{aligned} s &= \sqrt{(x-6)^2 + (y-3)^2 + z^2} \\ &= \sqrt{(x-6)^2 + (y-3)^2 + 2x^2 + 2y^2} \end{aligned}$$



$$s = \sqrt{(x-6)^2 + (y-3)^2 + 2x^2 + 2y^2}$$

$$s^2 = (x-6)^2 + (y-3)^2 + 2x^2 + 2y^2$$

$$2s\frac{\partial s}{\partial x} = 2(x-6) + 4x = 6x - 12$$

$$2s\frac{\partial s}{\partial y} = 2(y-3) + 4y = 6y - 6$$

Set $\frac{\partial s}{\partial x} = 0$ and $\frac{\partial s}{\partial y} = 0$

$$2s\frac{\partial s}{\partial x} = 6x - 12 \qquad 2s\frac{\partial s}{\partial y} = 6y - 6$$

Set $\frac{\partial s}{\partial x} = 0$ and $\frac{\partial s}{\partial y} = 0$

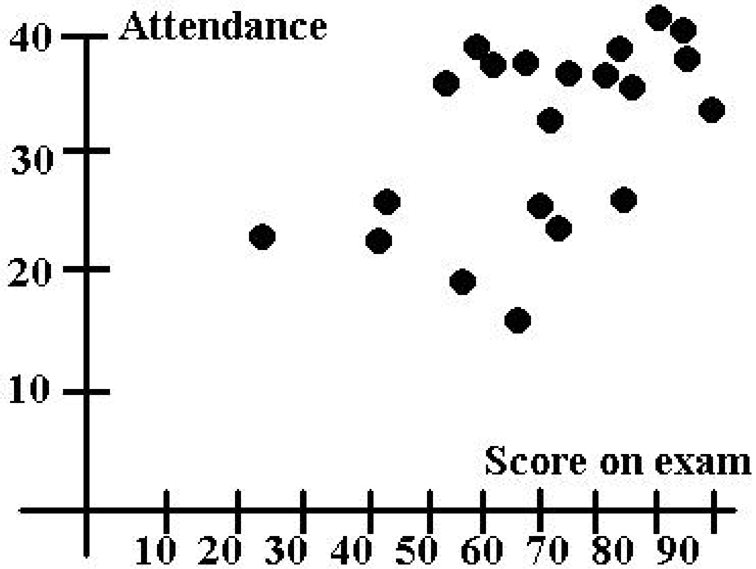
$$6x - 12 = 0 \qquad 6y - 6 = 0$$

$$x = 2 \qquad y = 1$$

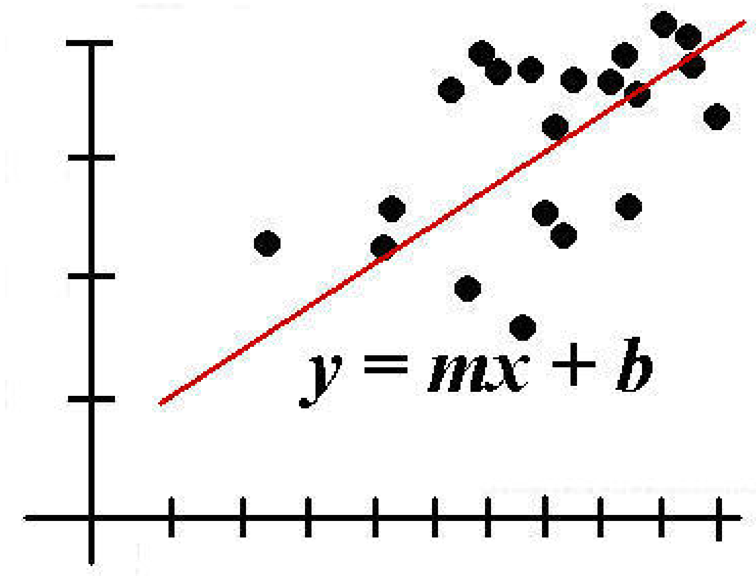
$$z = \sqrt{2x^2 + 2y^2} = \sqrt{10}$$

$(2,\ 1,\ \sqrt{10})$ is the point on the cone that is closest to $(6,\ 3,\ 0)$

Scores vs. Attendance



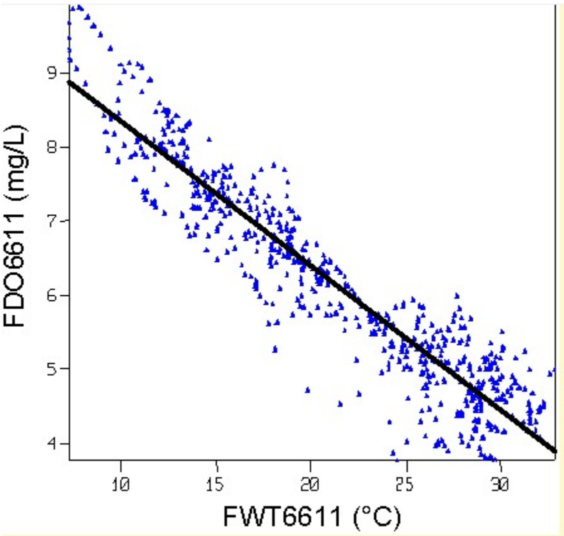
Straight Line Approx



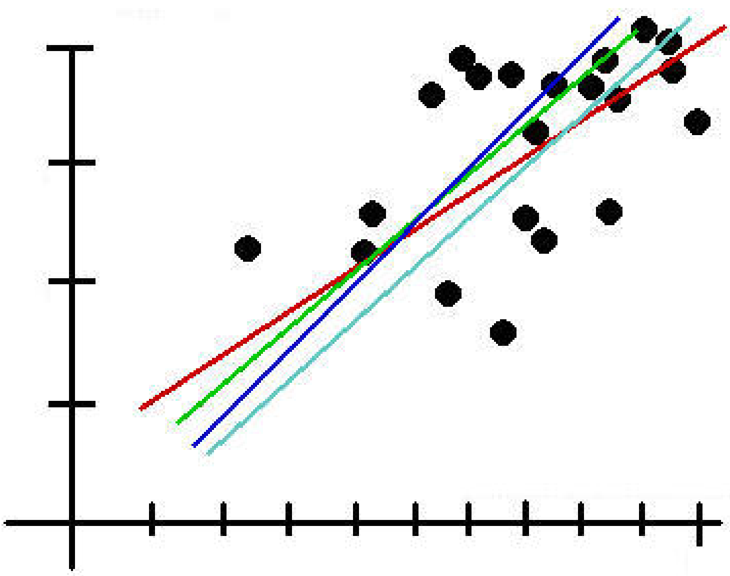
Amazon Prime:

Amazon Prime:

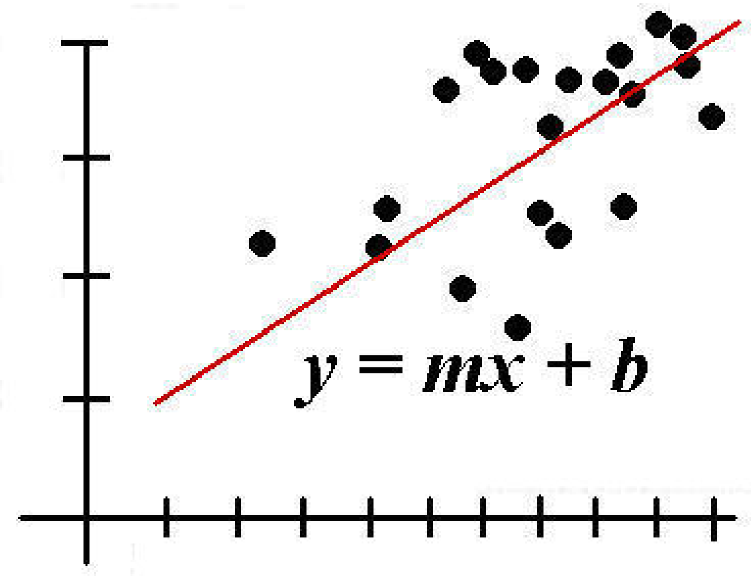
Dissolved Oxygen vs. Water Temperature for Beaufort River in South Carolina



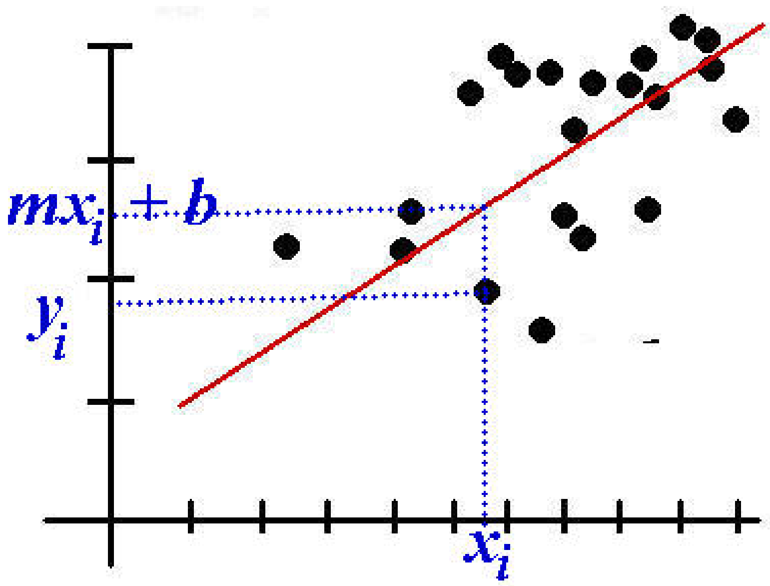
Many Possible Lines



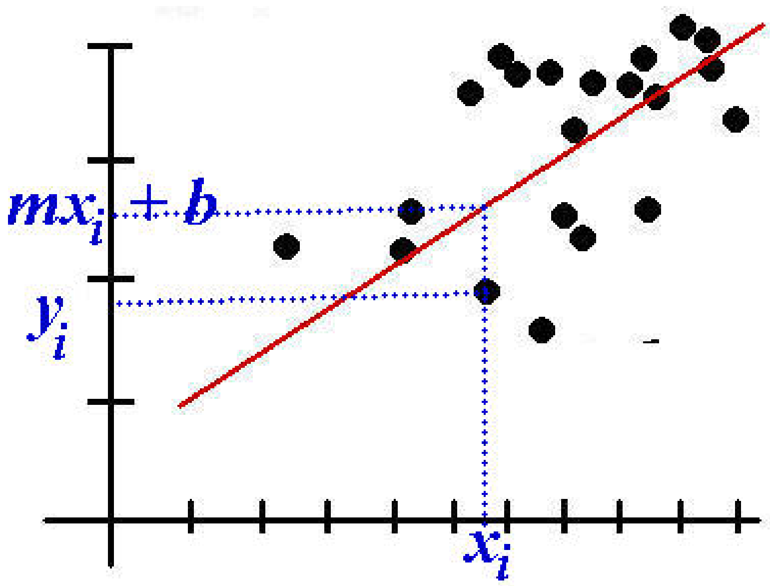
Find m and b



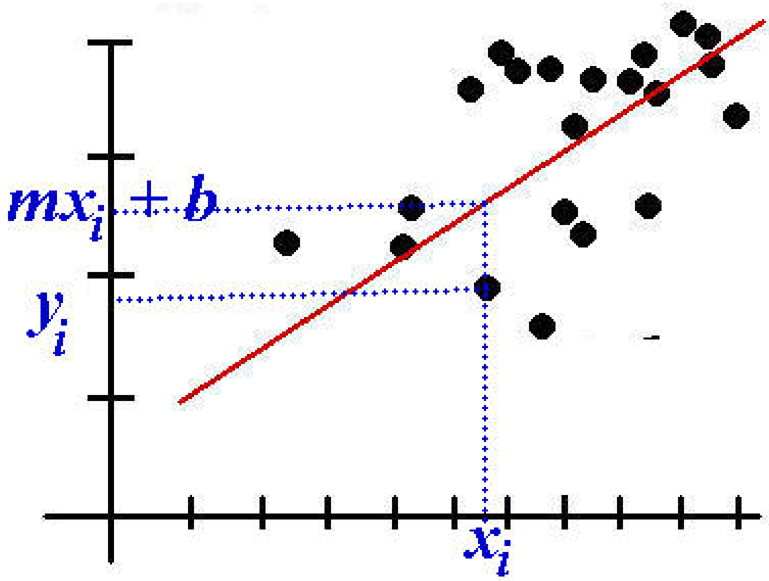
$$\text{Error} = mx_i + b - y_i$$



$$\text{Square of the Error} = (mx_i + b - y_i)^2$$



$$\text{Sum of the Squares} = \sum_{i=1}^{n}(mx_i + b - y_i)^2$$



$$f(m,\ b) = \sum_{i=1}^{n} (mx_i + b - y_i)^2$$

To minimize, set $\frac{\partial f}{\partial m}$ and $\frac{\partial f}{\partial b}$ to 0

$$f(m,\ b) = \sum_{i=1}^{n} (mx_i + b - y_i)^2$$

$$\frac{\partial f}{\partial m} = \sum_{i=1}^{n} 2(mx_i + b - y_i)x_i$$

$$f(m,\ b) = \sum_{i=1}^{n} (mx_i + b - y_i)^2$$

$$\frac{\partial f}{\partial m} = \sum_{i=1}^{n} 2(mx_i + b - y_i)x_i$$

$$\frac{\partial f}{\partial b} = \sum_{i=1}^{n} 2(mx_i + b - y_i)$$

$$\sum_{i=1}^{n} 2(mx_i + b - y_i)x_i = 0$$

$$\sum_{i=1}^{n} 2(mx_i + b - y_i) = 0$$

$$\sum_{i=1}^{n}(mx_i + b - y_i)x_i = 0$$

$$\sum_{i=1}^{n}(mx_i + b - y_i) = 0$$

$$\sum_{i=1}^{n}(mx_i + b - y_i)x_i = 0$$

$$m\sum_{i=1}^{n}x_i^2 + b\sum_{i=1}^{n}x_i - \sum_{i=1}^{n}x_iy_i = 0$$

$$\sum_{i=1}^{n}(mx_i + b - y_i)x_i = 0$$

$$m\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_i y_i = 0$$

$$m\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i y_i$$

$$\sum_{i=1}^{n}(mx_i + b - y_i) = 0$$

$$\sum_{i=1}^{n} mx_i + \sum_{i=1}^{n} b - \sum_{i=1}^{n} y_i = 0$$

$$
\begin{aligned}
\sum_{i=1}^{n} b &= b + b + \cdots + b \quad (n \text{ times}) \\
&= bn
\end{aligned}
$$

$$\sum_{i=1}^{n}(mx_i + b - y_i) = 0$$

$$\sum_{i=1}^{n} mx_i + \sum_{i=1}^{n} b - \sum_{i=1}^{n} y_i = 0$$

$$m\sum_{i=1}^{n} x_i + bn - \sum_{i=1}^{n} y_i = 0$$

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$$m\sum_{i=1}^{n} x_i + bn - \sum_{i=1}^{n} y_i = 0$$

$$m\sum_{i=1}^{n} x_i + bn = \sum_{i=1}^{n} y_i$$

$$m\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i y_i$$

$$m\sum_{i=1}^{n} x_i + bn = \sum_{i=1}^{n} y_i$$

$$\left(\sum x_i^2\right) m + \left(\sum x_i\right) b = \sum x_i y_i$$

$$\left(\sum x_i\right) m \quad + \qquad\qquad nb = \sum y_i$$

$$AX + BY = U$$
$$CX + DY = V$$

$$X = \frac{UD - BV}{AD - BC} \qquad Y = \frac{VA - UC}{AD - BC}$$

$$AX + BY = U$$
$$CX + DY = V$$

$$X = \frac{\begin{vmatrix} U & B \\ V & D \end{vmatrix}}{\begin{vmatrix} A & B \\ C & D \end{vmatrix}} \qquad Y = \frac{\begin{vmatrix} A & U \\ C & V \end{vmatrix}}{\begin{vmatrix} A & B \\ C & D \end{vmatrix}}$$

$$\left(\sum x_i^2\right) m + \left(\sum x_i\right) b = \sum x_i y_i$$
$$\left(\sum x_i\right) m \quad + \qquad\qquad nb = \sum y_i$$

$$m = \frac{\begin{vmatrix} \sum x_i y_i & \sum x_i \\ \sum y_i & n \end{vmatrix}}{\begin{vmatrix} \sum x_i^2 & \sum x_i \\ \sum x_i & n \end{vmatrix}} \qquad b = \frac{\begin{vmatrix} \sum x_i^2 & \sum x_i y_i \\ \sum x_i & \sum y_i \end{vmatrix}}{\begin{vmatrix} \sum x_i^2 & \sum x_i \\ \sum x_i & n \end{vmatrix}}$$

Find the least squares line through the points:

$$(0,0) \qquad (1,2) \qquad (2,1) \qquad (3,2)$$

x_i	y_i
0	0
1	2
2	1
3	2

Find the least squares line through the points:

$$(0,0) \qquad (1,2) \qquad (2,1) \qquad (3,2)$$

x_i	y_i	x_i^2	$x_i y_i$
0	0	0	0
1	2	1	2
2	1	4	2
3	2	9	6

Find the least squares line through the points:

$$(0,0) \qquad (1,2) \qquad (2,1) \qquad (3,2)$$

x_i	y_i	x_i^2	$x_i y_i$
0	0	0	0
1	2	1	2
2	1	4	2
3	2	9	6
$\sum x_i = 6$	$\sum y_i = 5$	$\sum x_i^2 = 14$	$\sum x_i y_i = 10$

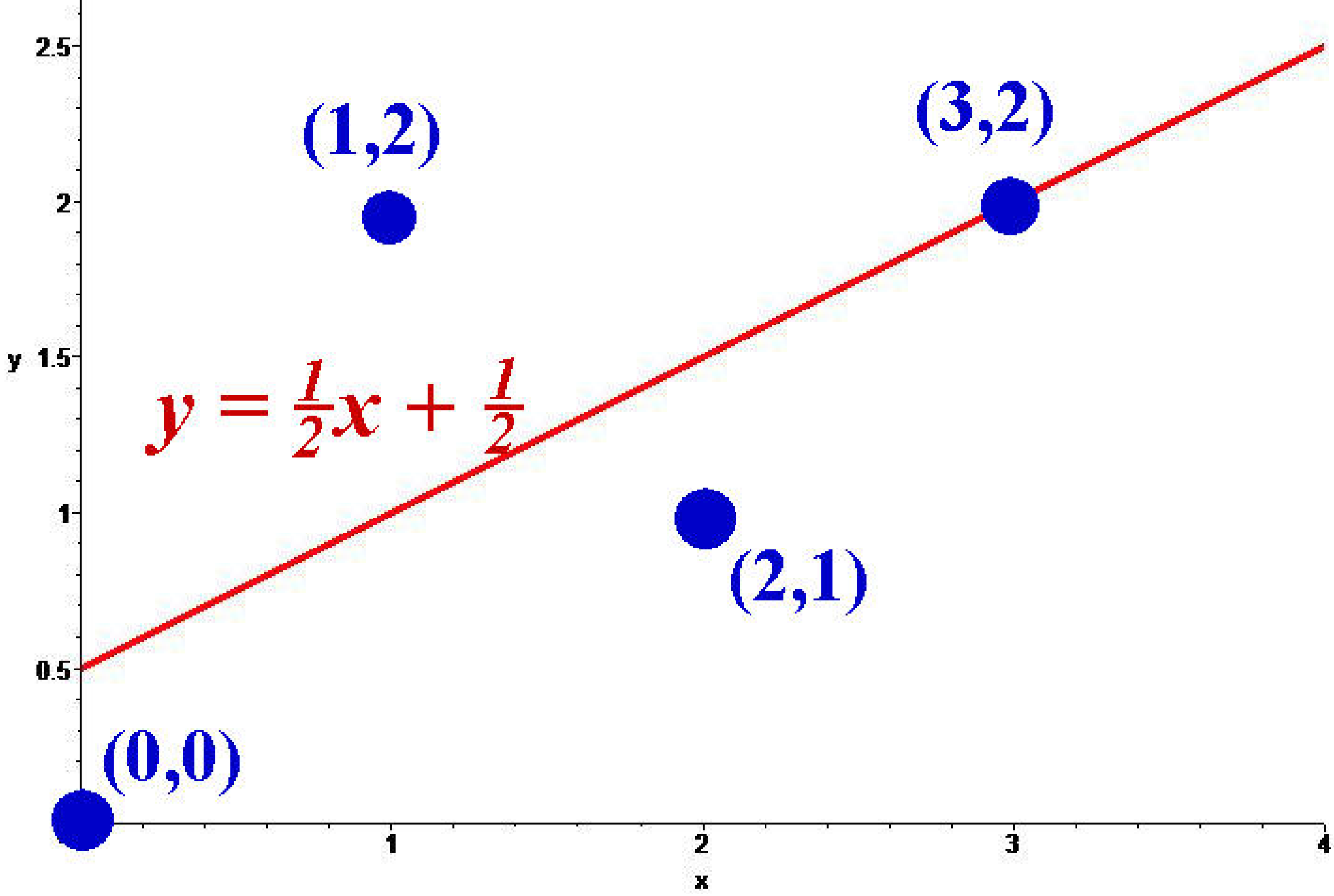
$$\sum x_i = 6 \qquad \sum y_i = 5 \qquad \sum x_i^2 = 14 \quad \sum x_i y_i = 10$$

$$m = \frac{\begin{vmatrix} \sum x_i y_i & \sum x_i \\ \sum y_i & n \end{vmatrix}}{\begin{vmatrix} \sum x_i^2 & \sum x_i \\ \sum x_i & n \end{vmatrix}} = \frac{\begin{vmatrix} 10 & 6 \\ 5 & 4 \end{vmatrix}}{\begin{vmatrix} 14 & 6 \\ 6 & 4 \end{vmatrix}} = \frac{1}{2}$$

$$\sum x_i = 6 \qquad \sum y_i = 5 \qquad \sum x_i^2 = 14 \quad \sum x_i y_i = 10$$

$$m = \frac{\begin{vmatrix} \sum x_i y_i & \sum x_i \\ \sum y_i & n \end{vmatrix}}{\begin{vmatrix} \sum x_i^2 & \sum x_i \\ \sum x_i & n \end{vmatrix}} = \frac{\begin{vmatrix} 10 & 6 \\ 5 & 4 \end{vmatrix}}{\begin{vmatrix} 14 & 6 \\ 6 & 4 \end{vmatrix}} = \frac{1}{2}$$

$$b = \frac{\begin{vmatrix} \sum x_i^2 & \sum x_i y_i \\ \sum x_i & \sum y_i \end{vmatrix}}{\begin{vmatrix} \sum x_i^2 & \sum x_i \\ \sum x_i & n \end{vmatrix}} = \frac{\begin{vmatrix} 14 & 10 \\ 6 & 5 \end{vmatrix}}{\begin{vmatrix} 14 & 6 \\ 6 & 4 \end{vmatrix}} = \frac{1}{2}$$



2.5
2
y 1.5
1
0.5
1
2
3
4
x
(1,2)
(3,2)
(2,1)
(0,0)
y = 1/2 x + 1/2

$$f(m,\ b) = \sum_{i=1}^{n} (mx_i + b - y_i)^2$$

$$\frac{\partial f}{\partial m} = \sum_{i=1}^{n} 2(mx_i{+}b{-}y_i)x_i \quad \frac{\partial f}{\partial b} = \sum_{i=1}^{n} 2(mx_i{+}b{-}y_i)$$

$$f(m,\ b) = \sum_{i=1}^{n} (mx_i + b - y_i)^2$$

$$\frac{\partial f}{\partial m} = \sum_{i=1}^{n} 2(mx_i + b - y_i)x_i \quad \frac{\partial f}{\partial b} = \sum_{i=1}^{n} 2(mx_i + b - y_i)$$

$$\frac{\partial^2 f}{\partial m^2} = \sum_{i=1}^{n} 2x_i^2 \qquad \frac{\partial^2 f}{\partial b^2} = 2n \qquad \frac{\partial^2 f}{\partial m \partial b} = \sum_{i=1}^{n} 2x_i$$

$$f(m,\ b) = \sum_{i=1}^{n} (mx_i + b - y_i)^2$$

$$\frac{\partial f}{\partial m} = \sum_{i=1}^{n} 2(mx_i{+}b{-}y_i)x_i \quad \frac{\partial f}{\partial b} = \sum_{i=1}^{n} 2(mx_i{+}b{-}y_i)$$

$$\frac{\partial^2 f}{\partial m^2} = \sum_{i=1}^{n} 2x_i^2 \qquad \frac{\partial^2 f}{\partial b^2} = 2n \qquad \frac{\partial^2 f}{\partial m \partial b} = \sum_{i=1}^{n} 2x_i$$

$$\mathcal{H} = f_{mm} f_{bb} - f_{mb}^2 = 4n \sum_{i=1}^{n} x_i^2 - 4 \left(\sum_{i=1}^{n} x_i \right)^2$$

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \mathrm{Var} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$$

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \mathrm{Var} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$$

$$\begin{aligned}
\mathrm{Var} &= \frac{1}{n}\sum_{i=1}^{n}(x_i^2 - 2\mu x_i + \mu^2) \\
&= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - 2\mu \cdot \frac{1}{n}\sum_{i=1}^{n} x_i + \frac{1}{n}\sum_{i=1}^{n} \mu^2
\end{aligned}$$

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \mathrm{Var} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$$

$$\begin{aligned}
\mathrm{Var} &= \frac{1}{n}\sum_{i=1}^{n}(x_i^2 - 2\mu x_i + \mu^2) \\
&= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - 2\mu \cdot \frac{1}{n}\sum_{i=1}^{n} x_i + \frac{1}{n}\sum_{i=1}^{n}\mu^2 \\
&= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - 2\mu \cdot \mu + \frac{1}{n} \cdot n \cdot \mu^2
\end{aligned}$$

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \mathrm{Var} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$$

$$\begin{aligned}
\mathrm{Var} &= \frac{1}{n}\sum_{i=1}^{n}(x_i^2 - 2\mu x_i + \mu^2) \\
&= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - 2\mu \cdot \frac{1}{n}\sum_{i=1}^{n} x_i + \frac{1}{n}\sum_{i=1}^{n} \mu^2 \\
&= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - 2\mu \cdot \mu + \frac{1}{n} \cdot n \cdot \mu^2 \\
&= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \mu^2
\end{aligned}$$

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \mathrm{Var} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2 > 0$$

$$\mathrm{Var} = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \mu^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2$$

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \mathrm{Var} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2 > 0$$

$$\mathrm{Var} = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \mu^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2$$

$$\mathcal{H} = 4n\sum_{i=1}^{n} x_i^2 - 4\left(\sum_{i=1}^{n} x_i\right)^2$$

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \mathrm{Var} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2 > 0$$

$$\mathrm{Var} = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \mu^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2$$

$$\begin{aligned}\mathcal{H} &= 4n\sum_{i=1}^{n} x_i^2 - 4\left(\sum_{i=1}^{n} x_i\right)^2 \\ &= 4n^2\left(\frac{1}{n}\sum x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2\right) = 4n^2\mathrm{Var}\end{aligned}$$

$$m = \frac{\begin{vmatrix} \sum x_i y_i & \sum x_i \\ \sum y_i & n \end{vmatrix}}{\begin{vmatrix} \sum x_i^2 & \sum x_i \\ \sum x_i & n \end{vmatrix}} \qquad b = \frac{\begin{vmatrix} \sum x_i^2 & \sum x_i y_i \\ \sum x_i & \sum y_i \end{vmatrix}}{\begin{vmatrix} \sum x_i^2 & \sum x_i \\ \sum x_i & n \end{vmatrix}}$$