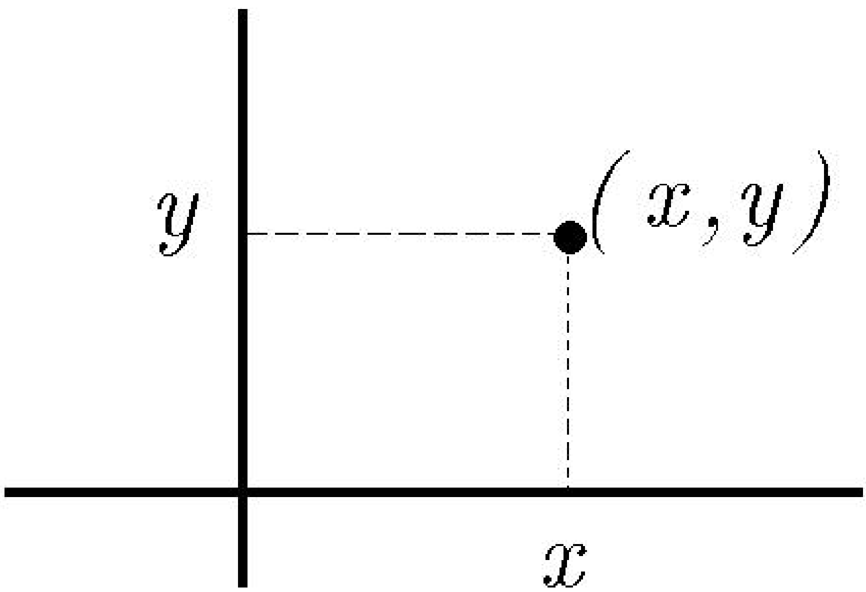
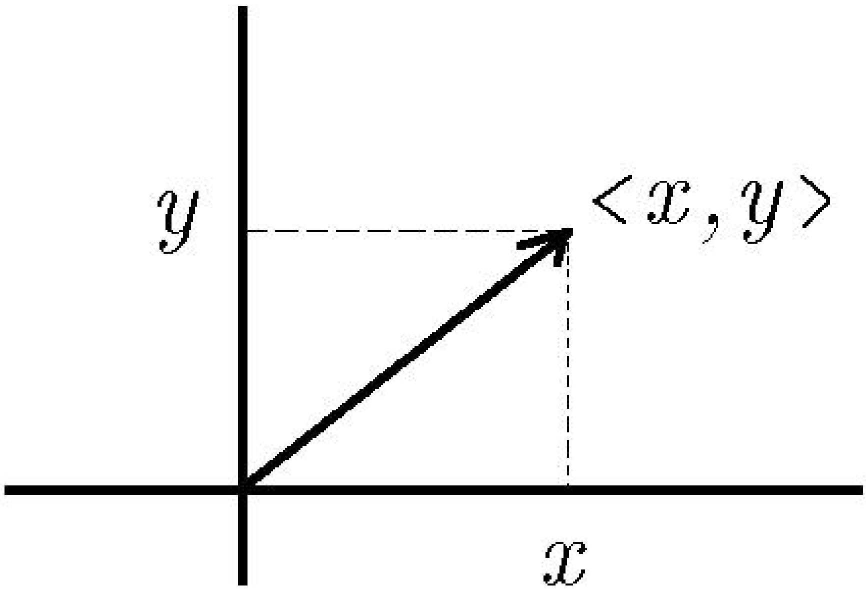
$$(x, y)$$

(x, y) can be represented as a point in the plane



A pair of numbers can be represented by an arrow.



A quantity with both magnitude and direction is called a *vector*

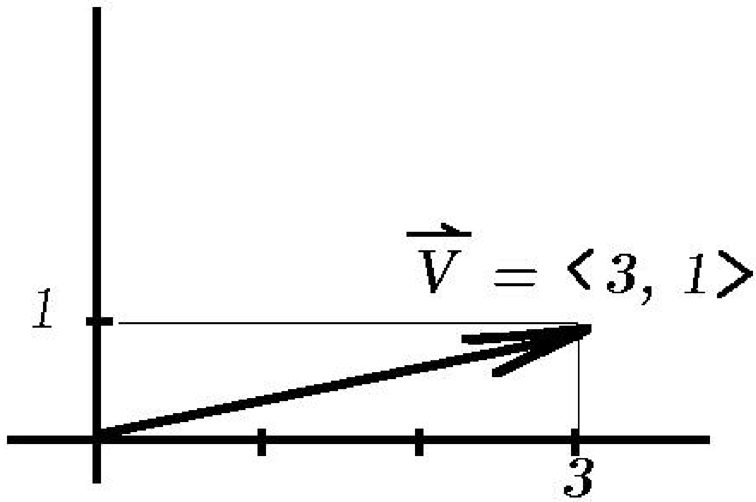
The vector $\vec{\mathbf{v}}$ from the origin to the point $(3, 1)$ is denoted by:

$$\vec{\mathbf{v}} = \langle 3,\ 1 \rangle$$

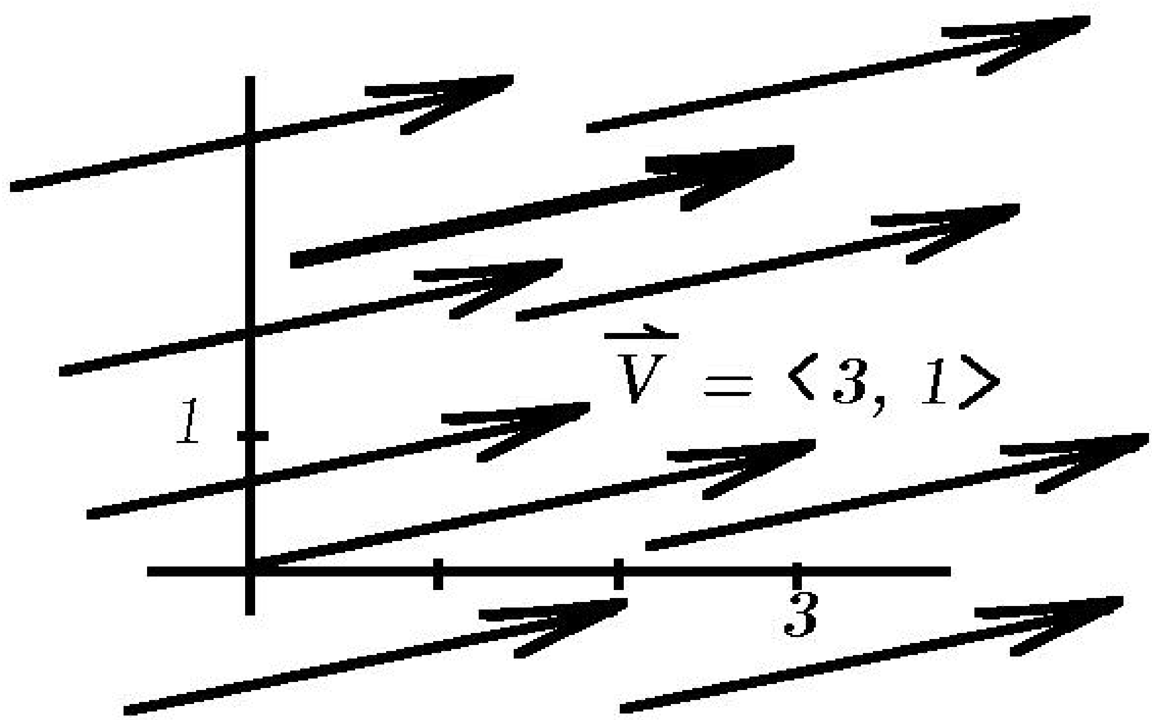
The length of the vector is denoted by the notation $|\vec{\mathbf{v}}|$. The notation $\|\vec{\mathbf{v}}\|$ is also used sometimes.

The length of $\vec{\mathbf{v}}$ is calculated using the distance formula

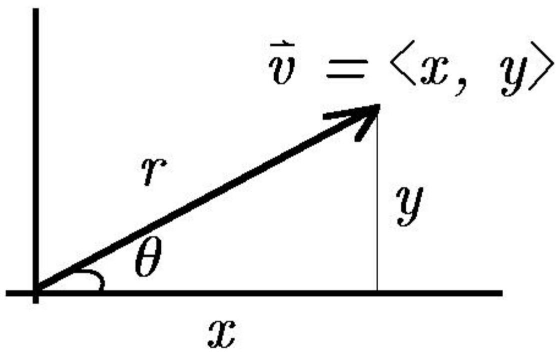
$$|\vec{\mathbf{v}}| = \sqrt{3^2 + 1^2} = \sqrt{10}$$



Any vector parallel to $\vec{\mathbf{v}}$ having the same length is considered to be the same as $\vec{\mathbf{v}}$, even if the tail of the vector does not begin at the origin.

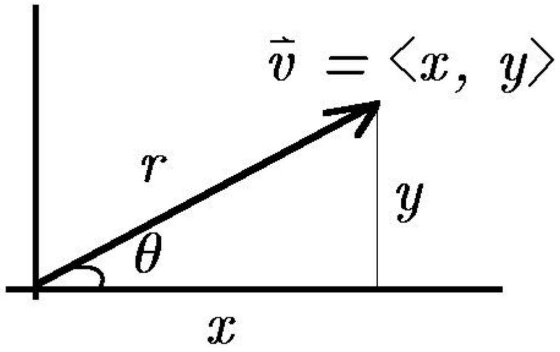


Let θ be the angle that a vector $\vec{\mathbf{v}} = \langle x,\, y \rangle$ makes with the x-axis. $r = |\vec{\mathbf{v}}| = \sqrt{x^2 + y^2}$ is the length.



$$\cos\theta = \frac{x}{r} \qquad \sin\theta = \frac{y}{r}$$

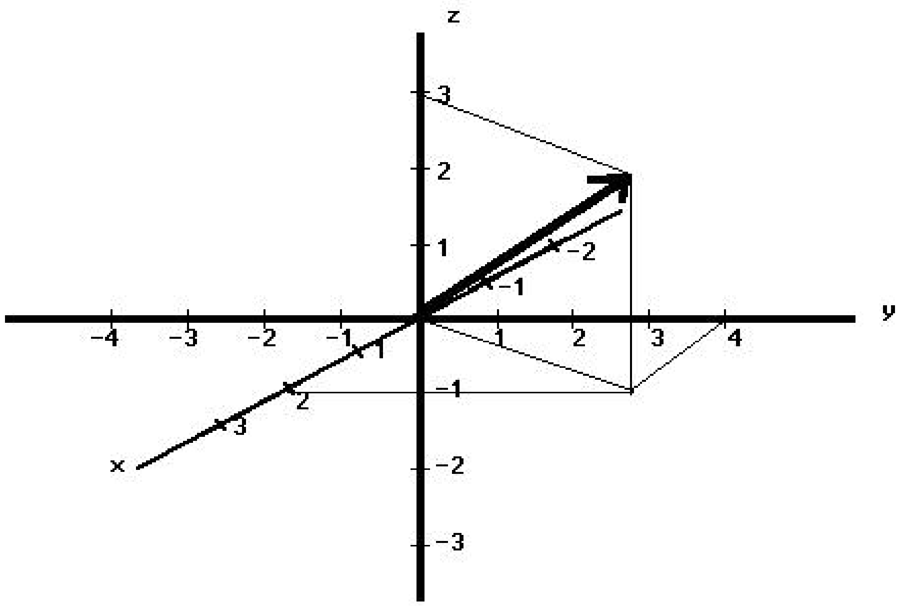
Let θ be the angle that a vector $\vec{\mathbf{v}} = \langle x,\, y \rangle$ makes with the x-axis. $r = |\vec{\mathbf{v}}| = \sqrt{x^2 + y^2}$ is the length.



$$x = r\cos\theta \qquad y = r\sin\theta$$

$$\vec{\mathbf{v}} = \langle x,\, y \rangle = \langle r\cos\theta,\, r\sin\theta \rangle$$

Vectors are easily generalized to higher dimension.



$$\vec{\mathbf{v}} = \langle 2,\ 4,\ 3 \rangle$$

Definition of Scalar Multiplication

Let $\vec{\mathbf{v}} = \langle x, y \rangle$.
Let c be a *scalar* (in other words, c is a number).

$$c\vec{\mathbf{v}} = c \langle x, y \rangle = \langle cx, cy \rangle$$

Let $\vec{\mathbf{v}} = \langle x,\, y,\, z \rangle$

$$c\vec{\mathbf{v}} = c\langle x,\, y,\, z \rangle = \langle cx,\, cy,\, cz \rangle$$

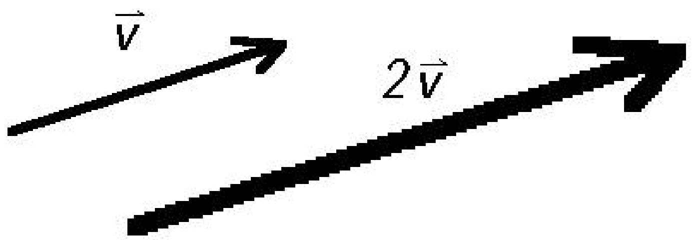
More generally, if $\vec{\mathbf{v}} = \langle v_1, v_2, \ldots, v_n \rangle$ then

$$c\vec{\mathbf{v}} = \langle cv_1, cv_2, \ldots, cv_n \rangle$$

Definition of Scalar Multiplication

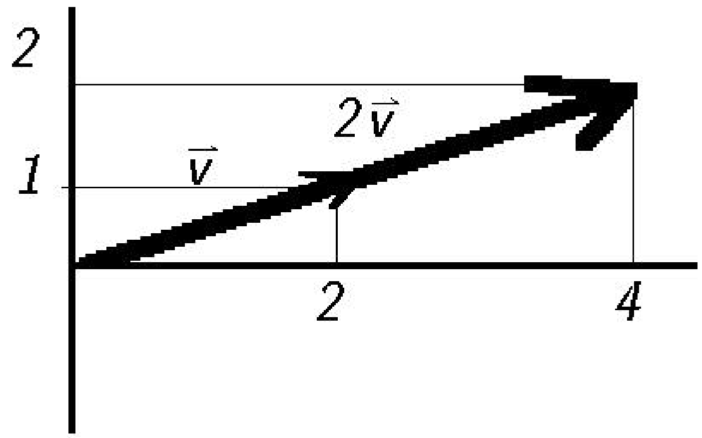
Motivation

Let $\vec{\mathbf{v}} = \langle 2, 1\rangle$. We would like to define the vector $2\vec{\mathbf{v}}$ so that it is in the same direction as $\vec{\mathbf{v}}$ but twice as long.



By similar triangles, the first coordinate of $2\vec{\mathbf{v}}$ must be twice the first coordinate of $\vec{\mathbf{v}}$.
Again by similar triangles, the second coordinate of $2\vec{\mathbf{v}}$ must be twice the second coordinate of $\vec{\mathbf{v}}$.

$$\vec{\mathbf{v}} = \langle 2,\ 1\rangle \qquad 2\vec{\mathbf{v}} = \langle 4,\ 2\rangle$$



A vector of length $= 1$ is a *unit vector*

If $\vec{\mathbf{v}}$ is any vector, then

$$\frac{1}{|\vec{\mathbf{v}}|}\,\vec{\mathbf{v}}$$

is always a unit vector that points in the same direction as $\vec{\mathbf{v}}$.

Example:

$$\vec{\mathbf{v}} = \langle 3,\ 4 \rangle$$

$$|\vec{\mathbf{v}}| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Therefore, the vector $\frac{1}{5}\vec{\mathbf{v}} = \left\langle \frac{3}{5},\ \frac{4}{5} \right\rangle$ is a unit vector

Check:

$$
\begin{aligned}
\left|\left\langle \frac{3}{5}, \frac{4}{5} \right\rangle\right| &= \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} \\
&= \sqrt{\frac{9}{25} + \frac{16}{25}} \\
&= \sqrt{1} = 1
\end{aligned}
$$

Vector Addition

If $\vec{\mathbf{u}} = \langle u_1, u_2 \rangle$ and $\vec{\mathbf{v}} = \langle v_1, v_2 \rangle$, then the *vector sum* is defined to be:

$$\vec{\mathbf{u}} + \vec{\mathbf{v}} = \langle u_1 + v_1, u_2 + v_2 \rangle$$

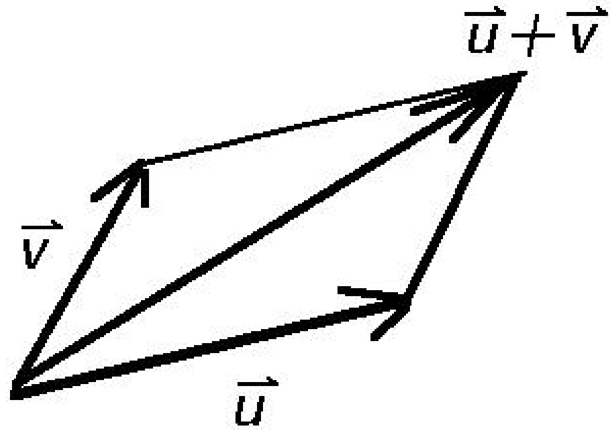
If $\vec{\mathbf{u}} = \langle u_1,\, u_2,\, u_3 \rangle$ and $\vec{\mathbf{v}} = \langle v_1,\, v_2,\, v_3 \rangle$ then

$$\vec{\mathbf{u}} + \vec{\mathbf{v}} = \langle u_1 + v_1,\, u_2 + v_2,\, u_3 + v_3 \rangle$$

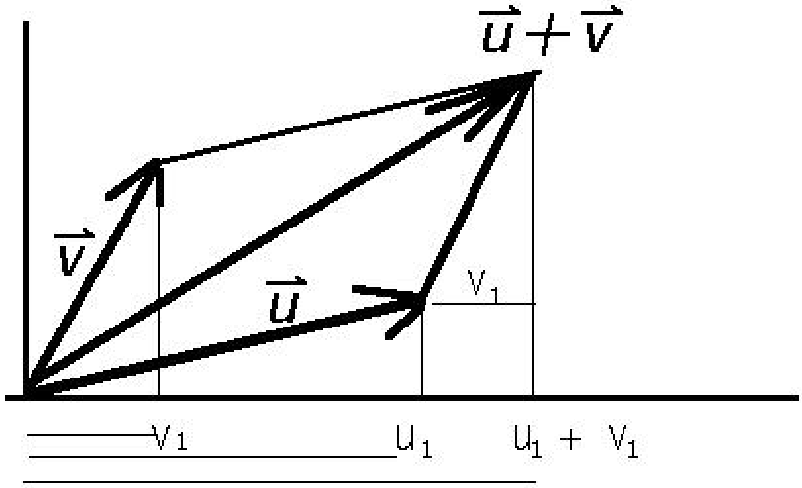
Vector Addition

Motivation

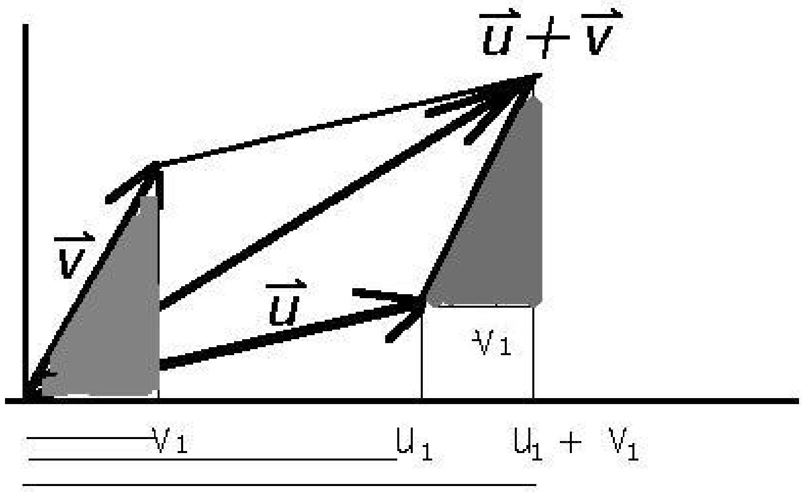
Vector addition is defined so that the vector sum is the diagonal of a parallelogram.



$$\vec{\mathbf{u}} = \langle u_1,\, u_2 \rangle \qquad \vec{\mathbf{v}} = \langle v_1,\, v_2 \rangle$$



$$\vec{\mathbf{u}} = \langle u_1, u_2 \rangle \qquad \vec{\mathbf{v}} = \langle v_1, v_2 \rangle$$



$$\vec{\mathbf{u}} = \langle u_1,\, u_2 \rangle \qquad \vec{\mathbf{v}} = \langle v_1,\, v_2 \rangle$$

$$\vec{\mathbf{u}} + \vec{\mathbf{v}} = \langle u_1 + v_1,\, u_2 + v_2 \rangle$$