MA 345   April 2020   **Final Exam - Jacobs**

I certify that the work I am submitting is my own and I have not accepted help on this exam from anybody else.

Name :______________________________

Sign here (legibly please)

Print out this test and write your solutions in the space provided for each problem. Show all work on problems 1 - 4. Express all answers in simplest form. When you are done, scan your solutions and e-mail the scanned pages to me at **jacobs@totcon.com** *no later than* **Monday, April 27.**

The scanned work that you send to me must be neat and easy to read. If not, I will reject your exam.

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1. *(15 points)* Suppose $y = y(t)$ is the solution of the differential equation:

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} = 2 \qquad \text{where } y(0) = 0 \text{ and } y'(0) = 0$$

On Exam 3, you used variation of parameters to solve $y'' - y' = 2$. This time, use the method of *Laplace transforms*.

2. *(15 points)* Let $y = y(x)$ be the solution of the differential equation:

$$\left(D^2 - 8D + 16\right) y = 2e^{4x}$$

On Exam 3, you solved this equation using Laplace transforms. This time, use the method of *undetermined coefficients* to obtain the general solution.

3. *(15 points)* A tank holds 16 liters of a brine solution. Brine is being added through an inflow pipe at the rate of 4 liters/minute. The brine coming through the inflow pipe contains salt at a concentration of $\frac{1}{8}$ gram/liter. At the same time, brine is flowing out of the tank through an outflow pipe at the rate of 4 liters/minute.

a) Let $y(t)$ denote the number of grams of salt in the tank after t minutes. Using the $\binom{\text{rate}}{\text{in}} - \binom{\text{rate}}{\text{out}}$ principle, write a differential equation that correctly determines y as a function of t.

b) Solve the differential equation that you have set up in part (a). Assume that $y(0) = 0$ (in other words, you are starting with only pure water in the tank).

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4. *(15 points)* Find the solution of the following differential equation:

$$\frac{dy}{dx} + 2y = e^{-2x} \qquad \text{where } y(0) = 1$$

Problems 5 - 12. *(40 points)*

For each of the following questions, simply circle the correct answer.

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5. Let $y = \sum a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$
We can find coefficients a_n so that y solves $\frac{dy}{dx} - 4y = 0$ where $y(0) = 1$.
Find the coefficient a_2

a) $a_2 = 0$ **b)** $a_2 = 1$ **c)** $a_2 = 2$ **d)** $a_2 = 4$ **e)** $a_2 = 8$

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6. The matrix $\begin{pmatrix} 4 & 1 \\ 0 & 4 \end{pmatrix}$ has an eigenvalue of $\lambda = 4$. Which of the following is the corresponding eigenvector?

a) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ **b)** $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ **c)** $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ **d)** $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ **e)** $\begin{pmatrix} 1 \\ -4 \end{pmatrix}$

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7. According to the Method of Variation of Parameters, we can find functions $v_1 = v_1(x)$ and $v_2 = v_2(x)$ so that the solution of the differential equation $y'' - y = \tan x$ has the form: $y = v_1 e^x + v_2 e^{-x}$. Which of the following integrals equals v_1 ?

a) $\frac{1}{2}\int \tan x\, dx$ **b)** $\frac{1}{2}\int e^{-x} \tan x\, dx$ **c)** $-\frac{1}{2}\int e^{x} \tan x\, dx$
d) $\int e^{-x} \tan(x)\, dx$ **e)** $\int \tan\left(e^x\right)\, dx$

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8. Calculate the following inverse Laplace transform: $\mathcal{L}^{-1}\left(\frac{3s-5}{(s-1)(s-2)}\right)$

a) $2e^t + e^{2t}$ **b)** $e^t - e^{2t}$ **c)** $e^t + e^{2t}$
d) $e^t - 2e^{2t}$ **e)** $e^t - 2e^{2t}$

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9. Find $\mathcal{L}(y)$ if y solves the following differential equation:

$$\frac{d^2y}{dt^2} - y = 0 \qquad \text{where } y(0) = 1 \text{ and } y'(0) = 1$$

a) $\frac{1}{s+1}$ **b)** $\frac{1}{s-1}$ **c)** $\frac{1}{(s-1)(s-2)}$ **d)** $\frac{s}{(s-1)(s-2)}$ **e)** $\frac{s+2}{(s-1)(s-2)}$

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10. Which of the following operators will be the *annihilator* of $y = 2x + e^x$

a) $D^2(D-1)$ **b)** $D^2(D+1)$ **c)** $D^2 + D - 1$
d) $(D-1)^2$ **e)** $(D-1)^3$

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11. The general form of the particular solution of $y'' - 2y' + y = \sin x$ is $y_p = A\cos x + B\sin x$. Find the coefficient A.

a) $A = 0$ **b)** $A = 1$ **c)** $A = 2$ **d)** $A = \frac{1}{2}$ **e)** $A = -\frac{1}{2}$

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12. Which of the following is the solution of $(D^2 - 2D + 5)y = 0$

a) $y = a + be^{-4x}$ **b)** $y = a\cos 2x + b\sin 2x$ **c)** $y = ae^{2x} + bxe^{2x}$
d) $y = ae^{2x} + be^{-2x}$ **e)** $y = ae^{x}\cos 2x + be^{x}\sin 2x$
