Embry-Riddle Aeronautical University — C. Jacobs
MA 345 Differential Equations — Spring 2020

Exam II

Questions 1 - 6 (24 points). Match each differential equation to its solution by putting the appropriate letter next to the equation number.

1. $\left(D^2 - 16D\right) y = 0$ **a)** $y = c_1 e^{4x} + c_2 e^{-4x}$

2. $\left(D^2 - 16\right) y = 0$ **b)** $y = c_1 + c_2 e^{16x}$

3. $\left(D^2 + 16\right) y = 0$ **c)** $y = c_1 + c_2 e^{-16x}$

4. $\left(D^2 + 16D\right) y = 0$ **d)** $y = c_1 e^{8x} + c_2 x e^{8x}$

5. $\left(D^2 - 2D + 17\right) y = 0$ **e)** $y = c_1 \cos(4x) + c_2 \sin(4x)$

6. $\left(D^2 - 16D + 64\right) y = 0$ **f)** $y = e^x \left(c_1 \cos(4x) + c_2 \sin(4x)\right)$

Answers: 1 - b 2 - a 3 - e 4 - c 5 - f 6 - d

Questions 7 - 11 (25 points). Answer each of the following multiple choice questions by circling the correct choice.

7. *(5 points)* Suppose $z = e^{-\frac{3\pi}{2} i}$. Then z equals:

a) -1 **b)** $-i$ $\boxed{\textbf{c)}}$ i **d)** 1 **e)** none of these

8. *(5 points)* Which of the following is the annihilator of $x^2 + e^{2x}$?

a) $D^4 - 3D^2$ $\boxed{\textbf{b)}}$ $D^4 - 2D^3$ **c)** $(D-2)^3$ **d)** $(D-3)^2$ **e)** none of these

9. *(5 points)* Which of the following will equal $(D-1)^3 \left(x^2 e^x\right)$?

$\boxed{\textbf{a)}}$ 0 **b)** $6e^x$ **c)** $6xe^x$ **d)** $3x^2 e^x$ **e)** none of these

10. *(5 points)* Find the general form of the particular solution y_p of the equation $(D-1)^2 y = e^x$

a) a **b)** ae^x **c)** axe^x $\boxed{\textbf{d)}}$ $ax^2 e^x$ **e)** none of these

11. *(5 points)* Suppose the matrix equation $\mathbf{A}\vec{\mathbf{X}} = \vec{\mathbf{b}}$ has at least one solution. If the determinant of $\mathbf{A}$ is 0, then which method should be used to solve for $\vec{\mathbf{X}}$?

a) Cramer's Rule Method
$\boxed{\textbf{b)}}$ Matrix Reduction Method
c) Matrix Inverse Method
d) Any of the above could be used successfully
e) None of the above.

12. *(30 points)* Let $\mathbf{A}$, $\vec{\mathbf{X}}$ and $\vec{\mathbf{b}}$ be defined as follows:

$$\mathbf{A} = \begin{pmatrix} -1 & 0 \\ 2 & -1 \end{pmatrix} \qquad \vec{\mathbf{X}} = \begin{pmatrix} x \\ y \end{pmatrix} \qquad \vec{\mathbf{b}} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

a) Find $\mathbf{A}^{-1}$ (the inverse of $\mathbf{A}$) or show that no inverse exists.

$$\mathbf{A}^{-1} = \begin{pmatrix} -1 & 0 \\ -2 & -1 \end{pmatrix}$$

b) Solve $\mathbf{A}\vec{\mathbf{X}} = \vec{\mathbf{b}}$

$$\vec{\mathbf{X}} = \mathbf{A}^{-1}\vec{\mathbf{b}} = \begin{pmatrix} -1 & 0 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ -5 \end{pmatrix}$$

c) Find a non-zero vector $\vec{\mathbf{X}}$ that solves the equation $\mathbf{A}\vec{\mathbf{X}} = -\vec{\mathbf{X}}$ or show that no such vector $\vec{\mathbf{X}}$ exists.

$\mathbf{A}\vec{\mathbf{X}} = -\vec{\mathbf{X}}$ can be rewritten as $(\mathbf{A} + \mathbf{I})\vec{\mathbf{X}} = \vec{\mathbf{0}}$ which means that $\left(\begin{smallmatrix} 0 & 0 \\ 2 & 0 \end{smallmatrix}\right) \left(\begin{smallmatrix} x \\ y \end{smallmatrix}\right) = \left(\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right)$. If we multiply out, we get $\left(\begin{smallmatrix} 2x \\ 0 \end{smallmatrix}\right) = \left(\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right)$. This can only happen if $x = 0$. There is no restriction on y, so the general solution is:

$$\vec{\mathbf{X}} = \begin{pmatrix} 0 \\ y \end{pmatrix} = y \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Thus, any nonzero multiple of $\left(\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right)$ will be a nonzero vector that solves $\mathbf{A}\vec{\mathbf{X}} = -\vec{\mathbf{X}}$

13. *(21 points)* Use the method of undetermined coefficients to solve the following differential equation. Show all work.

$$\left(D^2 + 1\right) y = e^x + e^{-x} \quad \text{where } y(0) = 0 \text{ and } y'(0) = 0$$

The homogeneous solution will be $y_h = c_1 \cos x + c_2 \sin x$. The particular solution will have the general form of $y_p = ae^x + be^{-x}$. Substitute this into the differential equation to find a and b

$$\left(D^2 + 1\right)\left(ae^x + be^{-x}\right) = e^x + e^{-x}$$
$$2ae^x + 2be^{-x} = e^x + e^{-x}$$

This can only happen if $a = \frac{1}{2}$ and $b = \frac{1}{2}$. The general solution is therefore:

$$y = c_1 \cos x + c_2 \sin x + \frac{1}{2}\left(e^x + e^{-x}\right)$$

The initial conditions imply that $c_1 = -1$ and $c_2 = 0$. This leaves us with:

$$y = -\cos x + \frac{1}{2}\left(e^x + e^{-x}\right) = -\cos x + \cosh x$$