

*I certify that the work I am submitting is my own and I have not accepted help on this exam from anybody else.*

Name : \_\_\_\_\_

*Sign here (legibly please)*

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*Print out this test and write your solutions in the space provided for each problem. Show all work. Express all answers in simplest form. Exact answers only, please. No decimal approximations. When you are done, scan your solutions and e-mail the scanned pages to me at [jacobs@totcon.com](mailto:jacobs@totcon.com). The scanned work that you send to me must be neat and easy to read. If not, I will reject your exam.*

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**1.** Suppose a spring is arranged horizontally. Let  $y = y(t)$  be the position of the mass at the end of the spring relative to the equilibrium position. Suppose the mass is  $m = 1$  kg, the damping constant is  $\beta = 5$  and the spring constant is  $k = 4$

**a)** (5 points)

Set up the differential equation that correctly determines  $y(t)$ .

**b)** (20 points) Use the method of Laplace transforms to solve your differential with the initial conditions  $y(0) = 0$  and  $y'(0) = 3$

**2.** (*25 points*) Use the method of variation of parameters to find the general solution of the following differential equation. Show all work.

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = 2$$

**3.** (25 points) Use the method of Laplace transforms to solve the following differential equation:

$$y'' - 8y' + 16y = 2e^{4t} \quad \text{where } y(0) = 0 \text{ and } y'(0) = 0$$

4. (25 points) Solve the following differential equation. The symbol  $\mathcal{U}$  stands for the unit step function.

$$y'' + y = 5e^{2t}\mathcal{U}(t - 2\pi) \quad \text{where } y(0) = 0 \text{ and } y'(0) = 0$$