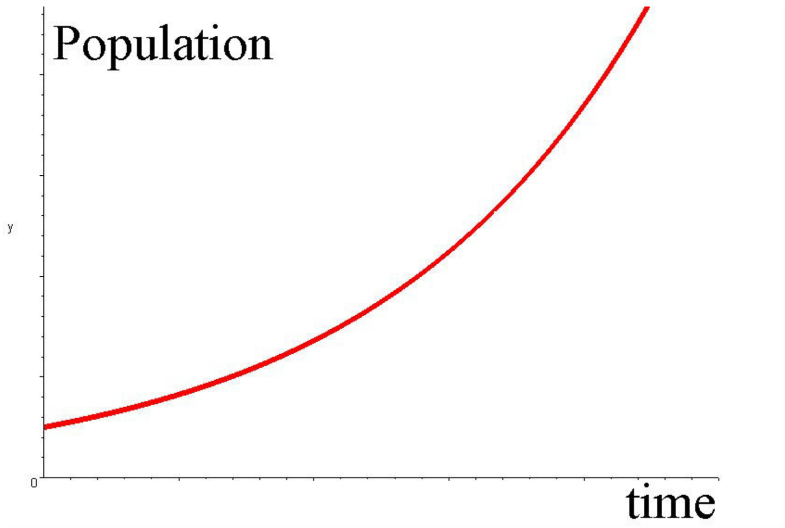
## World Population

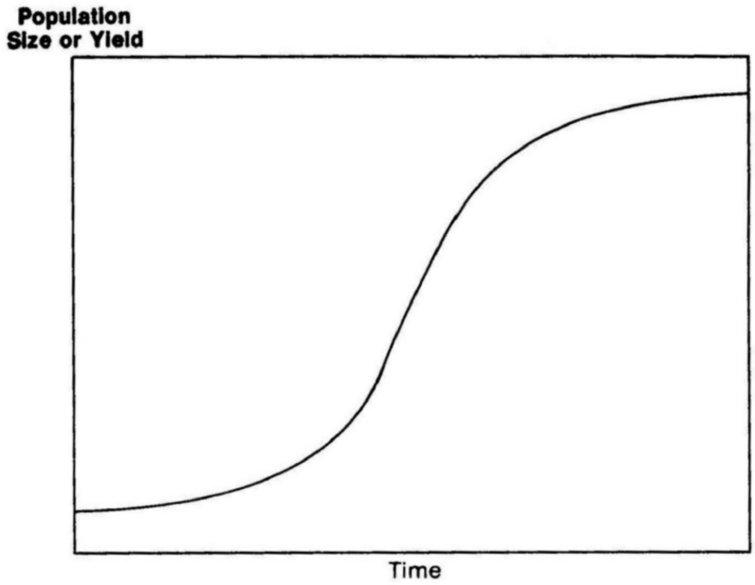
| | |
|---|---|
| 1800 | 1 billion |
| 1930 | 2 billion |
| 1960 | 3 billion |
| 1974 | 4 billion |
| 1987 | 5 billion |
| 1999 | 6 billion |
| 2011 | 7 billion |
| 2045 | 9 billion |

Malthusian Growth Model: $$\frac{dy}{dt} = ky$$

$$y = y_0 e^{kt}$$



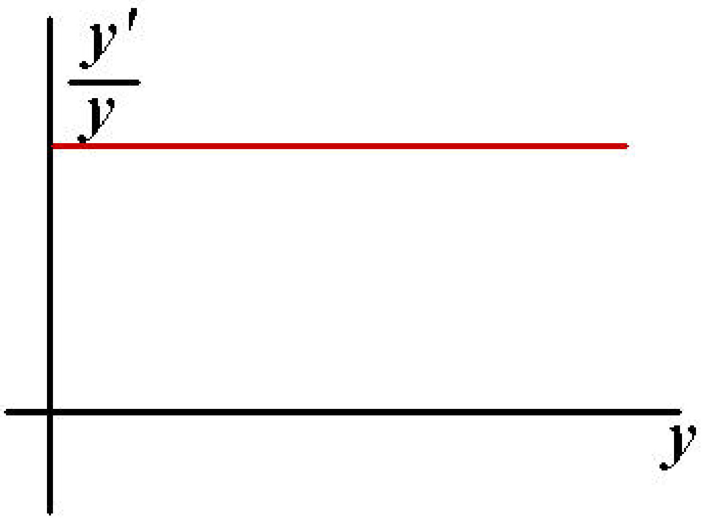
# Projected World Population



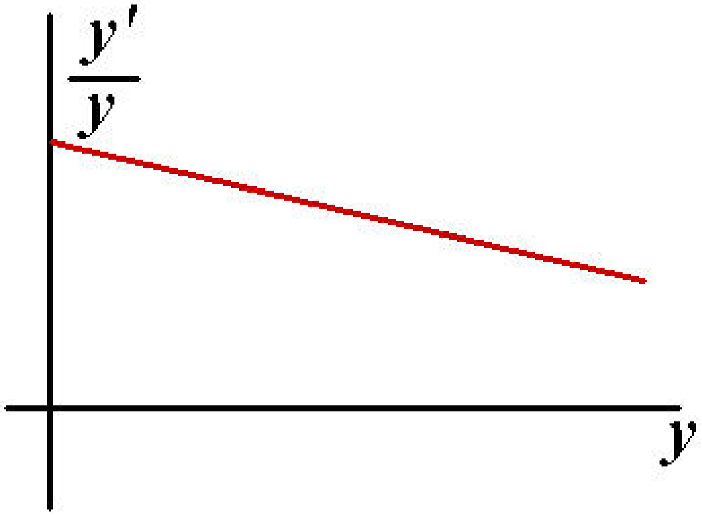
If $\frac{dy}{dt} = ky$ then:

$$\frac{y'}{y} = k$$

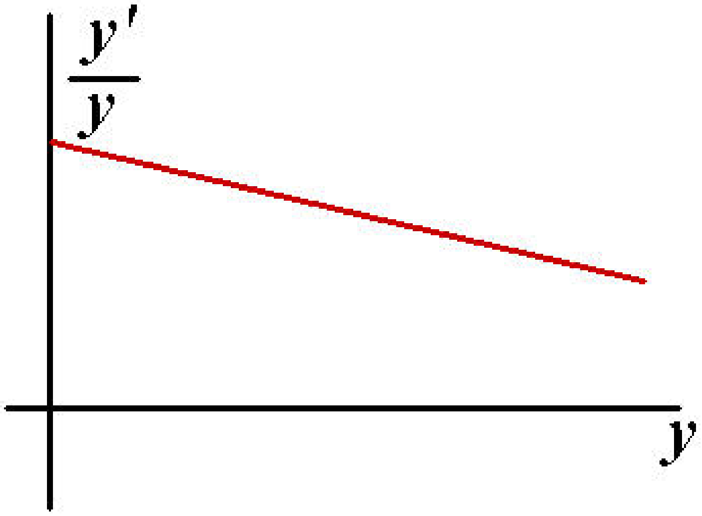
Thus, the Malthusian Model assumes that the rate of growth *per individual* remains constant no matter how large the population is.



More realistic model:

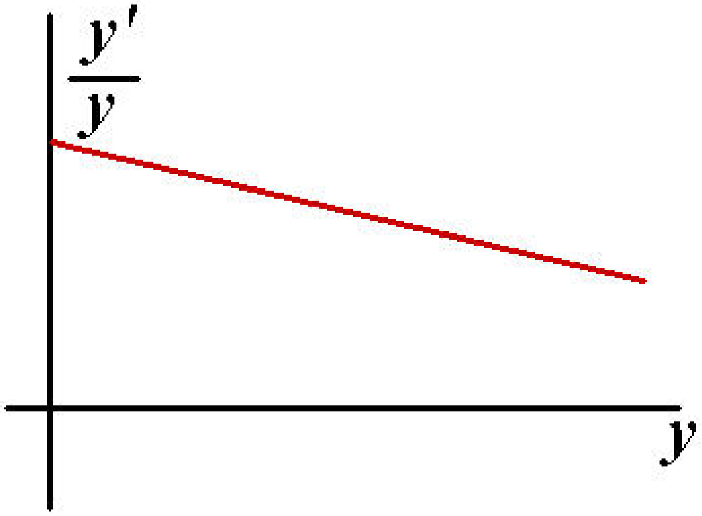


More realistic model:




$$\frac{y'}{y} = my + b$$

More realistic model:




$$\frac{y'}{y} = -ay + b$$

$$\frac{y'}{y} = -ay + b$$

$$\frac{y'}{y} = a\left(-y + \frac{b}{a}\right)$$

Let $L = \frac{b}{a}$

$$\frac{y'}{y} = a\left(L - y\right)$$

$$y' = a(L - y)y$$

This is called the *Logistics Equation*

$$\frac{dy}{dt} = a(L - y)y = aLy - ay^2$$

This is a special case of *Bernoulli's Equation*

Jakob Bernoulli

In 1696, Bernoulli solved the equation:

$$\frac{dy}{dt} + P(t)y = Q(t)y^n$$

The substitution

$$v = y^{1-n}$$

transforms $\frac{dy}{dt} + P(t)y = Q(t)y^n$ into a first order linear differential equation.

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The logistics equation

$$\frac{dy}{dt} = aLy - ay^2$$

is a special case of Bernoulli's equation where $n = 2$