$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t)\, dt$$

Let $F(s) = \mathcal{L}(f(t))$ and $g(s,\, t) = e^{-st} f(t)$

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$$F(s+h) - F(s) = \int_0^\infty (g(s+h,\, t) - g(s,\, t))\, dt$$

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$$\frac{F(s+h) - F(s)}{h} = \int_0^\infty \frac{g(s+h,\, t) - g(s,\, t)}{h}\, dt$$

$$\frac{F(s+h)-F(s)}{h}=\int_0^\infty \frac{g(s+h,\,t)-g(s,\,t)}{h}\,dt$$

Take the limit as $h \longrightarrow 0$

$$F'(s)=\int_0^\infty \frac{\partial g}{\partial s}\,dt$$

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Since $F(s) = \mathcal{L}(f(t))$ and $g(s,\, t) = e^{-st} f(t)$ then

$$\begin{aligned} \frac{d}{ds}(\mathcal{L}(f(t)) &= \int_0^\infty \frac{\partial}{\partial s} \left(e^{-st} f(t)\right)\, dt \\ &= \int_0^\infty -te^{-st} f(t)\, dt \end{aligned}$$

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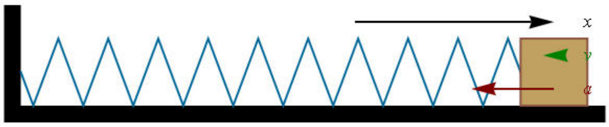

$$my'' + ky = 0$$

$$y = a\cos\sqrt{\frac{k}{m}}t \ + b\sin\sqrt{\frac{k}{m}}t$$

If we let $\omega = \sqrt{\frac{k}{m}}$ then $\quad y = a\cos\omega t + b\sin\omega t$

$$my'' + ky = \cos \gamma t$$

$$y'' + \frac{k}{m} y = \frac{1}{m} \cos \gamma t$$

Let $\omega = \sqrt{\frac{k}{m}}$ and $\gamma = \omega$

$$y'' + \omega^2 y = \frac{1}{m} \cos \omega t$$

$$y'' + \omega^2 y = \frac{1}{m}\cos\omega t$$

General initial conditions: $y(0) = A$ and $y'(0) = B$

$$\mathcal{L}(y'') + \omega^2\mathcal{L}(y) = \frac{1}{m}\mathcal{L}(\cos\omega t)$$

$$s^2\mathcal{L}(y) - sA - B + \omega^2\mathcal{L}(y) = \frac{1}{m}\cdot\frac{s}{s^2+\omega^2}$$

Solve for $\mathcal{L}(y)$

$$\mathcal{L}(y) = \frac{As}{s^2+\omega^2} + \frac{B}{s^2+\omega^2} + \frac{1}{m} \cdot \frac{s}{\left(s^2+\omega^2\right)^2}$$

$$\mathcal{L}(y) = A \cdot \frac{s}{s^2+\omega^2} + \frac{B}{\omega} \cdot \frac{\omega}{s^2+\omega^2} + \frac{1}{2m\omega} \cdot \frac{2\omega s}{\left(s^2+\omega^2\right)^2}$$

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$$y = A\cos\omega t + \frac{B}{\omega}\sin\omega t + \frac{1}{2m\omega} t \sin\omega t$$

$$y_p = \frac{1}{2m\omega} t \sin \omega t$$