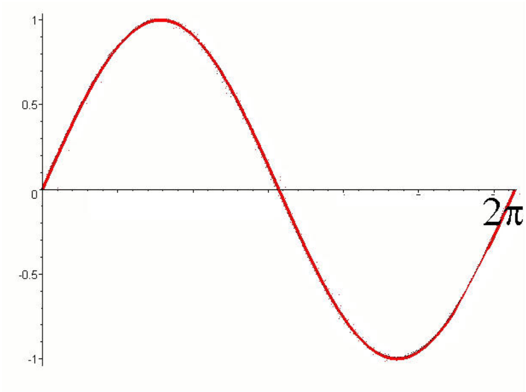
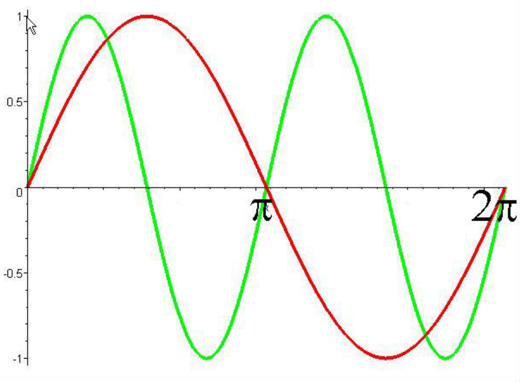
$$y = \sin t$$

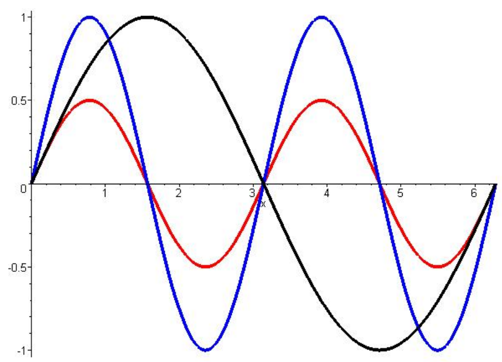


$$y = \sin t$$

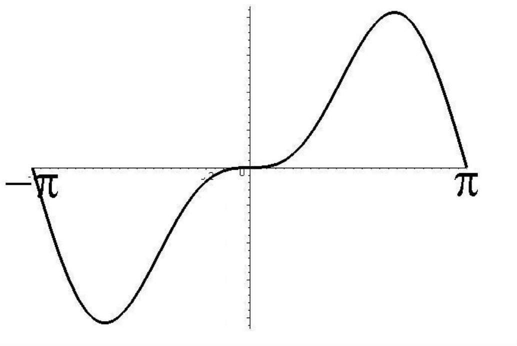
$$y = \sin 2t$$



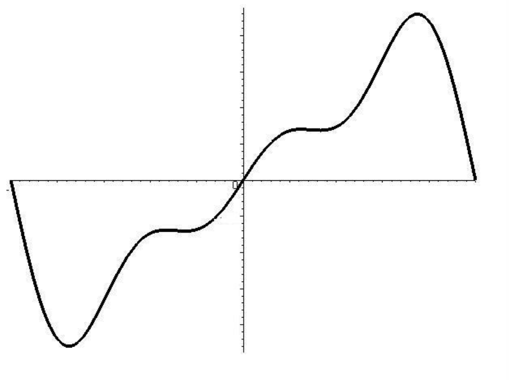
$$y = \sin t, \quad y = \sin 2t \quad \text{and} \quad y = \frac{1}{2}\sin 2t$$



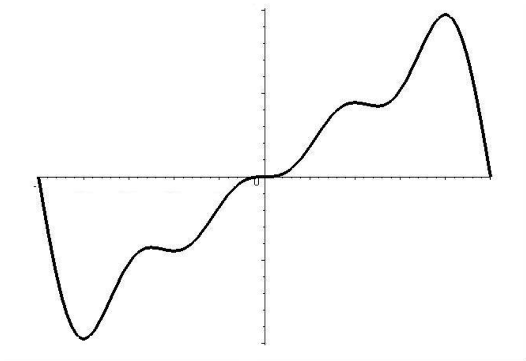
$$y = \sin t - \frac{1}{2}\sin 2t$$



$$y = \sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t$$

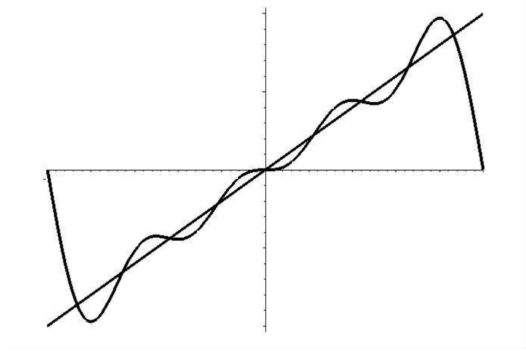


$$y = \sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t - \frac{1}{4}\sin 4t$$

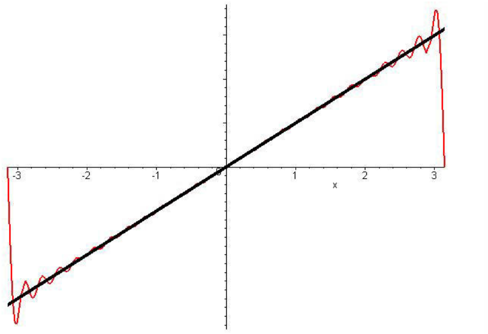


$$y = \sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t - \frac{1}{4}\sin 4t$$

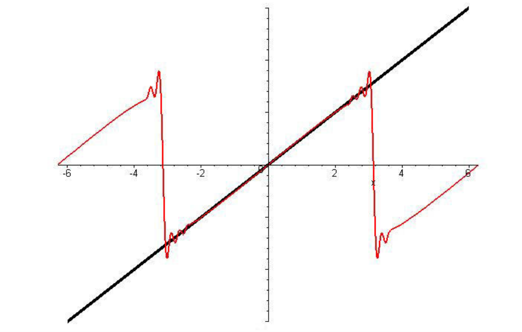
$$y = \frac{t}{2}$$



$$y = \sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t - \frac{1}{4}\sin 4t + \frac{1}{5}\sin 5t - \cdots$$



$$y = \sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t - \frac{1}{4}\sin 4t + \frac{1}{5}\sin 5t - \cdots$$



Fourier Sine Series

$$f(t) = b_1 \sin t + b_2 \sin 2t + b_3 \sin 3t + b_4 \sin 4t + \cdots$$

Fourier Sine Series

$$f(t) = \sum_{n=1}^{\infty} b_n \sin nt$$

Fourier Cosine Series

$$f(t) = \sum_{n=0}^{\infty} a_n \cos nt$$

More General Fourier Series

$$f(t) = \sum (a_n \cos nt + b_n \sin nt)$$

This sum will represent the function for $-\pi \le t \le \pi$

More General Fourier Series

$$f(t) = \sum \left(a_n \cos \frac{n\pi t}{L} + b_n \sin \frac{n\pi t}{L} \right)$$

This sum will represent the function for $-L \leq t \leq L$

More General Fourier Series

$$f(t) = \sum \left(a_n \cos \frac{n\pi t}{L} + b_n \sin \frac{n\pi t}{L} \right)$$

The formulas for a_n and b_n are given by *integrals*

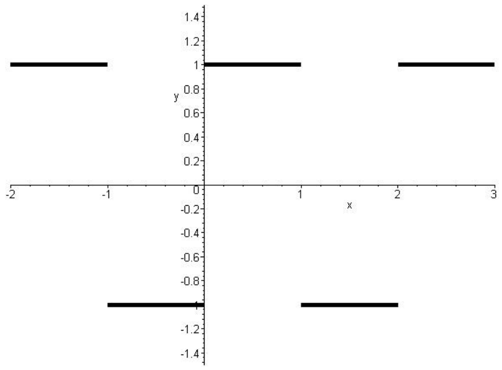
More General Fourier Series

$$f(t) = \sum \left(a_n \cos \frac{n\pi t}{L} + b_n \sin \frac{n\pi t}{L} \right)$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(t) \cos \frac{n\pi t}{L} \, dt$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(t) \sin \frac{n\pi t}{L} \, dt$$

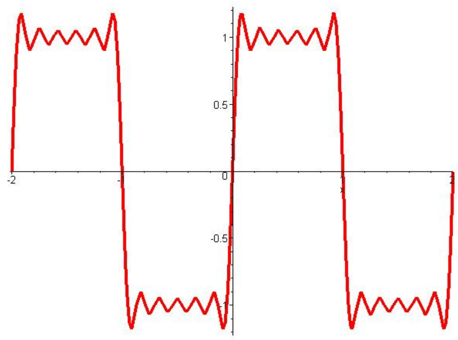
$$f(t) = \begin{cases} 1 & \text{for } 0 \le t \le \pi \\ -1 & \text{for } -\pi \le t < 0 \end{cases}$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos nt \; dt = 0$$
$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin nt \; dt = \frac{2}{n\pi} (1 - (-1)^n)$$

$$
\begin{aligned}
f(t) &= \sum_{n=1}^{\infty} \frac{2}{n\pi}\left(1-(-1)^{n}\right)\sin nt \\
&= \frac{4}{\pi}\sin t + \frac{4}{3\pi}\sin 3t + \frac{4}{5\pi}\sin 5t + \frac{4}{7\pi}\sin 7t + \cdots
\end{aligned}
$$

$$\mathcal{F}(t) = \frac{4}{\pi}\sin \pi t + \frac{4}{3\pi}\sin 3\pi t + \frac{4}{5\pi}\sin 5\pi t + \frac{4}{7\pi}\sin 7\pi t$$



$$f(t) = \sum_n \left(a_n \cos \frac{n\pi t}{L} + b_n \sin \frac{n\pi t}{L} \right)$$

where:

$$a_n = \frac{1}{L} \int_{-L}^{L} f(t) \cos \frac{n\pi t}{L} \, dt$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(t) \sin \frac{n\pi t}{L} \, dt$$

Define $\omega_n = \frac{n\pi}{L}$

If $\omega_n = \frac{n\pi}{L}$ then:

$$f(t) = \sum_n \left(a_n \cos(\omega_n t) + b_n \sin(\omega_n t)\right)$$

where:

$$a_n = \frac{1}{L}\int_{-L}^{L} f(t)\cos(\omega_n t)\,dt \qquad b_n = \frac{1}{L}\int_{-L}^{L} f(t)\sin(\omega_n t)\,dt$$

Let $L \to \infty$

$$f(t) = \frac{1}{2\pi} \int_0^\infty \left(A(\omega) \cos \omega t + B(\omega) \sin \omega t \right) \, d\omega$$

where:

$$A(\omega) = \int_{-\infty}^{\infty} f(t) \cos \omega t \, dt \qquad B(\omega) = \int_{-\infty}^{\infty} f(t) \sin \omega t \, dt$$

$$f(t) = \frac{1}{2\pi} \int_0^{\infty} \left(A(\omega) \cos \omega t + B(\omega) \sin \omega t\right) \, d\omega$$

where:

$$A(\omega) = \int_{-\infty}^{\infty} f(t) \cos \omega t \, dt \qquad B(\omega) = \int_{-\infty}^{\infty} f(t) \sin \omega t \, dt$$

Use Euler's Formula to obtain $\sin \omega t = \frac{1}{2i}\left(e^{i\omega t} - e^{-i\omega t}\right)$ and $\cos \omega t = \frac{1}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)$ and substitute this into the integrals.

$$
\begin{aligned}
f(t) &= \frac{1}{2\pi}\int_0^\infty \left(A(\omega)\cos\omega t + B(\omega)\sin\omega t\right)\,d\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^\infty C(\omega)e^{i\omega t}\,d\omega
\end{aligned}
$$

where:

$$
C(\omega) = \int_{-\infty}^\infty f(t)e^{-i\omega t}\,dt
$$

If $f(t) = 0$ for $t < 0$, then the amplitude $C(\omega)$ becomes:

$$\int_0^\infty f(t)e^{-i\omega t}\,dt$$

If $f(t) = 0$ for $t < 0$, then the amplitude $C(\omega)$ becomes:

$$\int_0^\infty f(t)e^{-(i\omega)t}\,dt$$

Let $s = i\omega$

$$\mathcal{L}(f(t)) = \int_0^\infty f(t)e^{-st}\,dt$$