$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{-i\theta} = \cos\theta - i\sin\theta$$

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$$e^{-i\theta} = \cos\theta - i\sin\theta$$

$$\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)$$

$$\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$$

$$\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$$

Define the hyperbolic sine as follows:

$$\sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$$

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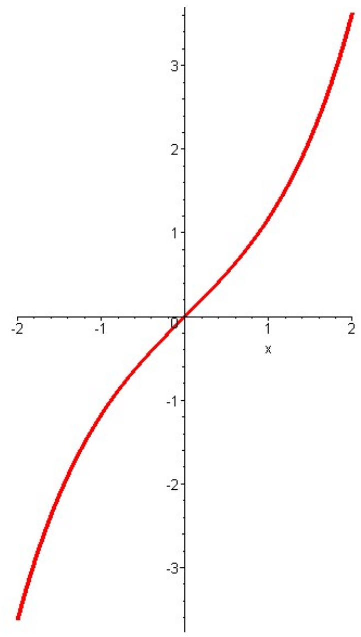

$$\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)$$

Define the hyperbolic cosine as follows:

$$\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$$

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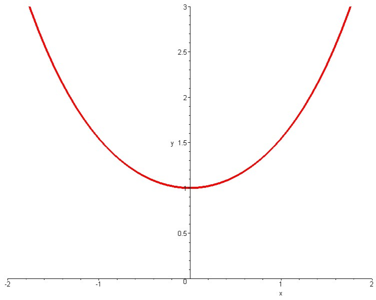

If $\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)$ then

$$\begin{aligned}
\cos(ix) &= \frac{1}{2}\left(e^{i \cdot ix} + e^{-i \cdot ix}\right) \\
&= \frac{1}{2}\left(e^{-x} + e^{x}\right) \\
&= \cosh x
\end{aligned}$$

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&= \frac{1}{2}\left(e^{-x} + e^{x}\right) \\
&= \cosh x
\end{aligned}$$

Similarly,

$$\sin(ix) = i\sinh x$$

$$\cos(ix) = \cosh x \qquad \sin(ix) = i \sinh x$$

$$\begin{aligned} \frac{d}{dx}(\cosh x) &= \frac{d}{dx}(\cos ix) = -i \sin ix \\ &= -i \cdot i \sinh x \\ &= \sinh x \end{aligned}$$

$$\cos(ix) = \cosh x \qquad \sin(ix) = i \sinh x$$

$$\begin{aligned} \frac{d}{dx}(\sinh x) &= \frac{d}{dx}(\frac{1}{i}\sin ix) \\ &= \frac{1}{i} \cdot i \cos ix \\ &= \cosh x \end{aligned}$$

$$\cos(ix) = \cosh x \qquad \sin(ix) = i \sinh x$$

$$\frac{d}{dx}(\cosh x) = \sinh x \qquad \frac{d}{dx}(\sinh x) = \cosh x$$

$$\int \sinh x \, dx = \cosh x + C$$

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$$\cos(ix) = \cosh x \qquad \sin(ix) = i \sinh x$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$(\cos(ix))^2 + (\sin(ix))^2 = 1$$

$$\cos(ix) = \cosh x \qquad \sin(ix) = i \sinh x$$

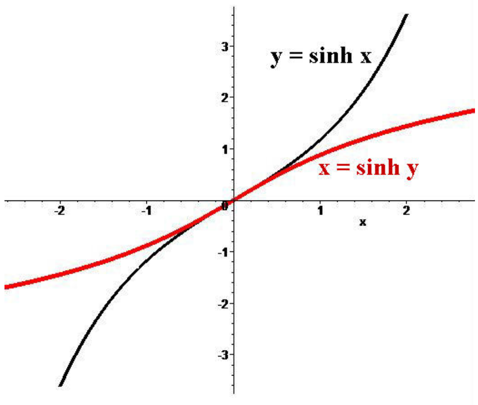
$$\cos^2 \theta + \sin^2 \theta = 1$$

$$(\cos(ix))^2 + (\sin(ix))^2 = 1$$

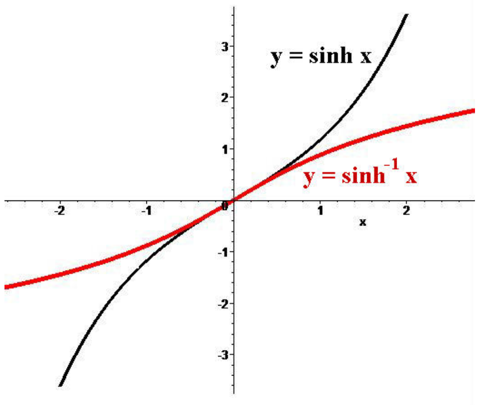
$$(\cosh x)^2 + (i \sinh x)^2 = 1$$

$$\cosh^2 x - \sinh^2 x = 1$$

Graph of $y = \sinh x$ in black
Graph of $x = \sinh y$ in red



Graph of $y = \sinh x$ in black
$y = \sinh^{-1} x$ means $\sinh y = x$. Graph in red.



$$y = \sinh^{-1} x$$

$$\sinh y = x$$

$$\frac{d}{dx}(\sinh y) = \frac{d}{dx}(x)$$

$$y = \sinh^{-1} x$$

$$\sinh y = x$$

$$\frac{d}{dx}(\sinh y) = \frac{d}{dx}(x)$$

$$\cosh y \, \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\cosh y}$$

$$\cosh^2 y - \sinh^2 y = 1$$

$$\cosh y = \sqrt{1 + \sinh^2 y}$$

$$y = \sinh^{-1} x$$

$$\sinh y = x$$

$$\frac{d}{dx}(\sinh y) = \frac{d}{dx}(x)$$

$$\cosh y \, \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\cosh y} = \frac{1}{\sqrt{1 + \sinh^2 y}} = \frac{1}{\sqrt{1 + x^2}}$$

$$\frac{d}{dx}\left(\sinh^{-1} x\right) = \frac{1}{\sqrt{1+x^2}}$$

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \sinh^{-1} x \ + C$$