MA 345 *Differential Equations* **Dr. E. Jacobs**

Exam I Practice

Note: This exam was given in Fall 2024

Problems 1 - 4 require that you show all work. All answers must be expressed in simplest form. Do your calculations on the blank sheets provided. The papers will be stapled together at the end of the test.

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1 - 3. *(60 points)* Find the solution of each of the following differential equation. Make sure you solve for y in terms of x. Don't forget about the given initial conditions.

1. $\dfrac{dy}{dx} = \dfrac{2y^2}{(x+1)^3}$ where $y(0) = 1$

2. $\dfrac{dy}{dx} + \dfrac{1}{2x}y = \dfrac{1}{\sqrt{x}}$ where $y(1) = 0$

3. $(2y - 3x)\,dx + x\,dy = 0$ where $y(1) = 1$

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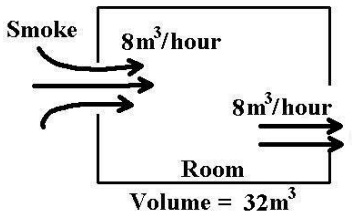
4. *(20 points)* Smoke is blowing through a broken window into a room. The air coming into the room is coming in at 8 cubic meters/hour with a concentration of smoke particles of 100 milligrams/cubic meter. At the same time, an exhaust fan on the other side of the room is removing air at the rate of 8 cubic meters/hour.
The volume of the room is 32 cubic meters.



Let $y(t)$ be the mass (in milligrams) of smoke particles in the room at time t hours.

a) Using the $\begin{pmatrix}\text{rate}\\ \text{in}\end{pmatrix} - \begin{pmatrix}\text{rate}\\ \text{out}\end{pmatrix}$ principle, write a differential equation that correctly determines y as a function of t.

b) Solve the differential equation in part (a). Assume that $y(0) = 0$

Problems 5 - 8. *(20 points)* The multiple choice questions are given on the reverse side of this page.

For each of the problems below, simply circle the correct choice directly on this page.

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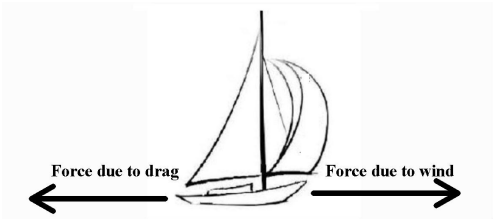
5. A 200 kilogram sailboat is floating motionless in the water. Suddenly, a wind with a constant force of 50 newtons begins to push the boat forward. The drag force, due to water resistance, is given by $F_R = 100v$ and acts in the opposite direction of the motion of the boat.



Let $v(t)$ be the velocity of the sailboat after t seconds. Use fact that (mass)(acceleration) = Net Force, write an appropriate differential equation that determines $v(t)$

a) $\frac{dv}{dt} = \frac{1}{4} - \frac{1}{2}v$ **b)** $\frac{dv}{dt} = \frac{1}{2} - \frac{1}{4}v$

c) $\frac{dv}{dt} = \frac{1}{4} + \frac{1}{2}v$ **d)** $\frac{dv}{dt} = \frac{1}{2} + \frac{1}{4}v$

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6. A differential equation is called an *exact differential equation* if it can be written in the form $dF = 0$. Only one of the following equations is exact. Which one?

a) $(x + 2y)dx + (3x + 4y)dy = 0$

b) $(3x + 2y)dx + (2x + 4y)dy = 0$

c) $(2x + 3y)\,dx + (4x + 2y)\,dy = 0$

d) $(2x + 3y)\,dx + (x + 2y)\,dy = 0$

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7. Which of the following would be an integrating factor μ for the differential equation $\frac{dy}{dx} + \left(2x + \frac{1}{x}\right) y = x^2$.

a) $\mu = \ln x$ **b)** $\mu = x\,e^{x^2}$ **c)** $\mu = x^2 + \ln x$ **d)** $\mu = x^2 e^{x^2}$

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8. The equation $\left(y^2 + 4xy\right) dx + \left(2xy + 2x^2\right) dy = 0$ can be written in the form $dF = 0$. Which of the following expressions is F ?

a) $F = 2x^2y^2 + x^2y + C$ **b)** $F = 2x^2 + y^2 + C$

c) $F = 2xy^2 + x^2y + C$ **d)** $F = xy^2 + 2x^2y + C$
